ECE 2713

HW 1 SOLUTION

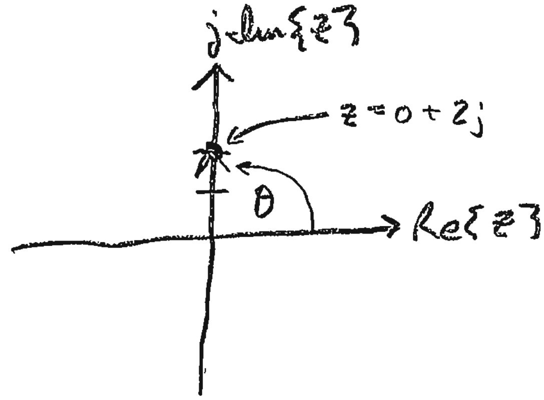
HAVLICEK

① P-A.1 : convert to polar :

$\boxed{\text{P-A.1-1}}$

a) $z = 0 + j2$

- The easy way to do this one is to plot z in the complex plane:



From this, we can read off the magnitude:

$$r = |z| = 2$$

and the angle:

$$\theta = \arg z = \frac{\pi}{2}$$

So $z = re^{j\theta} = \underline{\underline{2e^{j\pi/2}}}$

⟶

P-A.1-2

— But you can also work it analytically using the Euler formulas:

$$r = |z| = \sqrt{(\mathrm{Re}\{z\})^2 + (\mathrm{Im}\{z\})^2}$$

$$= \sqrt{0^2 + 2^2} = \sqrt{4} = \pm 2$$

→ Because it is a <u>magnitude</u>, it can't be negative, so take the non-negative solution...

$$r = 2$$

$$\theta = \measuredangle z = \arg z = \arctan \frac{\mathrm{Im}\{z\}}{\mathrm{Re}\{z\}}$$

→ But $\mathrm{Re}\{z\} = 0$, so we better be a little bit carefull...

$$\theta = \arg z = \lim_{\varepsilon \to 0} \arctan \frac{\mathrm{Im}\{z\}}{\varepsilon} = \lim_{\varepsilon \to 0} \arctan \frac{2}{\varepsilon}$$

$$= \lim_{x \to \infty} \arctan x$$

→ I look at the graph of arctan in my Math Handbook (or on wikipedia) and I see that it does converge to an asymptotic limit as the argument goes to infinity... it is given by $\lim_{x \to \infty} \arctan x = \pi/2$

So $z = re^{j\theta} = 2e^{j\pi/2}$ // (same answer as before)

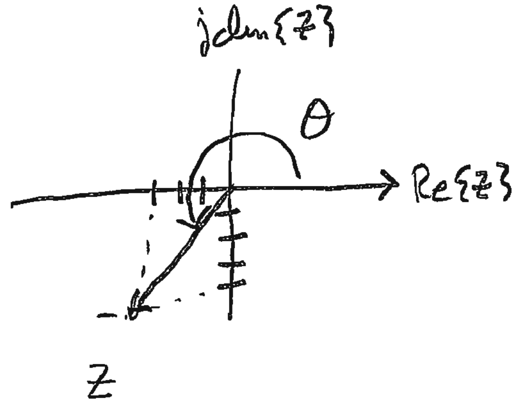
$\boxed{\text{P-A.1-3}}$

b) $z = -3 - j4$

$$|z| = \sqrt{(\mathrm{Re}\{z\})^2 + (\mathrm{Im}\{z\})^2}$$

$$= \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\theta = \arg z = \arctan \frac{-4}{-3}$$
$$= \arctan 4/3$$



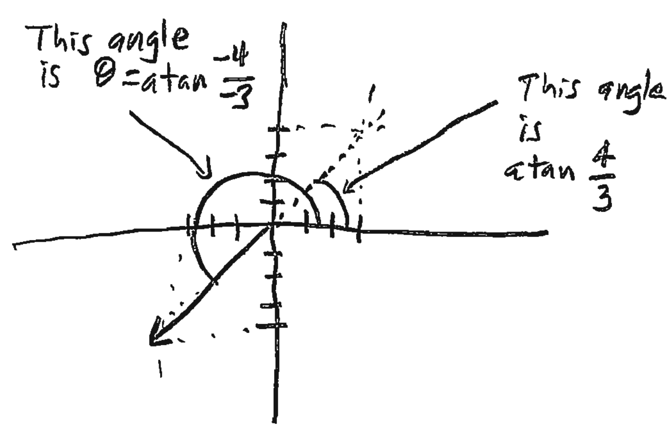
$$\Rightarrow \arctan \frac{4}{3} \approx 0.927295 \text{ rad}$$
$$\approx 53.1301 \text{ deg}$$

