ECE 2713
Test 1

Thursday, March 25 – Monday, March 29, 2021

Spring 2021 Name: SOLUTION

Dr. Havlicek Student Num: _____

Directions: This test is closed book and closed notes. Calculators are allowed. You may also use the formula sheet provided with the test All work must be your own. You may work the test on this test paper or you may use your own blank paper. Upload a scan or photograph of your test paper to the course Canvas page no later than noon on Monday, March 29, 2021.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) _____
2. (25) _____
3. (25) _____
4. (25) _____

_____

TOTAL (100):

_____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: _____ Date: _____

1. **25 pts.** Consider the discrete-time signal

$$x[n] = 5\cos\left(\frac{12\pi}{62}n\right). \qquad \omega_0 = \frac{12\pi}{62}$$

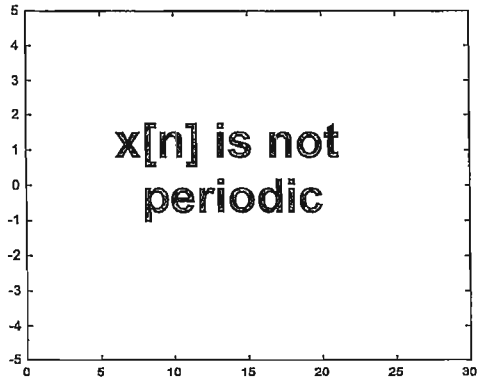
(a) **13 pts.** Is $x[n]$ periodic? If you say *no*, then explain why not. If you say *yes*, then find the fundamental period.

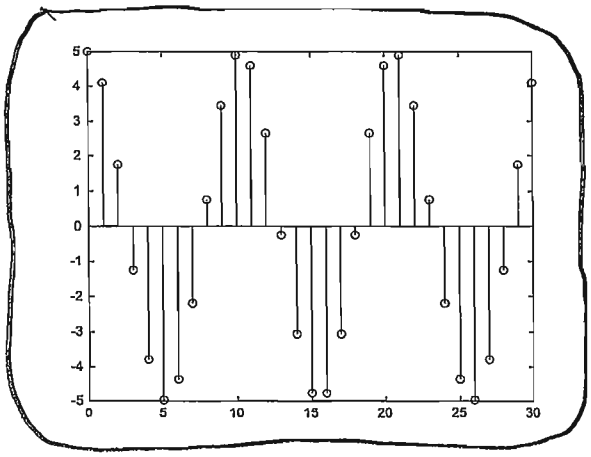
$\frac{\omega_0}{2\pi} = \frac{12\pi}{62} \cdot \frac{1}{2\pi} = \frac{12}{2\cdot 62} = \frac{6}{62} = \frac{3}{31} \in \mathbb{Q} \Rightarrow$ Therefore, $x[n]$ is periodic.

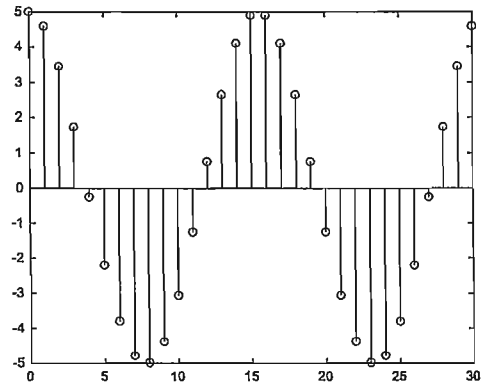
Now, $\frac{\omega_0}{2\pi} = \frac{3}{31} = \frac{m}{N}$ Fundamental period $= N = 31$

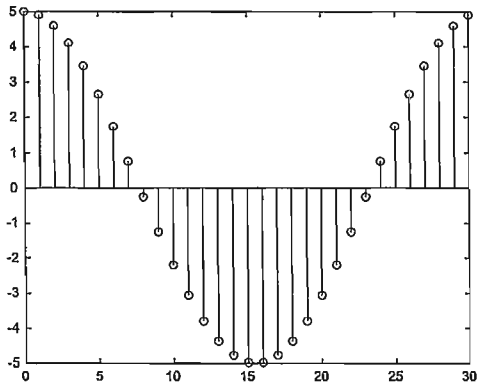
$m = 3$: graph goes around 3 times for one period

(b) **12 pts.** Circle the graph that shows one period of $x[n]$.









