

ECE 2713

Test 1

Thursday, March 24, 2022

12:00 PM - 1:15 PM

Spring 2022

Dr. Havlicek

Name: SOLUTION

Student Num: _____

Directions: This test is closed book and closed notes. Calculators are allowed. You may also use the formula sheet provided with the test. All work must be your own. You have 75 minutes to complete the test.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) _____

2. (25) _____

3. (25) _____

4. (25) _____

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: _____

Date: _____

1. 25 pts. Consider the discrete-time signal

$$x[n] = 4 \cos\left(\frac{3}{29}n\right).$$

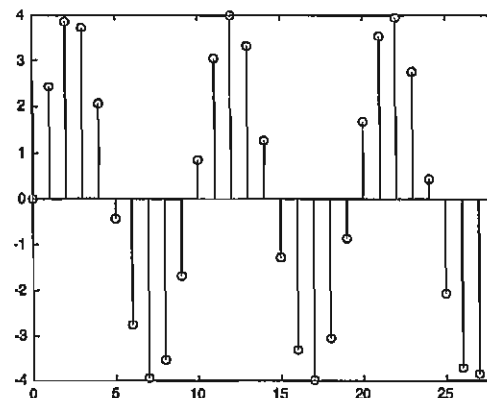
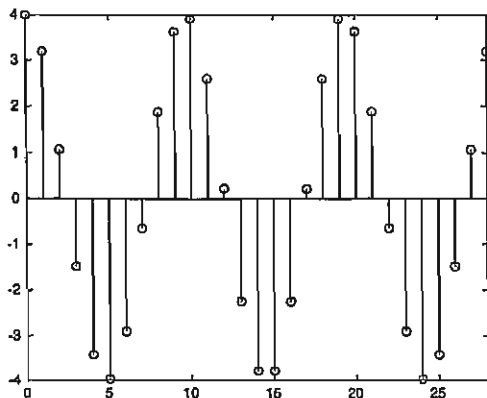
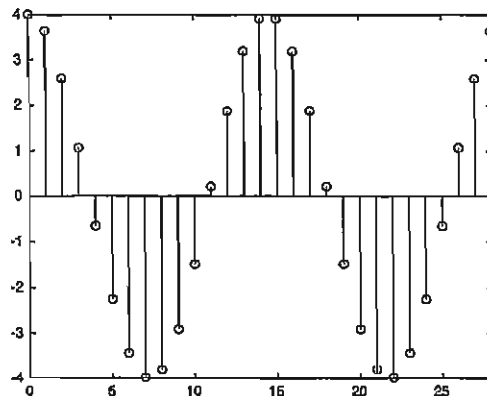
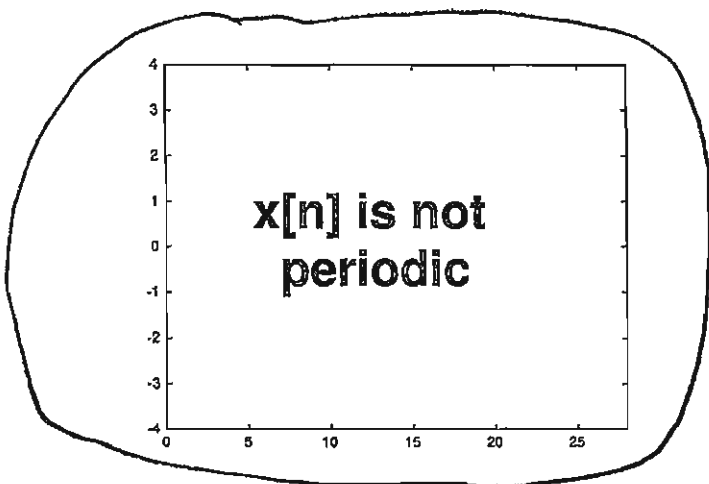
(a) 13 pts. Is $x[n]$ periodic? If you say *no*, then explain why not. If you say *yes*, then find the fundamental period.