$\Rightarrow$ True, but this angle is in the first quadrant, whereas θ must be in the third quadrant.

so $\theta = \arctan \frac{4}{3} + \pi$
≈ 4.06889 rad

$\left(\theta = \arctan \frac{4}{3} + 180 \text{ deg} \right.$
$\left. \approx 233.1301 \text{ deg} \right)$

$$\underline{\underline{z = 5e^{j4.06889}}}$$



c) $z = (-1, 1)$

→ This means $z = -1 + j1$

$$r = |z| = \sqrt{(-1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\theta = \arctan \frac{1}{-1} = \arctan -1 = \text{hmm....}$$

→ We again better be careful. Because arctan (-1) could have came from arctan $\left(\frac{1}{-1}\right)$, which is what we've got in this problem,

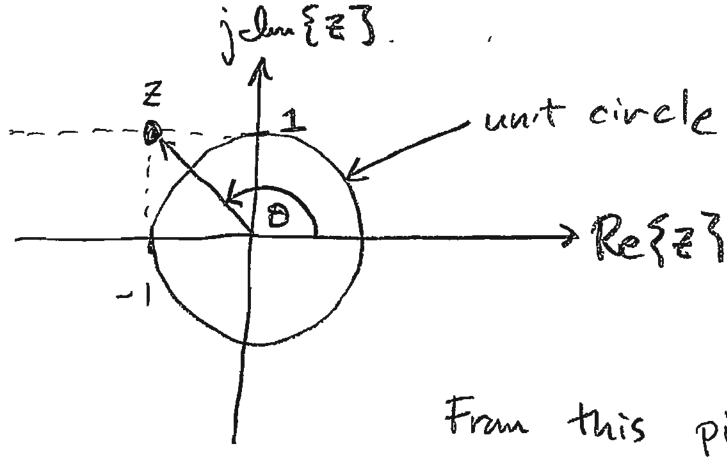
– or it could have came from arctan $\left(\frac{-1}{1}\right)$, which is <u>not</u> what we've got in this problem.

→ On a computer, you can call "atan2" to get the right answer like this: $\theta = \text{atan2}(1, -1)$

→ But on a test, you will not have a computer. So we better draw a picture...

⟶

a)... Graph $z = -1 + j1$ in the complex plane: $\boxed{\text{P-A.1-5}}$



From this picture, we can see that $\theta = \frac{3\pi}{4}$

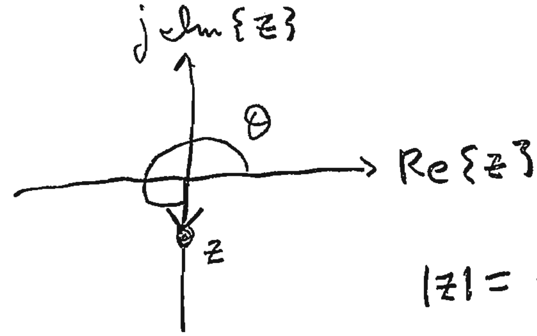
So $z = re^{j\theta} = \underline{\underline{\sqrt{2}\, e^{j3\pi/4}}}$

P-A.1-6

d) $z = (0, -1) = 0 - j$

$\mathrm{Re}\{z\} = 0 \qquad \mathrm{Im}\{z\} = -1$

→ The easiest way is to graph z:



$|z| = 1$

$\theta = \arg z = \frac{3\pi}{2}$

$$\underline{\underline{z = e^{j\,3\pi/2}}}$$

→ Alternatively, you can use the analytical formulas:

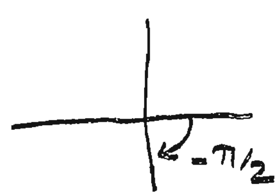
$$|z| = \sqrt{(\mathrm{Re}\{z\})^2 + (\mathrm{Im}\{z\})^2} = \sqrt{0^2 + (-1)^2}$$

$$= \sqrt{1} = 1$$

Similar to part (a), we have

$$\theta = \arctan \frac{\mathrm{Im}\{z\}}{\mathrm{Re}\{z\}} = \lim_{\varepsilon \to 0} \arctan \frac{-1}{\varepsilon}$$

$$= \lim_{x \to -\infty} \arctan x = -\frac{\pi}{2}$$

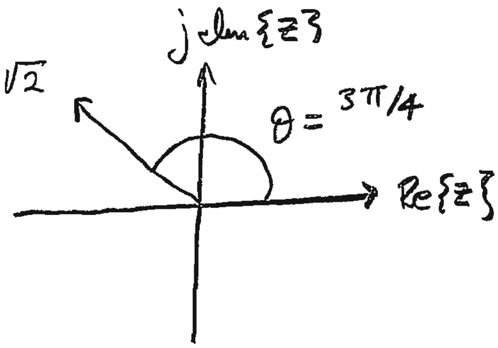


$$\underline{\underline{z = e^{-j\,\pi/2} = e^{+j\,3\pi/2}}}$$

② P-A.2 : Convert to rectangular:

$\boxed{\text{P-A.2-1}}$

a) $z = \sqrt{2}\, e^{j3\pi/4}$



$r = |z| = \sqrt{2}$

$\theta = \arg z = 3\pi/4$

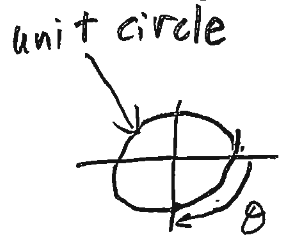
$a = \mathrm{Re}\{z\} = r\cos\theta = \sqrt{2}\cos\frac{3\pi}{4} = \sqrt{2}\;\frac{-\sqrt{2}}{2} = \frac{-2}{2} = -1$

$b = \mathrm{Im}\{z\} = r\sin\theta = \sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\;\frac{\sqrt{2}}{2} = \frac{2}{2} = 1$

$z = a + jb = -1 + j1$

$\boxed{\text{P-A.2-2}}$

b) $z = 3e^{-j\pi/2}$



$\cos\theta = 0$

$\sin\theta = -1$

$r = |z| = 3$

$\theta = \arg z = -\pi/2$

$a = \mathrm{Re}\{z\} = r\cos\theta = 3\cos(-\pi/2)$

$= 3 \cdot 0 = 0$

$b = \mathrm{Im}\{z\} = r\sin\theta = 3\sin(-\frac{\pi}{2})$

$= 3 \cdot (-1) = -3$

$z = a + jb = 0 - j3$

$c)\ z = 1.6 \measuredangle \pi/6 = 1.6\, e^{j\pi/6}$

P-A.2-3

$r = |z| = 1.6 \qquad \theta = \arg z = \pi/6$

$$a = \operatorname{Re}\{z\} = r\cos\theta = 1.6\cos \pi/6$$
$$= (1.6)\left(\tfrac{1}{2}\sqrt{3}\right) = \frac{16}{10}\cdot\frac{1}{2}\cdot\sqrt{3}$$
$$= \frac{16}{20}\sqrt{3} = \frac{4}{5}\sqrt{3} = \frac{4\sqrt{3}}{5}$$

$$b = \operatorname{Im}\{z\} = r\sin\theta = 1.6\sin\frac{\pi}{6}$$
$$= (1.6)\left(\tfrac{1}{2}\right) = \frac{16}{10}\cdot\frac{1}{2} = \frac{16}{20} = \frac{4}{5}$$

$$z = a + jb = \frac{4\sqrt{3}}{5} + j\,\frac{4}{5}$$

③ P–A.4 : Simplify in both Cartesian form and Polar form.

$\boxed{\text{P–A.4–1}}$

b) $(\sqrt{2}-j2)^{-1} = \dfrac{1}{\sqrt{2}-j2}$

"Rectangular" or "Cartesion" solution:

→ We need to clear the j out of the denominator.

→ To do this, conjugate the denominator and multiply the fraction by $1 = \dfrac{\text{denom}^*}{\text{denom}^*}$

$$\frac{1}{\sqrt{2}-j2} = \frac{1}{\sqrt{2}-j2}\cdot 1 = \frac{1}{\sqrt{2}-j2}\cdot\frac{\sqrt{2}+j2}{\sqrt{2}+j2}$$

$$= \frac{\sqrt{2}+j2}{(\sqrt{2}\sqrt{2})+(\sqrt{2})(j2)-(j2)\sqrt{2}-(j2)(j2)}$$

"foil" → "first" "outside" "inside" "last"

$$= \frac{\sqrt{2}+j2}{2+j2\sqrt{2}-j2\sqrt{2}-(-4)} = \frac{\sqrt{2}+j2}{2+4} = \frac{\sqrt{2}+j2}{6}$$

$$= \frac{\sqrt{2}}{6}+j\frac{2}{6} = \underline{\underline{\frac{\sqrt{2}}{6}+j\frac{1}{3}}} \approx 0.2357+j0.333$$

"Polar" solution: P-A.4-2

Let $z_1 = (\sqrt{2} - j2)^{-1}$ and $z_2 = \sqrt{2} - j2$.

