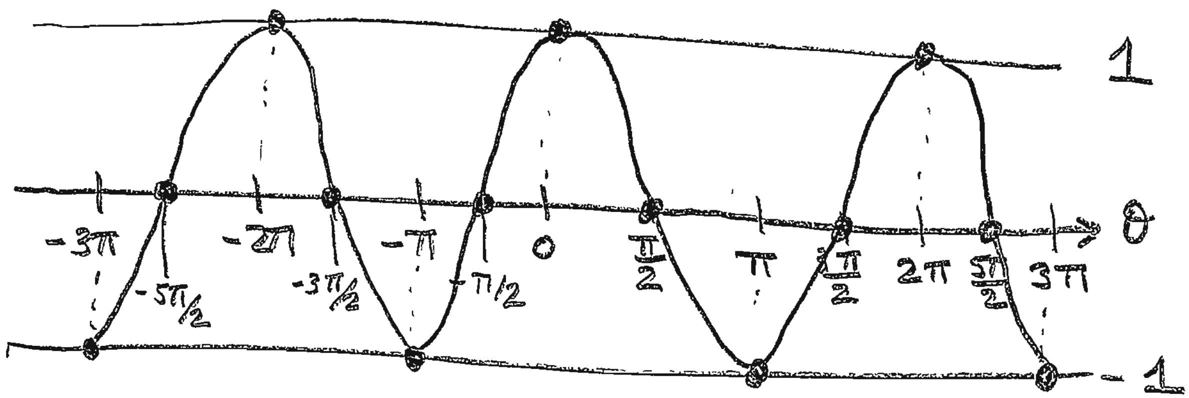
ECE 2713

HW 2 SOLUTION

HAVLICEK

① P-2.3 a) Sketch $\cos\theta$ for $-3\pi \le \theta \le 3\pi$

- The fundamental period of $\cos\theta$ is $T_0 = 2\pi$.
- $\cos\theta = 0$ at $\theta = -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$
- $\cos\theta = +1$ at $\theta = -2\pi, 0, 2\pi$
- $\cos\theta = -1$ at $\theta = -3\pi, -\pi, \pi, 3\pi$



P.2.3 c) We are asked to sketch two periods of $\cos\left(\frac{2\pi}{T_0}t + \frac{\pi}{2}\right)$.

- The book says that the graph should go from $t = -T_0$ to $t = +T_0$.
  - This implies that $2T_0$ is two periods... i.e., that the fundamental period is T_0.
  - But let's show this:
    - $\cos\left(\frac{2\pi}{T_0}t + \frac{\pi}{2}\right)$ goes through one period when $\frac{2\pi}{T_0}t$ goes through 2π radians.
    - When t goes from 0 to T_0, the argument $\frac{2\pi}{T_0}t$ goes from 0 to $\frac{2\pi}{T_0}T_0 = 2\pi$, i.e., $\frac{2\pi}{T_0}t$ goes through 2π radians when t goes through T_0 seconds ✓

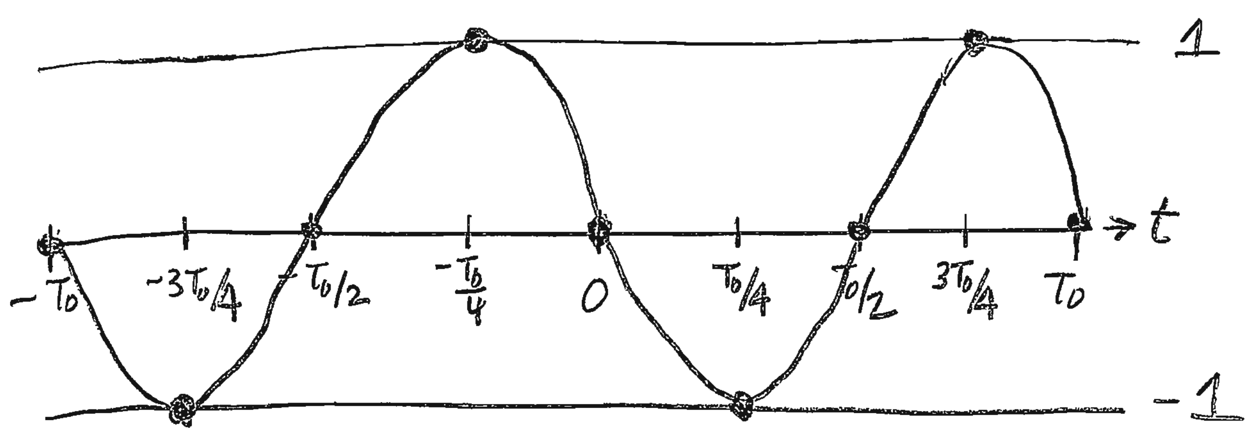
$\Rightarrow$ So the fundamental period is T_0. //

$\longrightarrow$

- Now let's make a table of values to use for graphing this function.