2. **25 pts.** A continuous-time signal $x(t)$ is given by

$$x(t) = 2\cos\left(120\pi t + \frac{5\pi}{3}\right) + 3\cos\left(120\pi t + \frac{17\pi}{25}\right).$$

Use phasor addition to express $x(t)$ in the form

$$x(t) = A\cos\left(120\pi t + \phi\right).$$

Hint: make sure your calculator is set for radians, not degrees!

Phasor for $2\cos(120\pi t + \frac{5\pi}{3})$: $X_1 = 2e^{j5\pi/3}$

Phasor for $3\cos(120\pi t + \frac{17\pi}{25})$: $X_2 = 3e^{j17\pi/25}$

Phasor for $x(t)$: $X = X_1 + X_2 = 2e^{j5\pi/3} + 3e^{j17\pi/25}$

$$= \left[2\cos\left(\tfrac{5\pi}{3}\right) + j2\sin\left(\tfrac{5\pi}{3}\right)\right] + \left[3\cos\left(\tfrac{17\pi}{25}\right) + j3\sin\left(\tfrac{17\pi}{25}\right)\right]$$

$$= \left[2\cos\left(\tfrac{5\pi}{3}\right) + 3\cos\left(\tfrac{17\pi}{25}\right)\right] + j\left[2\sin\left(\tfrac{5\pi}{3}\right) + 3\sin\left(\tfrac{17\pi}{25}\right)\right]$$

$$= [1 - 1.60748] + j[-1.73205 + 2.53298]$$

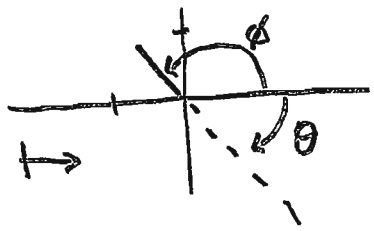
$$= -0.607480 + j0.800933$$

$$A = |X| = \sqrt{(-0.60748)^2 + (0.800933)^2} = \sqrt{1.01053} = 1.00525$$

$$\phi = \arg X = \arctan\left(\frac{0.800933}{-0.60748}\right)$$

→ since ϕ is a 2nd quadrant angle, "atan" will give the wrong answer. atan will give θ ↦

→ Then $\phi = \theta + \pi$



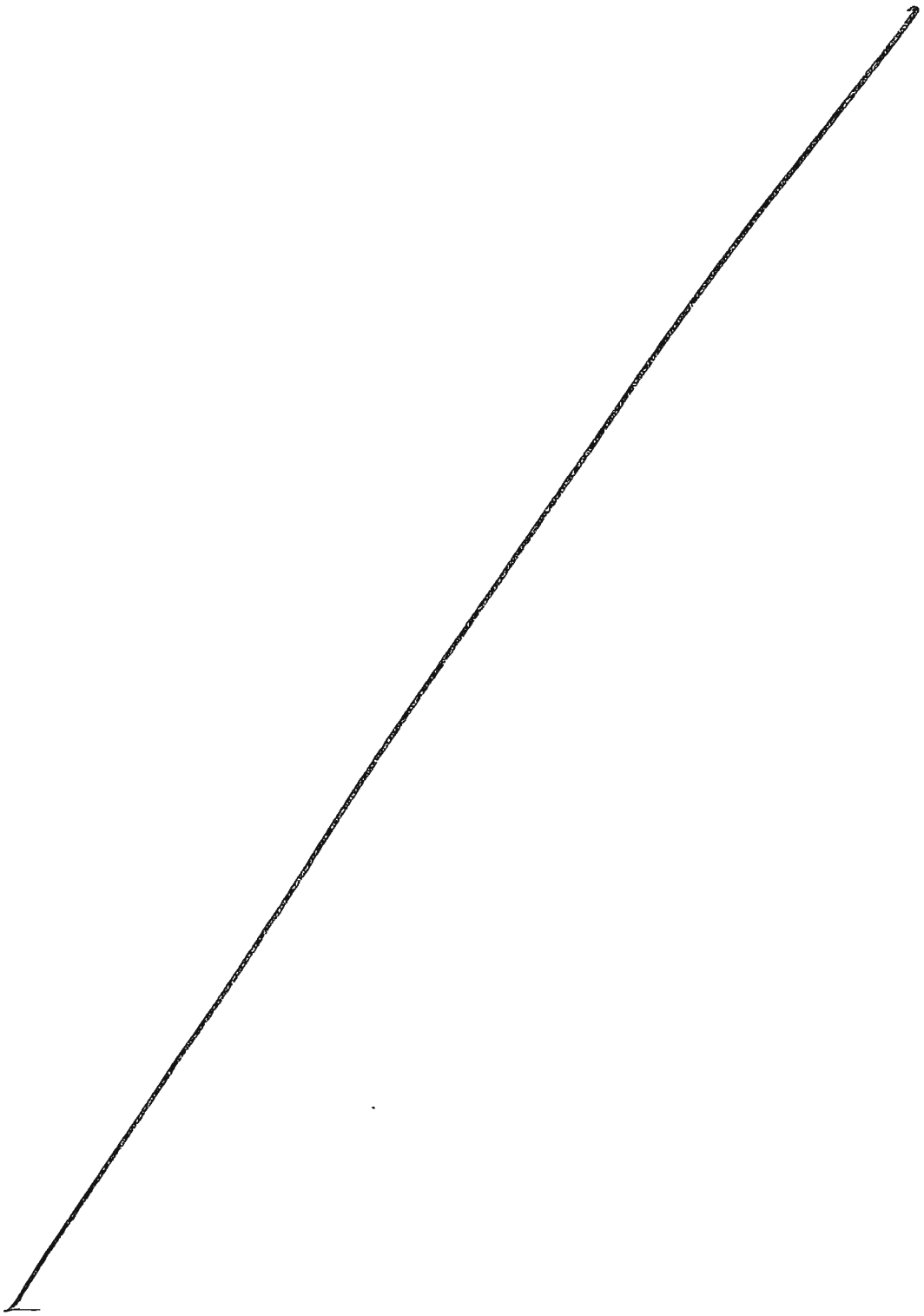
$$\theta = \text{atan}\left(\frac{0.800933}{-0.60748}\right) = -0.921899$$