Then $z_1 = z_2^{-1}$

So $|z_1| = \frac{1}{|z_2|}$ and $\arg z_1 = -\arg z_2$

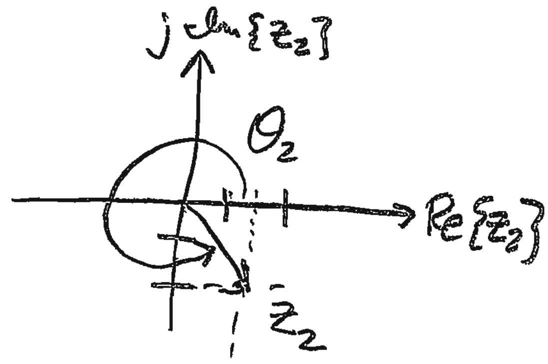
- Strategy: first, write z_2 in polar form. Then invert the magnitude and negate the angle to fip it upside down and get z_1. Finally, convert back to rectangular form.

$$z_2 = \sqrt{2} - j2 = r_2 e^{j\theta_2}$$

$$r_2 = \sqrt{(\sqrt{2})^2 + (-2)^2} = \sqrt{2+4} = \sqrt{6}$$

$$\theta_2 = \arg z_2 = \arctan\left(\frac{-2}{\sqrt{2}}\right) = \arctan(-\sqrt{2})$$

→ By graphing z_2, we can see that θ_2 is a <u>fourth quadrant</u> angle:



- In other words,

$$-\pi/2 \leq \theta_2 \leq \pi/2$$

- So "atan" or "$\tan^{-1}$" on your calculator or computer will give the right answer

$\theta_2 = \arctan(-\sqrt{2}) \approx -0.955317$ $\boxed{\text{P-A.4-3}}$

→ To get a positive angle, you can add 2π to get $-0.955317 + 2\pi \approx 5.32787$

→ You can use either version of θ_2 because they both have the same sine and the same cosine.

– So $z_2 = \sqrt{6}\, e^{-j0.955317} = \sqrt{6}\, e^{+j5.32787}$

$z_1 = z_2^{-1} = \frac{1}{\sqrt{6}} e^{+j0.955317}$ // polar

$= \frac{1}{\sqrt{6}} \cos(0.955317) + \frac{1}{\sqrt{6}} \sin(0.955317)$

$\approx 0.235702 + j0.333\overline{3}$ ✓ $\left(= \frac{\sqrt{2}}{6} + j\frac{1}{3} \right)$

Rectangular

c) $\left(\sqrt{2} - j2\right)^{1/2}$

P-A.4-4

Let $z_1 = \left(\sqrt{2} - j2\right)^{1/2}$ and $z_2 = \sqrt{2} - j2$.

Then $z_1 = z_2^{1/2}$.

So if $z_2 = r_2 e^{j\theta_2}$, then

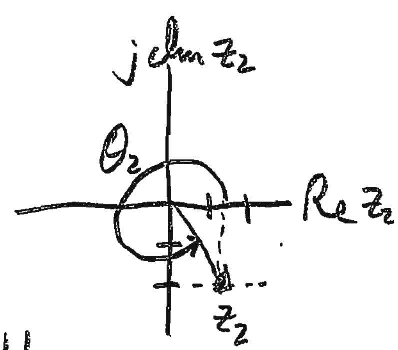
$$z_1 = \left(r_2 e^{j\theta_2}\right)^{1/2} = \sqrt{r_2}\, e^{j\,\theta_2/2}.$$

$$r_2 = |z_2| = \sqrt{(\sqrt{2})^2 + (-2)^2} = \sqrt{2+4} = \sqrt{6}$$

$$\theta_2 = \arg z_2 = \arctan\left(\frac{-2}{\sqrt{2}}\right) = \arctan\left(-\sqrt{2}\right)$$

→ As we saw in part (c), graphing z_2 shows that θ_2 is a fourth quadrant angle.

- That means $-\pi/2 \le \theta_2 \le \pi/2$
- That means "atan" or "$\tan^{-1}$" on your calculator or computer will give the correct answer for θ_2.



→

$\theta_2 = \arctan(-\sqrt{2}) \approx -0.955317$

P-A, 4-5

(you can add 2π to θ_2 if you prefer to have a positive angle. Doing so will not change the sine or the cosine)

$$Z_1 = Z_2^{1/2} = \sqrt{r_2}\, e^{j\theta_2/2} = 6^{\frac{1}{4}} e^{-j0.47765}$$

$$\approx 1.56508\, e^{-j0.47765}$$

polar

$$= \sqrt{r_2}\cos\frac{\theta_2}{2} + j\sqrt{r_2}\sin\frac{\theta_2}{2}$$

$$\approx \sqrt{\sqrt{6}}\cos\left(\frac{-0.955317}{2}\right) + j\sqrt{\sqrt{6}}\sin\left(\frac{-0.955317}{2}\right)$$

$$\approx 1.38991 - j0.71947$$

Rectangular

P-A.4-6

d) we are asked to subtract two complex numbers. Addition and subtraction are best done in rectangular form.