$$\omega_0 = \frac{3}{29}$$

$$\frac{\omega_0}{2\pi} = \frac{3}{29(2\pi)} = \frac{3}{58\pi} \notin \mathbb{Q}$$

$x[n]$ is NOT periodic because $\frac{\omega_0}{2\pi}$ is irrational

(b) 12 pts. Circle the graph that shows one period of $x[n]$:



2. 25 pts. A continuous-time signal $x(t)$ is given by

$$x(t) = \cos\left(1,000\pi t + \frac{\pi}{4}\right) + 4\cos\left(1,000\pi t + \frac{9\pi}{8}\right).$$

Use phasor addition to express $x(t)$ in the form

$$x(t) = A \cos(1,000\pi t + \phi).$$

Hint: make sure your calculator is set for radians, not degrees!

Phasor for $\cos(1,000\pi t + \frac{\pi}{4})$: $X_1 = e^{j\pi/4}$

Phasor for $4\cos(1,000\pi t + \frac{9\pi}{8})$: $X_2 = 4e^{j9\pi/8}$

Phasor for $x(t)$: $X = X_1 + X_2$

$$= e^{j\pi/4} + 4e^{j9\pi/8}$$

$$= [\cos(\frac{\pi}{4}) + j\sin(\frac{\pi}{4})] + [4\cos(\frac{9\pi}{8}) + j4\sin(\frac{9\pi}{8})]$$

$$= [\cos(\frac{\pi}{4}) + 4\cos(\frac{9\pi}{8})] + j[\sin(\frac{\pi}{4}) + 4\sin(\frac{9\pi}{8})]$$

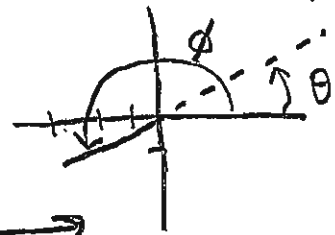
$$= [0.707107 - 3.69552] + j[0.707107 - 1.53073]$$

$$= -2.98841 - j0.823627$$

$$A = |X| = \sqrt{(-2.98841)^2 + (-0.823627)^2} = \sqrt{9.60896} = 3.09983$$

$$\phi = \arg X = \arctan\left[\frac{-0.823627}{-2.98841}\right]$$

→ since ϕ is a 3rd quadrant angle, \arctan will give the wrong answer; atan will give θ



