ECE 5283
Kalman Filtering
Homework 3

Fall 2005 Dr. Havlicek

Consider again the game show *Let's Make a Deal* described in problem (1) of Homework One. Recall that the two possible strategies for the player are:

1. **STICK:** that is, keep the door you initially picked.
2. **SWITCH:** that is, give up the door you initially picked and switch to the door that Monty *did not show.*

Write a computer program to simulate the game and empirically evaluate the probability of winning the prize for a player who adheres always to the STICK strategy and for a player who adheres always to the SWITCH strategy.

Your program will need to use three independent uniform $[0, 1]$ pseudo-random number generators. To obtain three independent pseudo-random sequences, you need to use three independent seeds for the generator. The details of this depend on exactly which generator you use, so you will need to read the documentation for your generator!

For each trial of the game, use the next number from the first pseudo-random sequence to decide which door holds the prize, e.g., if the number is between zero and 1/3, the prize is behind door one; if between 1/3 and 2/3, behind door two, and if between 2/3 and one, behind door three.

Use the next number from the second pseudo-random sequence to decide which door you initially pick, e.g., if the number is between zero and 1/3, you pick door one; if between 1/3 and 2/3, you pick door two, and if between 2/3 and one, you pick door three.

If there is a goat behind the door you initially picked, then it is clear which door Monty shows: he shows the one that does not have the prize, i.e., the one that you did not pick and that also holds a goat. However, if the prize is behind the door you initially picked, then Monty must choose one of the other two doors to show you. Since they both have goats, his choice is not clear. You can use the next number from the third pseudo-random sequence to make this decision. (NOTE: since in this case Monty's choice of which "goat door" to show you has *no effect* on the win/loose outcome of the game, it isn't really necessary to simulate this with a pseudo-random sequence; you will get the same result if Monty just always shows the "goat door" with the lower number. It is your choice whether or not you want to actually implement the simulation for this part of the game).

For each trial of the game that your computer program simulates, figure out if a player adhering to the STICK strategy wins the prize and also if a player adhering to the SWITCH strategy wins. Have your program save running counts of the number of trials, the number of times that the STICK strategy wins, and the number of times that the SWITCH strategy wins.

Approximate the probability $P(\text{win} \mid \text{STICK})$ with the ratio

$$P(\text{win} \mid \text{STICK}) \approx \frac{\textit{Number of Times STICK wins}}{\textit{Number of Trials}}.$$

Likewise, approximate the probability $P(\text{win} \mid \text{SWITCH})$ with the ratio

$$P(\text{win} \mid \text{SWITCH}) \approx \frac{\textit{Number of Times SWITCH wins}}{\textit{Number of Trials}}.$$

Plot your empirical estimates of $P(\text{win} \mid \text{STICK})$ and $P(\text{win} \mid \text{SWITCH})$ as a function of the number of trials. As the number of trials becomes large, we hope and expect to see these estimates converge to their theoretical values of 1/3 and 2/3, respectively.

DUE: FRIDAY 9/30/05, 5:00 PM (can be turned into the ECE office, obtain a timestamp from the receptionist and place in the instructor's mailbox)

NOTE: Homework two is postponed until 10/4/05.