$$\phi = \theta + \pi = \pi - 0.921899 = 2.21969$$

$$X = Ae^{j\phi} = 1.00525\, e^{j2.21969}$$

$$\boxed{x(t) = 1.00525\cos(120\pi t + 2.21969)}$$

More Workspace for Problem 2...



3. **25 pts.** A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-2].$$

The system input is given by

$$x[n] = \delta[n+1] - 2\delta[n] + \delta[n-1].$$

Find the system output $y[n]$.

$y[n] = x[n] * h[n] = x[n] * (3\delta[n] + 2\delta[n-1] + \delta[n-2])$

$= 3x[n] * \delta[n] + 2x[n] * \delta[n-1] + x[n] * \delta[n-2]$

$= 3x[n] + 2x[n-1] + x[n-2]$

$= 3\delta[n+1] - 6\delta[n] + 3\delta[n-1]$
$+ 2\delta[n] - 4\delta[n-1] + 2\delta[n-2]$
$+ \delta[n-1] - 2\delta[n-2] + \delta[n-3]$

$= 3\delta[n+1] - 4\delta[n] + 0\delta[n-1] + 0\delta[n-2] + \delta[n-3]$

$$\boxed{y[n] = 3\delta[n+1] - 4\delta[n] + \delta[n-3]}$$

OTHER WAY: $y[n] = x[n] * h[n] = (\delta[n+1] - 2\delta[n] + \delta[n-1]) * h[n]$

$= \delta[n+1] * h[n] - 2\delta[n] * h[n] + \delta[n-1] * h[n]$

$= h[n+1] - 2h[n] + h[n-1]$

$= 3\delta[n+1] + 2\delta[n] + \delta[n-1]$
$- 6\delta[n] - 4\delta[n-1] - 2\delta[n-2]$
$+ 3\delta[n-1] + 2\delta[n-2] + \delta[n-3]$

$= 3\delta[n+1] - 4\delta[n] + 0\delta[n-1] + 0\delta[n-2] + \delta[n-3]$

$$\boxed{y[n] = 3\delta[n+1] - 4\delta[n] + \delta[n-3]}$$

4. **25 pts.** A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

The system input is given by

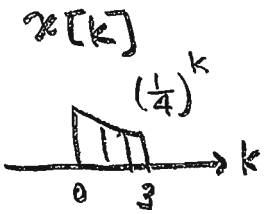
$$x[n] = \left(\frac{1}{4}\right)^n (u[n] - u[n-4]) = \begin{cases} \left(\frac{1}{4}\right)^n, & 0 \le n \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

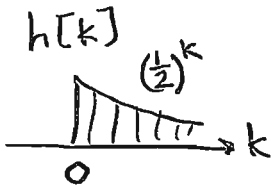
Find the system output $y[n]$.