→ Then $\phi = \theta \pm \pi$

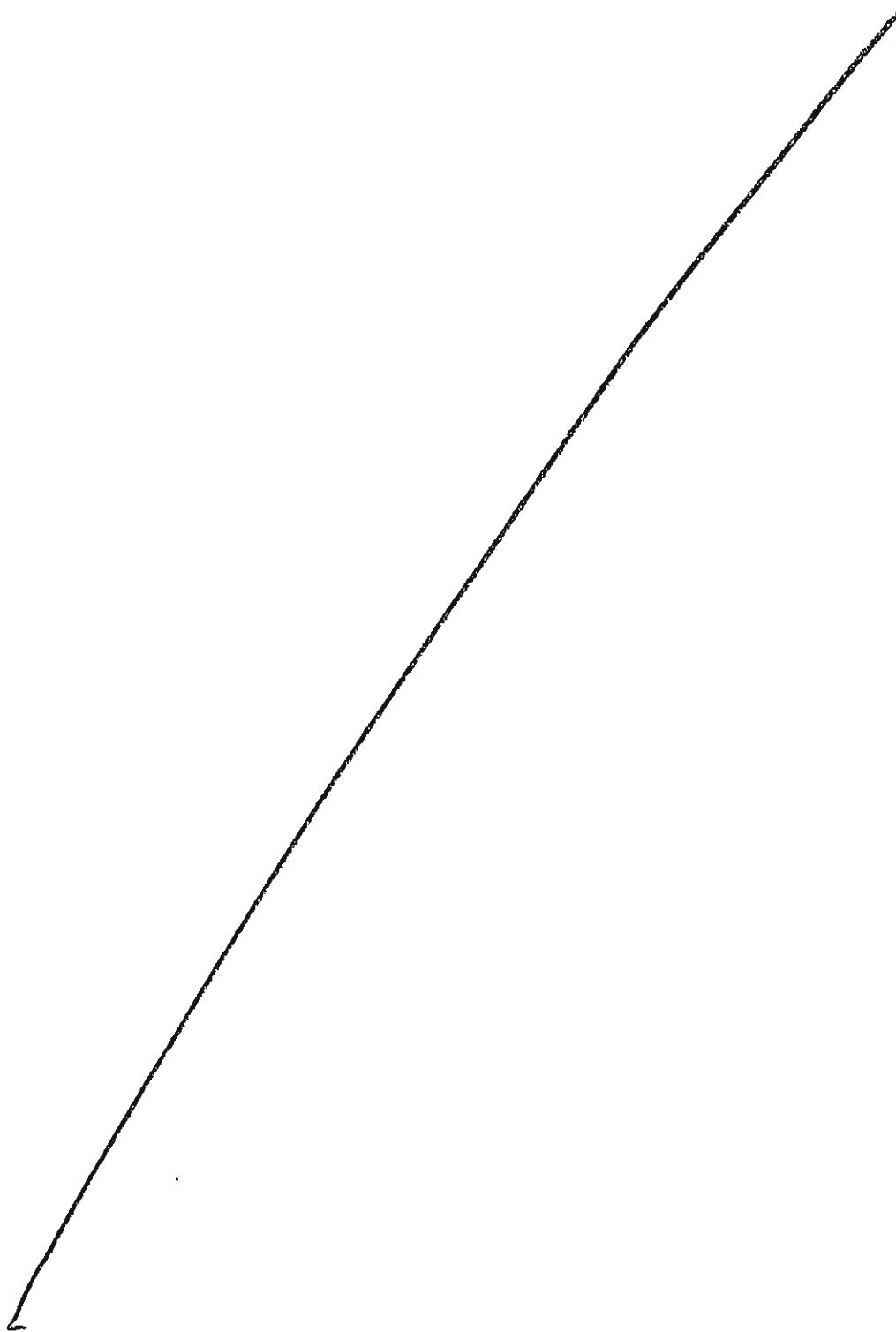
$$\theta = \text{atan}\left[\frac{-0.823627}{-2.98841}\right] = 0.268930$$

$$\phi = \theta + \pi = 0.268930 + \pi = 3.41052 \quad (\text{or } \theta - \pi = -2.87266)$$

$$X = Ae^{j\phi} = 3.09983 e^{j3.41052}$$

$$x(t) = 3.09983 \cos(1,000\pi t + 3.41052)$$

More Workspace for Problem 2...



3. 25 pts. A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2].$$

The system input is given by

$$x[n] = 3\delta[n+2] + 2\delta[n+1] - 3\delta[n].$$

Find the system output $y[n]$.

$$\begin{aligned} y[n] &= x[n] * h[n] = x[n] * (\delta[n] - 2\delta[n-1] + \delta[n-2]) \\ &= x[n] * \delta[n] - 2x[n] * \delta[n-1] + x[n] * \delta[n-2] \\ &= x[n] - 2x[n-1] + x[n-2] \\ &= 3\delta[n+2] + 2\delta[n+1] - 3\delta[n] \\ &\quad - 6\delta[n+1] - 4\delta[n] + 6\delta[n-1] \\ &\quad + 3\delta[n] + 2\delta[n-1] - 3\delta[n-2] \\ &= 3\delta[n+2] - 4\delta[n+1] - 4\delta[n] + 8\delta[n-1] - 3\delta[n-2] \end{aligned}$$

$$y[n] = 3\delta[n+2] - 4\delta[n+1] - 4\delta[n] + 8\delta[n-1] - 3\delta[n-2]$$

