5. **25/20 pts.** Use the window design method with an appropriate fixed window from Table 10.2 (p. 540 of the text) to design a causal lowpass FIR digital filter that meets the following specifications:

Passband Edge Freq.	$\omega_p = 0.5\pi$ rad/sample
Stopband Edge Freq.	$\omega_s = 0.7\pi$ rad/sample
Max. Passband Ripple	$\delta_p = 0.010$
Max. Stopband Ripple	$\delta_s = 0.010$

Give the filter impulse response $h[n]$.

$$\alpha_s = -20\log_{10}\delta_s = -20\log_{10} 0.01 = 40 \text{ dB}$$

Table 10.2: Hann, Hamming, and Blackman can meet the stopband spec.

$$\Delta\omega = \omega_s - \omega_p = 0.7\pi - 0.5\pi = 0.2\pi$$

Hann: $M = \left\lceil \frac{3.11\pi}{\Delta\omega} \right\rceil = \left\lceil \frac{3.11}{0.2} \right\rceil = \lceil 15.55 \rceil = 16$

Hamming: $M = \left\lceil \frac{3.32\pi}{\Delta\omega} \right\rceil = \left\lceil \frac{3.32}{0.2} \right\rceil = \lceil 16.6 \rceil = 17$

Blackman: $M = \left\lceil \frac{5.56\pi}{\Delta\omega} \right\rceil = \left\lceil \frac{5.56}{0.2} \right\rceil = \lceil 27.8 \rceil = 28$

Hann meets the spec with the lowest order. $\Rightarrow$ use Hann.

$M = 16$, order $= N = 2M = 32$, length $= 2M+1 = 33$

$$\omega_c = \frac{\omega_p + \omega_s}{2} = \frac{0.5\pi + 0.7\pi}{2} = \frac{1.2\pi}{2} = 0.6\pi$$

(10.14): $h_{LP}[n] = \frac{\sin \omega_c n}{\pi n} = \frac{\sin(0.6\pi n)}{\pi n}$

(10.33): $w[n] = \frac{1}{2}\left[1 + \cos\left(\frac{\pi}{16} n\right)\right], \quad -16 \le n \le 16$

$$w[n]h_{LP}[n] = \frac{1}{2}\left[1 + \cos\left(\frac{\pi}{16} n\right)\right] \frac{\sin(0.6\pi n)}{\pi n}, \quad -16 \le n \le 16$$

Shift to make causal:

$$\boxed{h[n] = \frac{1}{2}\left[1 + \cos\frac{\pi(n-16)}{16}\right] \frac{\sin[0.6\pi(n-16)]}{\pi(n-16)}, \quad 0 \le n \le 32}$$

3. **25/20 pts.** Use the window design method with an appropriate fixed window from Table 10.2 (p. 540 of the text) to design a causal lowpass FIR digital filter that meets the following specifications:

Passband Edge Freq.	$\omega_p = 0.2\pi$ rad/sample
Stopband Edge Freq.	$\omega_s = 0.9\pi$ rad/sample
Max. Passband Ripple	$\delta_p = 0.005$
Max. Stopband Ripple	$\delta_s = 0.005$

Give the filter impulse response $h[n]$.

$$\alpha_s = -20\log_{10}\delta_s = -20\log_{10}0.005 = 46.0206 \text{ dB}$$

Table 10.2: Hamming and Blackman can meet the spec.

$$\Delta\omega = \omega_s - \omega_p = 0.9\pi - 0.2\pi = 0.7\pi$$

Hamming: $\Delta\omega = \frac{3.32\pi}{M}$; $M = \left\lceil \frac{3.32\pi}{\Delta\omega} \right\rceil = \left\lceil \frac{3.32\pi}{0.7\pi} \right\rceil = \lceil 4.74 \rceil = 5$

Blackman: $\Delta\omega = \frac{5.56\pi}{M}$: $M = \left\lceil \frac{5.56\pi}{\Delta\omega} \right\rceil = \left\lceil \frac{5.56\pi}{0.7\pi} \right\rceil = \lceil 7.94 \rceil = 8$

$\Rightarrow$ Hamming meets the spec with a lower order $\rightarrow$ Use Hamming.

$M = 5$. Order $= N = 2M = 10$. Length $= 2M+1 = 11$.

$$\omega_c = \frac{\omega_p + \omega_s}{2} = \frac{0.2\pi + 0.9\pi}{2} = \frac{1.1\pi}{2} = 0.55\pi$$

$$h_{LP}[n] = \frac{\sin \omega_c n}{\pi n} = \frac{\sin 0.55\pi n}{\pi n}$$

(10.34) ; $w[n] = 0.54 + 0.46\cos\left(\frac{\pi}{M} n\right) = 0.54 + 0.46\cos\left(\frac{\pi}{5} n\right)$ $\quad -5 \le n \le 5$

$$w[n]h_{LP}[n] = \left\{0.54 + 0.46\cos\left(\frac{\pi}{5}n\right)\right\} \frac{\sin(0.55\pi n)}{\pi n}, \quad -5 \le n \le 5$$

Shift to make causal:

$$\boxed{h[n] = \frac{\sin[0.55\pi(n-5)]}{\pi(n-5)} \left\{0.54 + 0.46\cos\left[\frac{\pi}{5}(n-5)\right]\right\} \quad 0 \le n \le 10}$$