Hint: you should follow these steps:

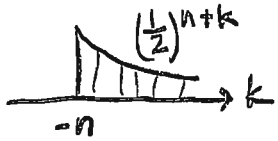
1. write $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$.
2. Use the definition of $x[n]$ given above to draw the graph of $x[k]$.
3. Use the definition of $h[n]$ given above to draw the graph of $h[k]$.
4. Slide the graph of $h[k]$ to the right by $-n$ to get the graph of $h[k-(-n)] = h[n+k]$.
5. Flip the graph of $h[n+k]$ with respect to k to get the graph of $h[-k-(-n)] = h[n-k]$.
6. For the n's in each region, multiply the graph of $x[k]$ with the graph of $h[n-k]$ and add up the product graph to get $y[n]$.

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$



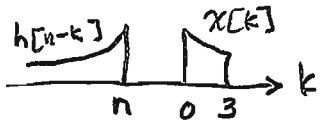


$h[k-(-n)] = h[n+k]$



$h[-k-(-n)] = h[n-k]$

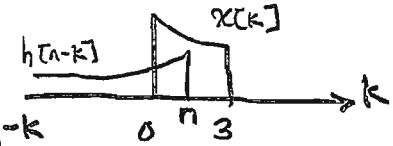
Case I) $n < 0$



product graph is all zeros

$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$

Case II) $n \ge 0$ and $n < 3$ $\therefore 0 \le n < 3$



$$y[n] = \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^{n-k} = \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{-k}$$

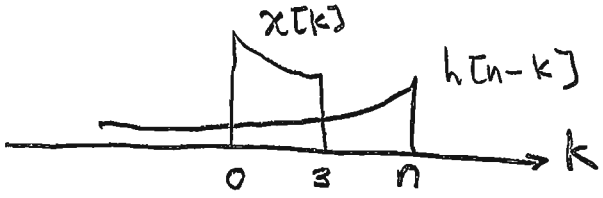
$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k 2^k = \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{2}{4}\right)^k = \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \frac{\left(\frac{1}{2}\right)^0 - \left(\frac{1}{2}\right)^{n+1}}{1 - 1/2} = \left(\frac{1}{2}\right)^n \frac{1 - \frac{1}{2}\left(\frac{1}{2}\right)^n}{1/2} = \left(\frac{1}{2}\right)^n \left[2 - \left(\frac{1}{2}\right)^n\right]$$

$$= 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^n = 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{2} \cdot \frac{1}{2}\right)^n$$

$$= 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n$$

More Workspace for Problem 4...



Case III) $n \geq 3$

$$y[n] = \sum_{k=0}^{3} \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^{n-k}$$

$$= \sum_{k=0}^{3} \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{-k} = \left(\frac{1}{2}\right)^n \sum_{k=0}^{3} \left(\frac{1}{4}\right)^k 2^k = \left(\frac{1}{2}\right)^n \sum_{k=0}^{3} \left(\frac{2}{4}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{3} \left(\frac{1}{2}\right)^k = \left(\frac{1}{2}\right)^n \frac{\left(\frac{1}{2}\right)^0 - \left(\frac{1}{2}\right)^4}{1 - 1/2} = \left(\frac{1}{2}\right)^n \frac{1 - \frac{1}{16}}{1/2}$$

$$= \left(\frac{1}{2}\right)^n \cdot 2\left(\frac{16}{16} - \frac{1}{16}\right) = 2 \cdot \frac{15}{16}\left(\frac{1}{2}\right)^n = \frac{15}{8}\left(\frac{1}{2}\right)^n$$

All Together:

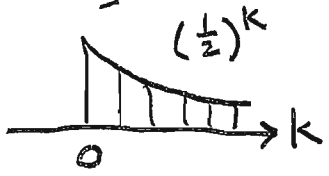
$$y[n] = \begin{cases} 0, & n < 0 \\ 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n, & 0 \leq n < 3 \\ \frac{15}{8}\left(\frac{1}{2}\right)^n, & n \geq 3 \end{cases}$$

OTHER WAY:

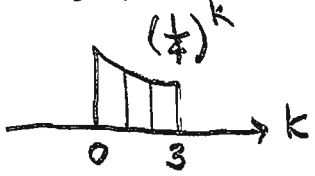
More Workspace for Problem 4...