OTHER WAY: $y[n] = x[n] * h[n] = (3\delta[n+2] + 2\delta[n+1] - 3\delta[n]) * h[n]$

$$\begin{aligned} &= 3\delta[n+2] * h[n] + 2\delta[n+1] * h[n] - 3\delta[n] * h[n] \\ &= 3h[n+2] + 2h[n+1] - 3h[n] \\ &= 3\delta[n+2] - 6\delta[n+1] + 3\delta[n] \\ &\quad + 2\delta[n+1] - 4\delta[n] + 2\delta[n-1] \\ &\quad - 3\delta[n] + 6\delta[n-1] - 3\delta[n-2] \\ &= 3\delta[n+2] - 4\delta[n+1] - 4\delta[n] + 8\delta[n-1] - 3\delta[n-2] \end{aligned}$$

$$y[n] = 3\delta[n+2] - 4\delta[n+1] - 4\delta[n] + 8\delta[n-1] - 3\delta[n-2]$$

4. 25 pts. A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \left(\frac{1}{3}\right)^n u[n].$$

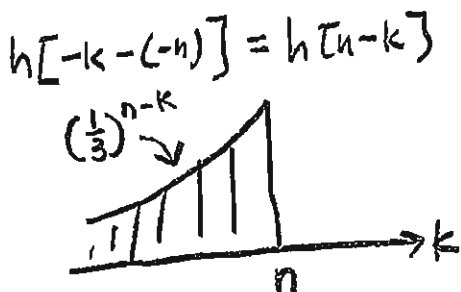
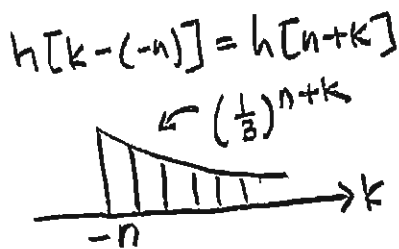
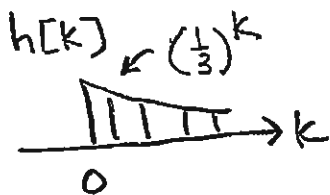
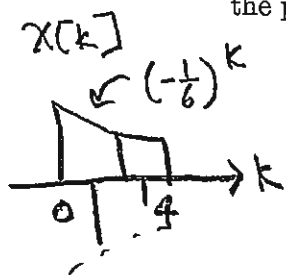
The system input is given by

$$x[n] = \left(-\frac{1}{6}\right)^n (u[n] - u[n-5]) = \begin{cases} \left(-\frac{1}{6}\right)^n, & 0 \leq n \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

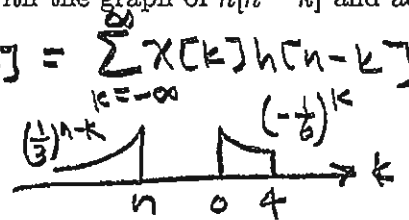
Find the system output $y[n]$.

Hint: you should follow these steps:

1. write $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$.
2. Use the definition of $x[n]$ given above to draw the graph of $x[k]$.
3. Use the definition of $h[n]$ given above to draw the graph of $h[k]$.
4. Slide the graph of $h[k]$ to the right by $-n$ to get the graph of $h[k - (-n)] = h[n+k]$.
5. Flip the graph of $h[n+k]$ with respect to k to get the graph of $h[-k - (-n)] = h[n-k]$.
6. For the n 's in each region, multiply the graph of $x[k]$ with the graph of $h[n-k]$ and add up the product graph to get $y[n]$.



case I) $n < 0$
 $y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$



case II) $n \geq 0$ and $n < 4$: $0 \leq n < 4$

$$y[n] = \sum_{k=0}^n \left(-\frac{1}{6}\right)^k \left(\frac{1}{3}\right)^{n-k}$$

$$= \sum_{k=0}^n \left(-\frac{1}{6}\right)^k \left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right)^{-k}$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^n \left(-\frac{1}{6}\right)^k 3^k = \left(\frac{1}{3}\right)^n \sum_{k=0}^n \left(-\frac{1}{2}\right)^k = \left(\frac{1}{3}\right)^n \frac{\left(-\frac{1}{2}\right)^0 - \left(-\frac{1}{2}\right)^{n+1}}{1 - \left(-\frac{1}{2}\right)}$$

$$= \left(\frac{1}{3}\right)^n \frac{1 - \left(-\frac{1}{2}\right)^{n+1}}{1 + \frac{1}{2}} = \left(\frac{1}{3}\right)^n \frac{1 + \frac{1}{2}\left(-\frac{1}{2}\right)^n}{\frac{3}{2}}$$