4. **25/20 pts.** Use the window design method with an appropriate fixed window from Table 10.2 (p. 540 of the text) to design a causal lowpass FIR digital filter that meets the following design specification:

$$\omega_c = \frac{\omega_p + \omega_s}{2} = \frac{0.51\pi + 0.6\pi}{2}$$

$$\omega_c = 0.555\pi$$

Passband Edge Freq.	$\omega_p = 0.51\pi$ rad/sample
Stopband Edge Freq.	$\omega_s = 0.60\pi$ rad/sample
Max. Passband Ripple	$\delta_p = 0.001$
Max. Stopband Ripple	$\delta_s = 0.001$

From (9.4):

$$\alpha_s = -20\log_{10}\delta_s = -20\log_{10} 0.001$$

$$\alpha_s = 60 \text{ dB}.$$

Give the filter impulse response $h[n]$.

From Table 10.2, only the Blackman window can provide enough Stop band attenuation, $\Rightarrow$ Use Blackman.

Transition Bandwidth: $\Delta\omega = \omega_s - \omega_p = 0.6\pi - 0.51\pi = 0.09\pi$

Table 10.2: $\Delta\omega = 0.09\pi = \frac{5.56\pi}{M}$

$$M = \left\lceil \frac{5.56\pi}{0.09\pi} \right\rceil = \lceil 61.7778 \rceil = 62$$

Length $= 2M+1 = 125$

Order $= 2M = 124$

(10.14): $h_{LP}[n] = \frac{\sin \omega_c n}{\pi n} = \frac{\sin(0.555\pi n)}{\pi n}$

(10.35): $w[n] = 0.42 + 0.5\cos\left(\frac{\pi n}{M}\right) + 0.08\cos\left(\frac{2\pi n}{M}\right), \quad -62 \le n \le 62$

$= 0.42 + 0.5\cos\left(\frac{\pi}{62}n\right) + 0.08\cos\left(\frac{\pi}{31}n\right), \quad -62 \le n \le 62$

$w[n]h_{LP}[n] = \left[0.42 + 0.5\cos\left(\frac{\pi}{62}n\right) + 0.08\cos\left(\frac{\pi}{31}n\right)\right]\frac{\sin(0.555\pi n)}{\pi n}, \quad -62 \le n \le 62$

Shift to make causal:

$$\boxed{h[n] = \left\{0.42 + 0.5\cos\left[\frac{\pi}{62}(n-62)\right] + 0.08\cos\left[\frac{\pi}{31}(n-62)\right]\right\}\frac{\sin[0.555\pi(n-62)]}{\pi(n-62)}, \quad 0 \le n \le 124}$$

Here is another example problem for Windowed FIR filter design. I am going to follow the steps on page 6 of our ECE 2713 Final Exam formula sheet.

Use the window design method to design a causal lowpass FIR digital filter that meets the following specifications:

Passband Edge Freq.	$\omega_p = 0.3\pi$ rad/sample
Stopband Edge Freq.	$\omega_s = 0.35\pi$ rad/sample
Max. Passband Ripple	$\delta_p = 0.01$
Max. Stopband Ripple	$\delta_s = 0.01$

Give the filter impulse response $h[n]$.

① $\alpha_s = -20\log_{10}\delta_s = -20\log_{10} 0.01 = 40\text{ dB}$

② The Hann, Hamming, and Blackman windows can all provide more than 40 dB of stopband attenuation.
→ I know that Hann will give the smallest M because it is highest in the table, but let's go ahead and compute the M's for each window anyway.

③ $\Delta\omega = \omega_s - \omega_p = 0.35\pi - 0.3\pi = 0.05\pi$.

Rectangular: can not provide 40 dB of stopband attenuation

Hann: $\Delta\omega = 0.05\pi = \frac{3.11\pi}{M} \rightarrow M = \left\lceil \frac{3.11}{0.05} \right\rceil = \lceil 62.2 \rceil = 63$

Hamming: $\Delta\omega = 0.05\pi = \frac{3.32\pi}{M} \rightarrow M = \left\lceil \frac{3.32}{0.05} \right\rceil = \lceil 66.4 \rceil = 67$

Blackman: $\Delta\omega = 0.05\pi = \frac{5.56\pi}{M} \rightarrow M = \left\lceil \frac{5.56}{0.05} \right\rceil = \lceil 111.2 \rceil = 112$

→ Hann meets the stopband spec with the smallest M... which means lowest order.

(4) $\omega_c = \frac{\omega_p + \omega_s}{2} = \frac{0.3\pi + 0.35\pi}{2} = 0.325\pi$

(5) $h_{LP}[n] = \frac{\sin(0.325\pi n)}{\pi n}$

(6) $w[n]\, h_{LP}[n] = \frac{1}{2}\left[1 + \cos\left(\frac{\pi n}{63}\right)\right] \frac{\sin(0.325\pi n)}{\pi n}, \quad -63 \le n \le 63$

(7) Shift to make causal:

$$h[n] = \frac{1}{2}\left[1 + \cos\left(\frac{\pi(n-63)}{63}\right)\right] \frac{\sin[0.325\pi(n-63)]}{\pi(n-63)},$$

$$0 \le n \le 126$$