Let $z_1 = 3e^{j\,2\pi/3}$ $\qquad z_2 = 4e^{-j\,\pi/6}$

$r_1 = |z_1| = 3$

$\theta_1 = \arg z_1 = 2\pi/3$

$a_1 = \mathrm{Re}\{z_1\} = r_1 \cos\theta_1$

$= 3\cos\frac{2\pi}{3} = 3(-\frac{1}{2})$

$= -\frac{3}{2}$

$b_1 = \mathrm{Im}\{z_1\} = r_1 \sin\theta_1$

$= 3\sin\frac{2\pi}{3} = 3(\frac{1}{2}\sqrt{3})$

$= \frac{3}{2}\sqrt{3}$

$r_2 = |z_2| = 4$

$\theta_2 = \arg z_2 = -\pi/6$

$a_2 = \mathrm{Re}\{z_2\} = r_2 \cos\theta_2$

$= 4\cos(-\pi/6)$

$\rightarrow$ cosine is <u>even</u>

$= 4\cos(\pi/6) = 4(\frac{1}{2}\sqrt{3})$

$= 2\sqrt{3}$

$b_2 = \mathrm{Im}\{z_2\} = r_2 \sin\theta_2 = 4\sin(\frac{-\pi}{6})$

$\rightarrow$ sine is <u>odd</u>

$= -4\sin\frac{\pi}{6} = (-4)(\frac{1}{2}) = -2$

$$3e^{j\,2\pi/3} - 4e^{-j\,\pi/6} = z_1 - z_2 = (a_1 + jb_1) - (a_2 + jb_2)$$

$$= (a_1 - a_2) + j(b_1 - b_2) = \left(-\frac{3}{2} - 2\sqrt{3}\right) + j\left(\frac{3}{2}\sqrt{3} + 2\right)$$

$$\approx -4.96410 + j\,4.59808 \quad \text{Rectangular}$$

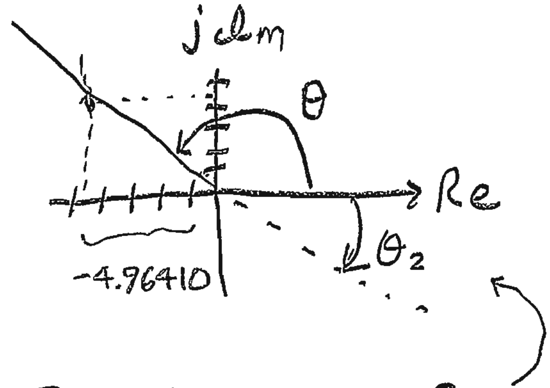
d)... Now to convert to polar:

P-A.4-7

$$r = \sqrt{(a_1-a_2)^2 + (b_1-b_2)^2}$$

$$= \sqrt{(-4.96410)^2 + (4.59808)^2}$$

$$= 6.76643$$



- To find θ, we realize that we have a second quadrant angle.
- So atan will give the <u>wrong</u> <u>answer</u>. It will give us θ_2.

$$\theta_2 = \arctan \frac{4.59808}{-4.96410} = -0.747138$$

$$\theta = \theta_2 + \pi = 2.39445$$

In polar form, our number is:

$$re^{j\theta} = 6.76643 e^{j2.39445}$$

// <u>polar</u>

Check: $r\cos\theta + jr\sin\theta = 6.76643\cos(2.39445) + j6.76643\sin(2.39445)$

$$= -4.96410 + j4.59808$$ ✓

④ P-A.5

a) $z_1 = -4 + j3, \quad z_2 = 1 - j$

$$z_1^* = Re\{z_1\} - j\,Im\{z_1\}$$
$$= \underline{\underline{-4 - j3}}$$

b) $z_2^2 = z_2 \cdot z_2$

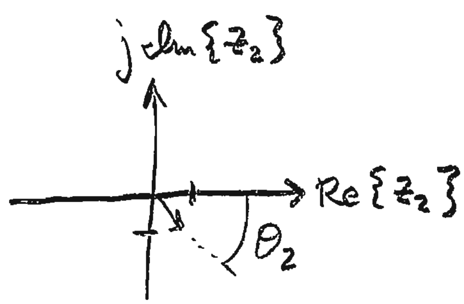
Method I: rectangular:

$$z_2 \cdot z_2 = (1-j)(1-j)$$
$$= 1 - j - j + (-j)(-j)$$
$$= 1 - 2j - 1 = \underline{\underline{-2j}}$$

