Formulas from Trigonometry:

$\sin^2 A + \cos^2 A = 1$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$\sin \frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{2}}$

$\cos \frac{A}{2} = \pm\sqrt{\frac{1 + \cos A}{2}}$

$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$

$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

$\cos A - \cos B = 2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(B - A)$

$\sin A \sin B = \frac{1}{2}\{\cos(A - B) - \cos(A + B)\}$

$\cos A \cos B = \frac{1}{2}\{\cos(A - B) + \cos(A + B)\}$

$\sin A \cos B = \frac{1}{2}\{\sin(A - B) + \sin(A + B)\}$

$\cos(\theta) = \sin(\theta + \pi/2)$

Differentiation Formulas:

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$

$\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$

$\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$

$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1 - u^2}} \frac{du}{dx}$, $\left(-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right)$

$\frac{d}{dx} \cos^{-1} u = \frac{-1}{\sqrt{1 - u^2}} \frac{du}{dx}$, $(0 < \cos^{-1} u < \pi)$

$\frac{d}{dx} \tan^{-1} u = \frac{1}{1 + u^2} \frac{du}{dx}$, $\left(-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right)$

$\frac{d}{dx} e^u = e^u \frac{du}{dx}$

$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$

$\frac{d}{dx} \log_a u = \frac{\log_a e}{u} \frac{du}{dx}$, $a \neq 0, 1$

Integration Formulas:

Integration by parts: $\int u\, dv = uv - \int v\, du$

$\int \frac{du}{u} = \ln |u|$

$\int e^u\, du = e^u$

$\int a^u\, du = \frac{a^u}{\ln a}, \quad a > 0,\ a \neq 1$

$\int \sin u\, du = -\cos u$

$\int \cos u\, du = \sin u$

$\int \tan u\, du = -\ln \cos u$

$\int \sin^2 u\, du = \frac{u}{2} - \frac{\sin 2u}{4} = \frac{1}{2}(u - \sin u \cos u)$

$\int \tan^2 u\, du = \tan u - u$

$\int \cos^2 u\, du = \frac{u}{2} + \frac{\sin 2u}{4} = \frac{1}{2}(u + \sin u \cos u)$

$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}$

$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln\left(\frac{u - a}{u + a}\right)$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a}$

$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2})$

$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2})$

$\int e^{ax} \sin bx\, dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$

$\int e^{ax} \cos bx\, dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$

$\int x \sin ax\, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$

$\int x^2 \sin ax\, dx = \frac{2x}{a^2} \sin ax + \left(\frac{2}{a^3} - \frac{x^2}{a}\right) \cos ax$

$\int \sin^2 ax\, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$

$\int x \cos ax\, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$

$\int x^2 \cos ax\, dx = \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right) \sin ax$

$\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$

$\int \tan^2 ax\, dx = \frac{\tan ax}{a} - x$

$\int x e^{ax}\, dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$

$\int \ln x\, dx = x \ln x - x$

$\int x \ln x\, dx = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right)$

Rules for Exponents:

$$a^{b+c} = a^b a^c \qquad (ab)^c = a^c b^c$$

$$(a^b)^c = a^{bc} \qquad a^{-b} = \left(\frac{1}{a}\right)^b = \frac{1}{a^b}$$

Taylor Series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\sin x = 1 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

Euler's Formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Rectangular and Polar Form of a Complex Number:

$$z = a + jb = re^{j\theta}$$

$$r = |z| = \sqrt{a^2 + b^2} \qquad a = \text{Re}\{z\} = r\cos\theta$$

$$r^2 = zz^* \qquad a = \text{Re}\{z\} = \frac{z + z^*}{2}$$

$$\theta = \arctan\frac{b}{a} \qquad b = \text{Im}\{z\} = r\sin\theta$$

$$b = \text{Im}\{z\} = \frac{z - z^*}{2j}$$

Phasors:

Complex Signal: $z(t) = Ae^{j(\omega_0 t + \phi)} = Ae^{j\phi}e^{j\omega_0 t}$
Real Signal: $x(t) = \text{Re}\{z(t)\} = A\cos(\omega_0 t + \phi)$
Phasor Representation: $X = Ae^{j\phi}$

Phasor Addition:
Let $x_1(t) = A_1\cos(w_0 t + \phi_1)$, $x_2(t) = A_2\cos(w_0 t + \phi_2)$, and $x(t) = x_1(t) + x_2(t)$.
Then $x(t) = A\cos(w_0 t + \phi)$ and:
the phasor representation for $x(t)$ is $X = Ae^{j\phi} = A_1 e^{j\phi_1} + A_2 e^{j\phi_2}$.