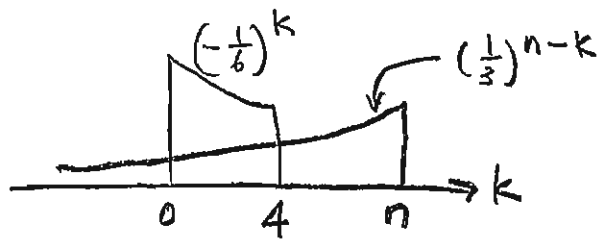
$$= \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^n \left[1 + \frac{1}{2}\left(-\frac{1}{2}\right)^n\right] = \frac{2}{3} \left(\frac{1}{3}\right)^n + \frac{1}{3} \left(\frac{1}{3}\right)^n \left(-\frac{1}{2}\right)^n$$

$$= \frac{2}{3} \left(\frac{1}{3}\right)^n + \frac{1}{3} \left(-\frac{1}{6}\right)^n$$



More Workspace for Problem 4...

case III) $n \geq 4$



$$\begin{aligned}
 y[n] &= \sum_{k=0}^4 \left(-\frac{1}{6}\right)^k \left(\frac{1}{3}\right)^{n-k} = \sum_{k=0}^4 \left(-\frac{1}{6}\right)^k \left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right)^{-k} = \left(\frac{1}{3}\right)^n \sum_{k=0}^4 \left(-\frac{1}{6}\right)^k 3^k \\
 &= \left(\frac{1}{3}\right)^n \sum_{k=0}^4 \left(-\frac{1}{2}\right)^k = \left(\frac{1}{3}\right)^n \frac{\left(-\frac{1}{2}\right)^0 - \left(-\frac{1}{2}\right)^5}{1 - \left(-\frac{1}{2}\right)} = \left(\frac{1}{3}\right)^n \frac{1 - \left(-\frac{1}{32}\right)}{1 + \frac{1}{2}} \\
 &= \left(\frac{1}{3}\right)^n \frac{1 + \frac{1}{32}}{\frac{3}{2}} = \frac{2}{3} \left(\frac{1}{3}\right)^n \left[\frac{32}{32} + \frac{1}{32} \right] + \frac{2}{3} \left(\frac{1}{3}\right)^n \frac{33}{32} \\
 &= \frac{11}{16} \left(\frac{1}{3}\right)^n
 \end{aligned}$$

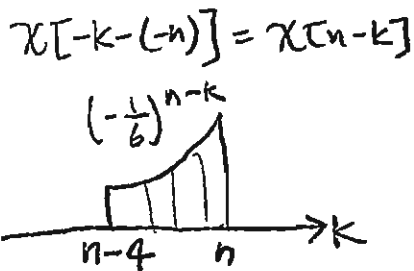
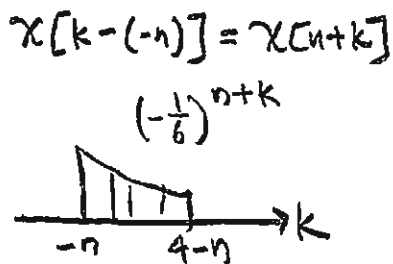
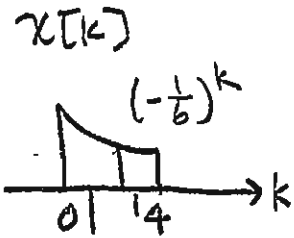
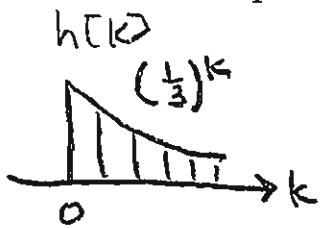
All Together:

$$y[n] = \begin{cases} 0 & , n < 0 \\ \frac{2}{3} \left(\frac{1}{3}\right)^n + \frac{1}{3} \left(-\frac{1}{6}\right)^n & , 0 \leq n < 4 \\ \frac{11}{16} \left(\frac{1}{3}\right)^n & , n \geq 4 \end{cases}$$

OTHER WAY:

More Workspace for Problem 4...