Method II: polar:

$$z_2 = r_2 e^{j\theta_2}$$

graph z_2:



From the graph, we see that $\theta_2 = -\frac{\pi}{4}$

$$r_2 = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

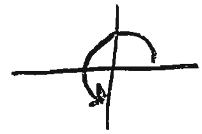
$$z_2 = \sqrt{2}\, e^{-j\pi/4} \longrightarrow$$

P-A.5-2

So $z_2^2 = \left(\sqrt{2}\, e^{-j\pi/4}\right)^2$

$= \left(\sqrt{2}\right)^2 \left(e^{-j\pi/4}\right)^2$

$= 2e^{-j\left(\frac{\pi}{4}\right)(2)} = 2e^{-j\pi/2}$



$e^{-j\pi/2} = -j$

$= \underline{\underline{-2j}}$

c) $z_1 = -4 + j3 \quad z_2 = 1 - j$

$z_1 + z_2^* = (-4+j3) + (1-j)^*$

$= (-4+j3) + (1+j)$

$= (-4+1) + j(3+1)$

$= \underline{\underline{-3+j4}}$

d) $jz_2 = j(1-j) = j - (j)(j) = j - (-1)$

$= \underline{\underline{1+j}}$

e) z_1^{-1} $\qquad z_1 = -4+j3 \qquad z_2 = 1-j$

$\boxed{\text{P-A.5-3}}$

Method I: rectangular:

$$z_1^{-1} = \frac{1}{z_1} = \frac{1}{-4+j3} \cdot \underbrace{\frac{-4-j3}{-4-j3}}_{\text{one written as } \frac{z_1^*}{z_1^*}}$$

$$= \frac{-4-j3}{(-4+j3)(-4-j3)}$$

$$= \frac{-4-j3}{(-4)^2+12j-12j+(j)(-j)9}$$

$$= \frac{-4-j3}{16+9} = \frac{-4-j3}{25} = \underline{\underline{\frac{-4}{25} - j\frac{3}{25}}}$$

$(= -0.16 - j0.12)$

$\longrightarrow$

$z_1 = -4 + j3$ $\quad$ $z_2 = 1 - j$

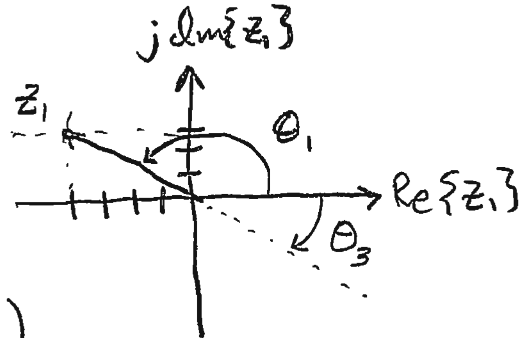
$\boxed{\text{P-A.5-4}}$

Method II : Polar :

$$z_1 = r_1 e^{j\theta_1}$$

$$r_1 = \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

- To find θ_1, graph z_1 :



- we see that θ_1 is in the second quadrant

→ $\theta_1 = \arctan \frac{3}{-4} = \arctan\left(-\frac{3}{4}\right)$

→ but "atan" or "$\tan^{-1}$" on your calculator or computer will give you an angle between $-\pi/2$ and $\pi/2$

(atan2 would give you θ_1 correctly).

→ In other words, atan or $\tan^{-1}$ will give you the angle θ_3 shown in the graph above.

→ Then $\theta_1 = \theta_3 + \pi$

→ Or you can alternatively use $\theta_3 - \pi$, which will be a negative angle with the same sine and cosine as $\theta_3 + \pi$

→ This happens because $\tan\theta_3 = -\frac{3}{4} = \tan\theta_1$ →

$z_1 = -4 + j3$ $\qquad z_2 = 1 - j$

P-A.5-5

- So our strategy is to use $\tan^{-1}$ to find θ_3, then add or subtract π to get θ_1:

$$\theta_3 = \arctan\left(-\tfrac{3}{4}\right) \approx -0.64350$$

$$\theta_1 = \theta_3 + \pi \approx 2.49809$$

(you could also use $\theta_1 = \theta_3 - \pi \approx -3.78509$)

