

Formulas from Trigonometry:

$$\sin^2 A + \cos^2 A = 1$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\sin A \sin B = \frac{1}{2} \{ \cos(A - B) - \cos(A + B) \}$$

$$\sin A \cos B = \frac{1}{2} \{ \sin(A - B) + \sin(A + B) \}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

$$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A - \cos B = 2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(B - A)$$

$$\cos A \cos B = \frac{1}{2} \{ \cos(A - B) + \cos(A + B) \}$$

$$\cos(\theta) = \sin(\theta + \pi/2)$$

Differentiation Formulas:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + \frac{du}{dx} v$$

$$\text{Chain rule: } \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}, \quad \left(-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right)$$

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}, \quad \left(-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right)$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

$$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$$

$$\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx} \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}, \quad (0 < \cos^{-1} u < \pi)$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx} \log_a u = \frac{\log_a e}{u} \frac{du}{dx}, \quad a \neq 0, 1$$

Integration Formulas:

$$\text{Integration by parts: } \int u dv = uv - \int v du$$

$$\int \frac{du}{u} = \ln |u|$$

$$\int a^u du = \frac{a^u}{\ln a}, \quad a > 0, a \neq 1$$

$$\int \cos u du = \sin u$$

$$\int \sin^2 u du = \frac{u}{2} - \frac{\sin 2u}{4} = \frac{1}{2}(u - \sin u \cos u)$$

$$\int \cos^2 u du = \frac{u}{2} + \frac{\sin 2u}{4} = \frac{1}{2}(u + \sin u \cos u)$$

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left(\frac{u-a}{u+a} \right)$$

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2})$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$$

$$\int x \sin ax dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int x^2 \cos ax dx = \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3} \right) \sin ax$$

$$\int \tan^2 ax dx = \frac{\tan ax}{a} - x$$

$$\int \ln x dx = x \ln x - x$$

$$\int e^u du = e^u$$

$$\int \sin u du = -\cos u$$

$$\int \tan u du = -\ln \cos u$$

$$\int \tan^2 u du = \tan u - u$$

$$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a}$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2})$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$$

$$\int x^2 \sin ax dx = \frac{2x}{a^2} \sin ax + \left(\frac{2}{a^3} - \frac{x^2}{a} \right) \cos ax$$

$$\int x \cos ax dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a} \left(x - \frac{1}{a} \right)$$

$$\int x \ln x dx = \frac{x^2}{2} \left(\ln x - \frac{1}{2} \right)$$

Rules for Exponents:

$$a^{b+c} = a^b a^c$$

$$(ab)^c = a^c b^c$$

$$(a^b)^c = a^{bc}$$

$$a^{-b} = \left(\frac{1}{a}\right)^b = \frac{1}{a^b}$$

Taylor Series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

Euler's Formula:

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Rectangular and Polar Form of a Complex Number:

$$z = a + jb = re^{j\theta}$$

$$r = |z| = \sqrt{a^2 + b^2}$$

$$a = \operatorname{Re}\{z\} = r \cos \theta$$

$$r^2 = zz^*$$

$$a = \operatorname{Re}\{z\} = \frac{z + z^*}{2}$$

$$\theta = \arctan \frac{b}{a}$$

$$b = \operatorname{Im}\{z\} = r \sin \theta$$

$$b = \operatorname{Im}\{z\} = \frac{z - z^*}{2j}$$

Phasors:

$$\text{Complex Signal: } z(t) = Ae^{j(\omega_0 t + \phi)} = Ae^{j\phi} e^{j\omega_0 t}$$

$$\text{Real Signal: } x(t) = \operatorname{Re}\{z(t)\} = A \cos(\omega_0 t + \phi)$$

$$\text{Phasor Representation: } X = Ae^{j\phi}$$

Phasor Addition:

Let $x_1(t) = A_1 \cos(\omega_0 t + \phi_1)$, $x_2(t) = A_2 \cos(\omega_0 t + \phi_2)$, and $x(t) = x_1(t) + x_2(t)$.

Then $x(t) = A \cos(\omega_0 t + \phi)$ and:

the phasor representation for $x(t)$ is $X = Ae^{j\phi} = A_1 e^{j\phi_1} + A_2 e^{j\phi_2}$.

Continuous-Time Unit Impulse and Unit Step:

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\int_{-\infty}^{\infty} x(t) \delta(t) dt = x(0)$$

$$\int_{-\infty}^{\infty} x(t) \delta(t - t_0) dt = x(t_0)$$

$$u(t) = \int_{-\infty}^t \delta(t) dt$$

Discrete-Time Unit Impulse and Unit Step:

$$\delta[n] = \begin{cases} 1, & n = 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

Complex Exponential Signals:

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct w_0	Identical signals for values of w_0 separated by multiples of 2π
Periodic for any choice of w_0	Periodic only if $w_0/(2\pi) = m/N \in \mathbb{Q}$
Fundamental frequency w_0	Fundamental frequency w_0/m
Fundamental period: $w_0 = 0$: undefined $w_0 \neq 0$: $2\pi/w_0$	Fundamental period: $w_0 = 0$: one $w_0 \neq 0$: $N = 2\pi m/w_0$

Periodicity of Discrete-Time Sinusoids:

$\cos(\omega_0 n)$, $\sin(\omega_0 n)$, and $e^{j\omega_0 n}$ are periodic if and only if $\frac{w_0}{2\pi}$ is a ratio of two integers.

If periodic, then write in reduced form: $\frac{w_0}{2\pi} = \frac{m}{N}$ (no common factors between m and N)

N : Fundamental Period

m : In each period of the discrete-time signal, the graph “goes around” m times.

Summation Formulas:

$$\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1 - \alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1 - a}, \quad |a| < 1$$

$$\sum_{k=0}^{\infty} k a^k = \frac{a}{(1 - a)^2}, \quad |a| < 1$$

$$\sum_{k=0}^n a^k = \frac{1 - a^{n+1}}{1 - a}, \quad a \neq 1$$

$$\sum_{k=0}^n k a^k = \frac{a\{1 - (n+1)a^n + n a^{n+1}\}}{(1 - a)^2}$$

Time Domain Representation of Discrete-Time Signals:

$$\begin{aligned} x[n] &= \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \cdots \\ &= \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]. \end{aligned}$$

Systems:

System H is linear if $H\{ax_1[n] + bx_2[n]\} = aH\{x_1[n]\} + bH\{x_2[n]\}$.

System H is time invariant if $H\{x[n - n_0]\} = y[n - n_0]$.

Impulse response: for LTI system H , $h[n] = H\{\delta[n]\}$.

Convolution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

Convolution with $\delta[n]$:

$$x[n] * \delta[n] = x[n] \qquad x[n] * \delta[n - n_0] = x[n - n_0].$$