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$



Case I) $n < 0$

$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$$



Case II) $n > 0$ and $n-4 < 0$: $n > 0$ and $n < 4$: $0 \leq n < 4$

$$y[n] = \sum_{k=0}^n \left(\frac{1}{3}\right)^k \left(-\frac{1}{6}\right)^{n-k}$$

$$= \sum_{k=0}^n \left(\frac{1}{3}\right)^k \left(-\frac{1}{6}\right)^n \left(-\frac{1}{6}\right)^{-k}$$

$$= \left(-\frac{1}{6}\right)^n \sum_{k=0}^n \left(\frac{1}{3}\right)^k \left(-6\right)^k = \left(-\frac{1}{6}\right)^n \sum_{k=0}^n (-2)^k$$

$$= \left(-\frac{1}{6}\right)^n \frac{(-2)^0 - (-2)^{n+1}}{1 - (-2)} = \left(-\frac{1}{6}\right)^n \frac{1 - (-2)(-2)^n}{1 + 2}$$

$$= \left(-\frac{1}{6}\right)^n \frac{1 + 2(-2)^n}{3} = \frac{1}{3} \left(-\frac{1}{6}\right)^n [1 + 2(-2)^n]$$

$$= \frac{1}{3} \left(-\frac{1}{6}\right)^n + \frac{2}{3} \left(-\frac{1}{6}\right)^n (-2)^n = \frac{1}{3} \left(-\frac{1}{6}\right)^n + \frac{2}{3} \left(\frac{1}{3}\right)^n$$

$$= \frac{2}{3} \left(\frac{1}{3}\right)^n + \frac{1}{3} \left(-\frac{1}{6}\right)^n$$

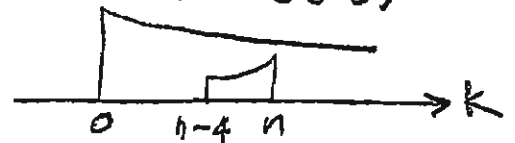
Case III) $n > 4$

$$y[n] = \sum_{k=n-4}^n \left(\frac{1}{3}\right)^k \left(-\frac{1}{6}\right)^{n-k} = \sum_{k=n-4}^n \left(\frac{1}{3}\right)^k \left(-\frac{1}{6}\right)^n \left(-\frac{1}{6}\right)^{-k}$$

$$= \left(-\frac{1}{6}\right)^n \sum_{k=n-4}^n \left(\frac{1}{3}\right)^k \left(-6\right)^k = \left(-\frac{1}{6}\right)^n \sum_{k=n-4}^n (-2)^k = \left(-\frac{1}{6}\right)^n \frac{(-2)^{n-4} - (-2)^{n+1}}{1 - (-2)}$$

$$= \left(-\frac{1}{6}\right)^n \frac{(-2)^{-4} (-2)^n - (-2)(-2)^n}{1 + 2} = \left(-\frac{1}{6}\right)^n (-2)^n \frac{(-\frac{1}{2})^4 + 2}{3}$$

$$= \left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right) \left[\frac{1}{16} + 2\right] = \left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right) \left[\frac{33}{16}\right] = \frac{11}{16} \left(\frac{1}{3}\right)^n$$



All Together:

$$y[n] = \begin{cases} 0 & , n < 0 \\ \frac{2}{3} \left(\frac{1}{3}\right)^n + \frac{1}{3} \left(-\frac{1}{6}\right)^n & , 0 \leq n < 4 \\ \frac{11}{16} \left(\frac{1}{3}\right)^n & , n > 4 \end{cases}$$