Now,

$$z_1^{-1} = (r_1 e^{j\theta_1})^{-1} = (r_1)^{-1}(e^{j\theta_1})^{-1}$$

$$= \frac{1}{r_1} e^{-j\theta_1} = \frac{1}{r_1}\cos(-\theta_1) + j\frac{1}{r_1}\sin(-\theta_1)$$

$$= \frac{1}{r_1}\cos\theta_1 - j\frac{1}{r_1}\sin\theta_1$$

(because cosine is <u>even</u>) (because sin is <u>odd</u>)

$$= \frac{1}{5}\cos(2.49809) - j\frac{1}{5}\sin(2.49809)$$

$$\approx \underline{\underline{-0.16 - j0.12}}$$

$z_1 = -4 + j3$ $\qquad z_2 = 1 - j$

P-A.5-6

f) z_1/z_2

Method I: rectangular: use z_2^* to clear "j" from denominator:

$$\frac{z_1}{z_2} = \frac{z_1}{z_2} \cdot 1$$

$$= \frac{z_1}{z_2} \cdot \frac{z_2^*}{z_2^*} = \frac{(-4+j3)(1+j)}{(1-j)(1+j)}$$

$$= \frac{-4 - j4 + j3 + (j)(j)3}{1 + j - j + (-j)(j)}$$

$$= \frac{-4 - j - 3}{1 + 1} = \frac{-7 - j}{2} = \underline{\underline{-\tfrac{7}{2} - j\tfrac{1}{2}}}$$

Method II: polar:

$z_1 = r_1 e^{j\theta_1}$

$$r_1 = \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

As we saw in part (e), $\theta_1 \approx 2.49809$

$\longrightarrow$

$z_1 = -4 + j3 \qquad z_2 = 1 - j$

P-A.5-7

$z_2 = r_2 e^{j\theta_2}$

$r_2 = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

As we saw in part (b), $\theta_2 = -\pi/4$

So $\frac{z_1}{z_2} = \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \frac{r_1}{r_2} e^{j\theta_1} \frac{1}{e^{j\theta_2}}$

$= \frac{r_1}{r_2} e^{j\theta_1} e^{-j\theta_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$

$\approx \frac{5}{\sqrt{2}} e^{j(2.49809 + \pi/4)}$

$\approx \frac{5}{\sqrt{2}} e^{j3.28349}$

$\approx \frac{5}{\sqrt{2}} \cos(3.28349) + j\frac{5}{\sqrt{2}} \sin(3.28349)$

$= -3.5 - j0.5 = -\frac{7}{2} - j\frac{1}{2}$

$z_1 = -4 + j3$ $\quad z_2 = 1 - j$

P-A.5-8

9) e^{z_2}

- You could solve this by plugging in z_2 for "x" in the Taylor series expansion for e^x given in the notes on page 1.26.

- But it's easier to plug in the rectangular form of z_2 in the expression e^{z_2} and then simplify:

$$e^{z_2} = e^{(1-j)} = e^1 e^{-j}$$

$$= e \cdot e^{j(-1)} = e\left[\cos(-1) + j\sin(-1)\right]$$

$$\simeq e\left[0.54030 - j0.84147\right]$$

$$\simeq 1.46869 - j2.28736$$

$z_1 = -4 + j3 \qquad z_2 = 1 - j$

h) $z_1 z_1^*$

$\boxed{\text{P-A.5-9}}$

- you could compute z_1^* and then multiply using the foil rule.

- But there is an easier way. For <u>any</u> complex number z, $zz^* = |z|^2$.

So $z_1 z_1^* = |z_1|^2$

$$|z_1| = \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$z_1 z_1^* = |z_1|^2 = \underline{\underline{25}}$$

i) $z_1 z_2$

Method I: rectangular:

$$z_1 z_2 = (-4+j3)(1-j)$$

$$= -4 + j4 + j3 - 3j^2$$

$$= -4 + j7 + 3 = \underline{\underline{-1 + j7}}$$

$\longrightarrow$

$z_1 = -4 + j3 \qquad z_2 = 1 - j$

P-A.5-10

Method II: polar:

- As we saw in part (e),

$z_1 = r_1 e^{j\theta_1}$ where $r_1 = 5$ and $\theta_1 \approx 2.49809$

- As we saw in part (b),

$z_2 = r_2 e^{j\theta_2}$ where $r_2 = \sqrt{2}$ and $\theta_2 = -\pi/4$

So:

$$
\begin{aligned}
z_1 z_2 &= (r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) \\
&= r_1 r_2 e^{j\theta_1} e^{j\theta_2} \\
&= r_1 r_2 e^{j(\theta_1+\theta_2)} \\
&\approx 5(\sqrt{2}) e^{j(2.49809 - \pi/4)} \\
&\approx 5(\sqrt{2}) e^{j1.71269} \\
&\approx 5(\sqrt{2}) \cos(1.71269) + j5\sqrt{2} \sin(1.71269) \\
&= \underline{\underline{-1 + j7}}
\end{aligned}
$$

(5) P-A.7 Simplify... give reduced polar form.