Continuous-Time Unit Impulse and Unit Step:

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \qquad \int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0) \qquad \int_{-\infty}^{\infty} x(t)\delta(t - t_0)\,dt = x(t_0)$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\,dt$$

Discrete-Time Unit Impulse and Unit Step:

$$\delta[n] = \begin{cases} 1, & n = 0, \\ 0, & \text{otherwise.} \end{cases} \qquad u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

Complex Exponential Signals:

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct w_0	Identical signals for values of w_0 separated by multiples of 2π
Periodic for any choice of w_0	Periodic only if $w_0/(2\pi) = m/N \in \mathbb{Q}$
Fundamental frequency w_0	Fundamental frequency w_0/m
Fundamental period: $w_0 = 0$: undefined $w_0 \neq 0$: $2\pi/w_0$	Fundamental period: $w_0 = 0$: one $w_0 \neq 0$: $N = 2\pi m/w_0$

Periodicity of Discrete-Time Sinusoids:

$\cos(\omega_0 n)$, $\sin(\omega_0 n)$, and $e^{j\omega_0 n}$ are periodic if and only if $\dfrac{w_0}{2\pi}$ is a ratio of two integers.

If periodic, then write in reduced form: $\dfrac{w_0}{2\pi} = \dfrac{m}{N}$ (no common factors between m and N)

N: Fundamental Period

m: In each period of the discrete-time signal, the graph “goes around” m times.

Summation Formulas:

$$\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1-\alpha}, \quad \alpha \neq 1 \qquad \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}, \quad |a| < 1$$

$$\sum_{k=0}^{\infty} k a^k = \frac{a}{(1-a)^2}, \quad |a| < 1 \qquad \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}, \quad a \neq 1$$

$$\sum_{k=0}^{n} k a^k = \frac{a\{1-(n+1)a^n + na^{n+1}\}}{(1-a)^2}$$

Time Domain Representation of Discrete-Time Signals:

$$\begin{aligned} x[n] &= \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \cdots \\ &= \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]. \end{aligned}$$

Systems:

- System H is linear if $H\{ax_1[n] + bx_2[n]\} = aH\{x_1[n]\} + bH\{x_2[n]\}$.
- System H is time invariant if $H\{x[n-n_0]\} = y[n-n_0]$.
- Impulse response: for LTI system H, $h[n] = H\{\delta[n]\}$.
- System H is causal if the current output does not depend on future inputs.
- LTI system H is causal iff $h[n] = 0 \ \forall \ n < 0$.
- System H is stable if every bounded input signal produces a bounded output signal.

Convolution:

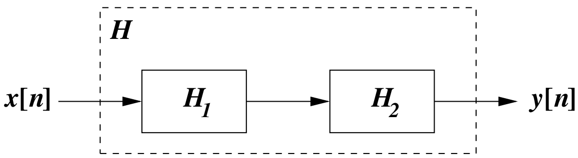
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

Convolution with $\delta[n]$:

$$x[n] * \delta[n] = x[n] \qquad\qquad x[n] * \delta[n-n_0] = x[n-n_0].$$

LTI System Interconnections:

Series/Cascade

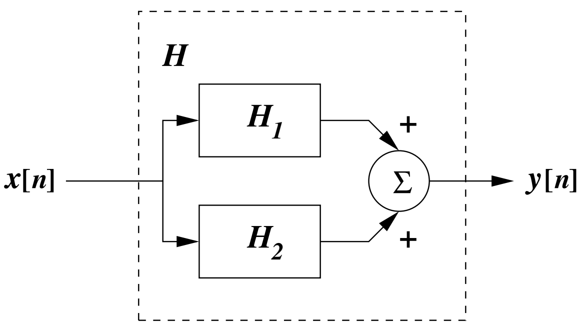


$$h[n] = h_1[n] * h_2[n]$$

$$H(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})$$

$$H(z) = H_1(z)H_2(z)$$

Parallel

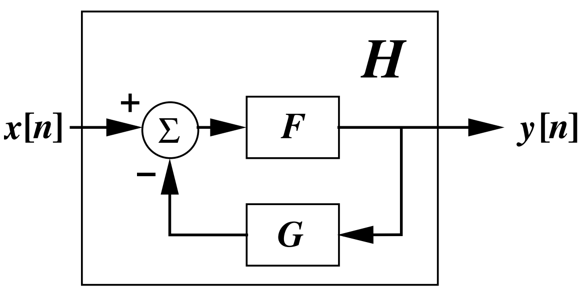


$$h[n] = h_1[n] + h_2[n]$$

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

$$H(z) = H_1(z) + H_2(z)$$

Negative Feedack



$h[n]$: *No general expression*

$$H(e^{j\omega}) = \frac{F(e^{j\omega})}{1 + F(e^{j\omega})G(e^{j\omega})}$$

$$H(z) = \frac{F(z)}{1 + F(z)G(z)}$$

ROC of $H(z)$:

- If H is a **causal** discrete-time LTI system, then the ROC of $H(z)$ is **exterior**.
- A discrete-time LTI system H is **stable** if and only if the ROC of $H(z)$ contains the unit-circle of the z-plane. (This is *assuming* that $H(z)$ is a ratio of two polynomials in z^{-1})

Sampling:

Ω: analog frequency

ω: digital frequency

$\omega = \Omega T_s = \Omega \frac{2\pi}{\Omega_s} = \frac{\Omega}{f_s}$

$\Omega = \frac{\omega}{T_s} = \omega \frac{\Omega_s}{2\pi} = \omega f_s$

$x[n] = x(nT_s)$

T_s: sampling interval

$\Omega_s = \frac{2\pi}{T_s}$: sampling frequency (radians)

$f_s = \frac{1}{T_s}$: sampling frequency (Hz)

$\Omega_N = \frac{\Omega_s}{2} = \frac{\pi}{T_s}$: Nyquist rate (radians)

$f_N = \frac{f_s}{2} = \frac{1}{2T_s}$: Nyquist rate (Hz)

If $X(\Omega) = \mathcal{F}\{x(t)\} = 0$ for $|\Omega| > \Omega_M$, then we say that the signal $x(t)$ is *bandlimited* to Ω_M. To avoid aliasing, you must sample $x(t)$ with a sampling frequency $\Omega_s > 2\Omega_M$. In other words, you must sample at a frequency that is at least twice the highest frequency in the signal. Another way of saying this is: to avoid aliasing, the highest frequency in the signal must be less than the Nyquist frequency: $\Omega_M < \Omega_N$.

When you sample $x(t)$ to get $x[n]$, the analog frequencies $\pm\Omega_N$ map to the digital frequencies $\pm\pi$.

- To convert analog Herzian frequency to analog radian frequency, multiply by 2π.
- To convert analog radian frequency to analog Herzian frequency, divide by 2π.
- To convert analog Herzian frequency to digital Herzian frequency, multiply by T_s.
- To convert analog Herzian frequency to digital radian frequency, multiply by $2\pi T_s$.
- To convert analog radian frequency to digital radian frequency, multiply by T_s.
- To convert analog radian frequency to digital Herzian frequency, multiply by $\frac{T_s}{2\pi}$.
- To convert digital radian frequency to normalized digital frequency, divide by π.

Common Window Functions for FIR filter design:

Rectangular: $w[n] = 1, \quad -M \leq n \leq M,$

Hann: $w[n] = \frac{1}{2}\left[1 + \cos\left(\frac{\pi n}{M}\right)\right], \quad -M \leq n \leq M,$

Hamming: $w[n] = 0.54 + 0.46\cos\left(\frac{\pi n}{M}\right), \quad -M \leq n \leq M,$

Blackman: $w[n] = 0.42 + 0.5\cos\left(\frac{\pi n}{M}\right) + 0.08\cos\left(\frac{2\pi n}{M}\right), \quad -M \leq n \leq M.$

Main Properties of the Window Functions:

Type of Window	Main Lobe Width Δ_{ML}	Relative Sidelobe Level $A_{s\ell}$	Minimum Stopband Attenuation	Transition Bandwidth $\Delta\omega$
Rectangular	$4\pi/(2M+1)$	13.3 dB	20.9 dB	$0.92\pi/M$
Hann	$8\pi/(2M+1)$	31.5 dB	43.9 dB	$3.11\pi/M$
Hamming	$8\pi/(2M+1)$	42.7 dB	54.5 dB	$3.32\pi/M$
Blackman	$12\pi/(2M+1)$	58.1 dB	75.3 dB	$5.56\pi/M$

Design Steps:

1. Convert the minimum stopband attenuation spec δ_s to dB using the formula $\alpha_s = -20\log_{10}\delta_s$.
2. Look in column 4 (Minimum Stopband Attenuation) of the table above to determine which window functions can provide at least α_s dB of stopband attenuation.
3. Let $\Delta\omega = \omega_s - \omega_p$. Use the last column of the table to figure out which window function $w[n]$ can meet the stopband spec with the smallest value M.
   - To do this, set $\Delta\omega$ equal to the formula in the last column of the table and solve for M. M must be an integer and you must always round up. For example, 2.001 means $M = 3$.
   - The order of your filter will be $2M$.
   - The length of $h[n]$ will be $2M + 1$.
4. Let $\omega_c = \frac{\omega_p + \omega_s}{2}$.
5. Let $h_{LP}[n] = \frac{\sin(\omega_c n)}{\pi n}$.
6. For the window function $w[n]$ that meets the stopband spec with the smallest M, compute the "centered" impulse response $h_1[n] = w[n]h_{LP}[n]$, $-M \leq n \leq M$.
7. Shift it right by M to make it causal: $h[n] = h_1[n-M] = w[n-M]h_{LP}[n-M]$, $0 \leq n \leq 2M$.

Basic Discrete-Time Fourier Transform Pairs:

Signal	Fourier transform
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l)$
Periodic square wave $x[n] = \begin{cases} 1, & \|n\| < N_1 \\ 0, & N_1 < \|n\| \leq N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$\sum_{k=-\infty}^{\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$
$a^n u[n], \quad \|a\| < 1$	$\frac{1}{1 - ae^{-j\omega}}$
$x[n] = \begin{cases} 1, & \|n\| \leq N_1 \\ 0, & \|n\| > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$
$\frac{\sin Wn}{\pi n}, \quad 0 < W < \pi$	$X(e^{j\omega}) = \begin{cases} 1, & 0 \leq \|\omega\| \leq W \\ 0, & W < \|\omega\| \leq \pi \end{cases}$ $X(e^{j\omega})$ is 2π-periodic
$\delta[n]$	1
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2\pi k)$
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$(n+1)a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$

Properties of the Discrete-Time Fourier Transform:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad\qquad x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

Property	Aperiodic signal	Fourier transform
Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Time Reversal	$x[-n]$	$X(e^{-j\omega})$
Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})\,d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - e^{-j\omega}}X(e^{j\omega})$
Differentiation in Frequency	$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \mathrm{Re}\{X(e^{j\omega})\} = \mathrm{Re}\{X(e^{-j\omega})\} \\ \mathrm{Im}\{X(e^{j\omega})\} = -\mathrm{Im}\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
Symmetry for Real and Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
Symmetry for Real and Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\mathrm{Re}\{X(e^{j\omega})\}$ $j\mathrm{Im}\{X(e^{j\omega})\}$

Parseval's Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(e^{j\omega})\rvert^2\,d\omega$$

Common z-Transform Pairs:

Signal	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z, except 0 (if $m > 0$) or ∞ (if $m < 0$)
$\alpha^n u[n]$	$\dfrac{1}{1-\alpha z^{-1}}$	$\|z\| > \|\alpha\|$
$-\alpha^n u[-n-1]$	$\dfrac{1}{1-\alpha z^{-1}}$	$\|z\| < \|\alpha\|$
$n\alpha^n u[n]$	$\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\|z\| > \|\alpha\|$
$-n\alpha^n u[-n-1]$	$\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\|z\| < \|\alpha\|$
$[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
$[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
$[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2 z^{-2}}$	$\|z\| > r$
$[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2 z^{-2}}$	$\|z\| > r$

Properties of the z-Transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad\qquad x[n] = \frac{1}{2\pi j}\oint_C X(z) z^{n-1}\, dz$$

Property	Signal	z-Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least $R_1 \cap R_2$
Time Shifting	$x[n - n_0]$	$z^{-n_0} X(z)$	R, except possibly $z = 0$
z-Domain Scaling	$e^{j\omega_0 n} x[n]$	$X(e^{-j\omega_0} z)$	R
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
	$a^n x[n]$	$X\left(\frac{z}{a}\right)$	$\lvert a\rvert R$
Time Reversal	$x[-n]$	$X(z^{-1})$	R^{-1}
Time Expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases} \quad r \in \mathbb{Z}$	$X(z^k)$	$R^{\frac{1}{k}}$
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Convolution	$x_1[n] * x_2[n]$	$X_1(z) X_2(z)$	At least $R_1 \cap R_2$
First Difference	$x[n] - x[n-1]$	$(1 - z^{-1}) X(z)$	At least $R \cap \{\lvert z\rvert > 0\}$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - z^{-1}} X(z)$	At least $R \cap \{\lvert z\rvert > 1\}$
z-Domain Differentiation	$nx[n]$	$-z \frac{d}{dz} X(z)$	R