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

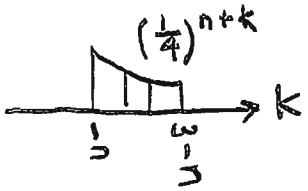
$h[k]$



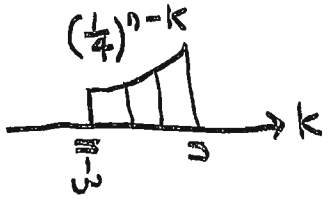
$x[k]$



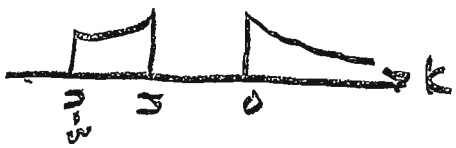
$x[k-(-n)] = x[n+k]$



$x[-k-(-n)] = x[n-k]$



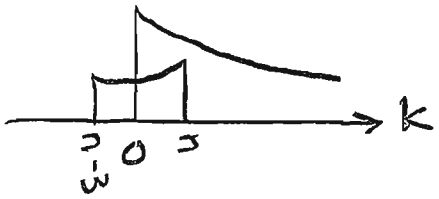
Case I) $n < 0$



$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$$

Case II) $n \geq 0$ and $n-3 < 0$

$0 \leq n < 3$



$$y[n] = \sum_{k=0}^{n} (\tfrac{1}{2})^k (\tfrac{1}{4})^{n-k}$$

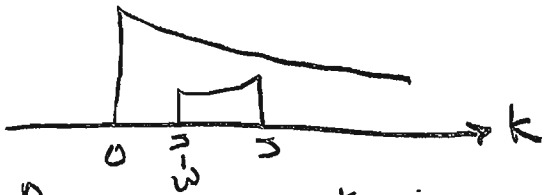
$$= \sum_{k=0}^{n} (\tfrac{1}{2})^k (\tfrac{1}{4})^n (\tfrac{1}{4})^{-k} = (\tfrac{1}{4})^n \sum_{k=0}^{n} (\tfrac{1}{2})^k 4^k$$

$$= (\tfrac{1}{4})^n \sum_{k=0}^{n} (\tfrac{4}{2})^k = (\tfrac{1}{4})^n \sum_{k=0}^{n} 2^k = (\tfrac{1}{4})^n \frac{2^0 - 2^{n+1}}{1-2}$$

$$= (\tfrac{1}{4})^n \frac{1 - 2\cdot 2^n}{-1} = (\tfrac{1}{4})^n [2\cdot 2^n - 1]$$

$$= 2(\tfrac{1}{4})^n 2^n - (\tfrac{1}{4})^n = 2(\tfrac{2}{4})^n - (\tfrac{1}{4})^n = 2(\tfrac{1}{2})^n - (\tfrac{1}{4})^n$$

Case III) $n \geq 3$



$$y[n] = \sum_{k=n-3}^{n} (\tfrac{1}{2})^k (\tfrac{1}{4})^{n-k} = \sum_{k=n-3}^{n} (\tfrac{1}{2})^k (\tfrac{1}{4})^n (\tfrac{1}{4})^{-k}$$

$$= (\tfrac{1}{4})^n \sum_{k=n-3}^{n} (\tfrac{1}{2})^k 4^k = (\tfrac{1}{4})^n \sum_{k=n-3}^{n} (\tfrac{4}{2})^k = (\tfrac{1}{4})^n \sum_{k=n-3}^{n} 2^k$$

$$= (\tfrac{1}{4})^n \frac{2^{n-3} - 2^{n+1}}{1-2} = (\tfrac{1}{4})^n \frac{2^n 2^{-3} - 2\cdot 2^n}{-1} = (\tfrac{1}{4})^n [2\cdot 2^n - \tfrac{1}{8} 2^n]$$

$$= (\tfrac{1}{4})^n \cdot 2^n [2 - \tfrac{1}{8}] = (\tfrac{2}{4})^n [\tfrac{16}{8} - \tfrac{1}{8}] = \frac{15}{8}(\tfrac{1}{2})^n$$

All Together:

$$y[n] = \begin{cases} 0 & , \ n < 0 \\ 2(\frac{1}{2})^n - (\frac{1}{4})^n & , \ 0 \leq n < 3 \\ \frac{15}{8}(\frac{1}{2})^n & , \ n \geq 3 \end{cases}$$