$\boxed{\text{P-A.7-1}}$

a) $z = -3+j4$, find $\frac{1}{z}$

Method I: rectangular:

$$\frac{1}{z} = \frac{1}{-3+j4}$$

. Clear "j" from denominator by multiplying times $1 = \frac{z^*}{z^*}$.

$$\frac{1}{z} = \frac{1}{-3+j4} = \frac{1}{-3+j4} \cdot 1 = \frac{1}{-3+j4} \frac{z^*}{z^*}$$

$$= \frac{1}{-3+j4} \cdot \frac{-3-j4}{-3-j4} = \frac{-3-j4}{9+j12-j12+(j)(-j)16}$$

$$= \frac{-3-j4}{9+16} = \frac{-3-j4}{25} = \frac{-3}{25} - j\frac{4}{25}$$

$$r = \sqrt{\left(\frac{-3}{25}\right)^2 + \left(\frac{-4}{25}\right)^2} = \sqrt{\frac{9+16}{(25)^2}} = \sqrt{\frac{25}{(25)^2}} = \sqrt{\frac{1}{25}}$$

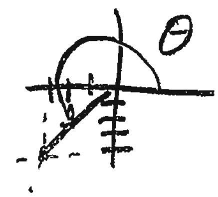
$$= \frac{\sqrt{1}}{\sqrt{25}} = \frac{1}{5}$$

$$\theta = \arctan \frac{-4/25}{-3/25} = \arctan\left(\frac{-4}{-3}\right)$$

$\longrightarrow$

P-A.7-2

Because the top and bottom are both negative in $\theta = \arctan\left(\frac{-4}{-3}\right)$, we know that $\sin\theta$ and $\cos\theta$ are both negative. This means that θ is in the third quadrant



→ atan will give a first quadrant angle as the answer... call it φ. We will need to add π to that (you could alternatively subtract π... which would give a negative angle with the same sine and cosine).

So let $\varphi = \arctan\frac{4}{3} \approx 0.927295$

Then $\theta = \varphi + \pi \approx 4.06889$

So $\frac{1}{z} = re^{j\theta} \approx \frac{1}{5}e^{j4.06889}$ //

$$\approx \tfrac{1}{5}\cos(4.06889) + j\tfrac{1}{5}\sin(4.06889)$$

$$= -0.12 - j0.16$$

In polar, $\boxed{\frac{1}{z} = \frac{1}{5}e^{j4.06889}}$

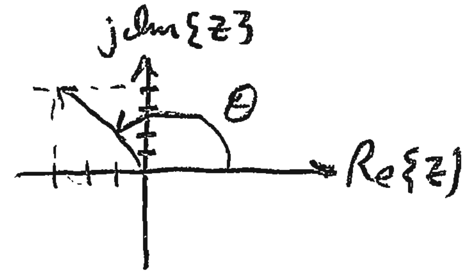
Method II: polar:

$z = -3 + j4 = re^{j\theta}$

$r = |z| = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Let $\theta = \arg z = \arctan\left(\frac{4}{-3}\right) = \arctan\left(-\frac{4}{3}\right)$

- By graphing z, we see that θ must be in the second quadrant:



- But atan will give us an answer that is between $-\pi/2$ and $\pi/2$. Call it φ.

Then we can take $\theta = \varphi + \pi$ or $\theta = \varphi - \pi$ (they both have the same sine and the same cosine).

So let $\varphi = \arctan\left(-\frac{4}{3}\right) \approx -0.927295$

Then $\theta = \varphi + \pi \approx 2.21430$

and $z = re^{j\theta} = 5e^{j2.21430}$

$\frac{1}{z} = z^{-1} = (re^{j\theta})^{-1} = (r)^{-1}(e^{j\theta})^{-1} = \frac{1}{r}e^{-j\theta}$

$= \frac{1}{5}e^{-j2.21430}$

→

- To make this look the same as the answer we got with Method I, add 2π to the angle to make it positive:

$$\frac{1}{z} = \frac{1}{5}e^{-j\,2.21430} = \frac{1}{5}e^{j(-2.21430 + 2\pi)}$$

$$\simeq \frac{1}{5}e^{j\,4.06889}$$ ✓ agrees with Method I

c) $z = -5 + j13$. Find $|z|^2$

→ $|z| = \sqrt{(\mathrm{Re}\{z\})^2 + (\mathrm{Im}\{z\})^2}$

so $|z|^2 = (\mathrm{Re}\{z\})^2 + (\mathrm{Im}\{z\})^2$

$= (-5)^2 + (13)^2$

$= 25 + 169 = 194 = 194 + j0$

→ This is a non-negative real number, but it is also a complex number. The magnitude is 194 and the angle is zero.

So, in polar form, $\boxed{|z|^2 = 194\,e^{j0}}$