P-2.3-3

t	$\frac{2\pi}{T_0}t + \frac{\pi}{2}$	$\cos\left(\frac{2\pi}{T_0}t + \frac{\pi}{2}\right)$
$-T_0$	$-2\pi + \frac{\pi}{2} = -\frac{3\pi}{2}$	0
$-3T_0/4$	$-\frac{3\pi}{2} + \frac{\pi}{2} = -\pi$	-1
$-T_0/2$	$-\pi + \frac{\pi}{2} = -\frac{\pi}{2}$	0
$-T_0/4$	$-\frac{\pi}{2} + \frac{\pi}{2} = 0$	1
0	$0 + \frac{\pi}{2} = \frac{\pi}{2}$	0
$T_0/4$	$\frac{\pi}{2} + \frac{\pi}{2} = \pi$	-1
$T_0/2$	$\pi + \pi/2 = 3\pi/2$	0
$3T_0/4$	$3\pi/2 + \pi/2 = 2\pi$	1
T_0	$2\pi + \pi/2 = 5\pi/2$	0



(2)

2.4) This problem is worked in the notes on pages 1.47 – 1.52.

P-2.4-1

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots + \frac{(-1)^n\,\theta^{2n}}{(2n)!} + \cdots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots + \frac{(-1)^n\,\theta^{2n+1}}{(2n+1)!} + \cdots$$

Plugging $x = j\theta$ into the Maclaurin series for e^x, we get

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \cdots$$

$$= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} - \frac{\theta^6}{6!} - j\frac{\theta^7}{7!} \cdots$$

$$= \underbrace{\left[1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots\right]}_{\text{Take every other term starting with the first term.}} + j\underbrace{\left[\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right]}_{\text{Take every other term starting with the second term}}$$

$$= \cos\theta + j\sin\theta$$

③ P 2.5

P-2.5-1

a) $\cos(\theta_1+\theta_2) = Re\{e^{j(\theta_1+\theta_2)}\}$

$= Re\{e^{j\theta_1}e^{j\theta_2}\}$

$= Re\{[\cos\theta_1 + j\sin\theta_1][\cos\theta_2 + j\sin\theta_2]\}$

$= Re\{\cos\theta_1\cos\theta_2 + j\cos\theta_1\sin\theta_2$

$+ j\cos\theta_2\sin\theta_1 - \sin\theta_1\sin\theta_2\}$

$= Re\{[\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2] + j[\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1]\}$

$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$ //

b) $\cos(\theta_1-\theta_2) = Re\{e^{j(\theta_1-\theta_2)}\} = Re\{e^{j\theta_1}e^{-j\theta_2}\}$

$= Re\{[\cos\theta_1 + j\sin\theta_1][\cos(-\theta_2) + j\sin(-\theta_2)]\}$

$= Re\{[\cos\theta_1 + j\sin\theta_1][\cos\theta_2 - j\sin\theta_2]\}$

$= Re\{\cos\theta_1\cos\theta_2 - j\cos\theta_1\sin\theta_2 + j\sin\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\}$

$= Re\{[\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2] + j[\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2]\}$

$= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$ //

④ P-2.9)

- We can read the amplitude off of the graph immediately... we get $A = 9$.

- For the frequency ω_0, we need the fundamental period T_0.

  ⇒ Note that the horizontal axis of the graph is in units of msec.

  - We see from the graph that $x(t) = 0$ at $t = -7$ msec $= -7\times10^{-3}$ sec and at $t = +1$ msec $= 1\times10^{-3}$ sec.

  ⇒ So the fundamental period is $T_0 = 8$ msec $= 8\times10^{-3}$ sec.

  - Then the frequency of $x(t)$ is

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{8\times10^{-3}} = 1000 \times \frac{2\pi}{8}$$

$$= \frac{1000}{4}\pi = 250\pi \text{ rad/sec.}$$

- To find ϕ, we note that $\cos(0) = 1$.

  - So $x(t) = A\cos(\omega_0 t + \phi)$

$$= 9\cos(250\pi t + \phi) = 9$$

when $250\pi t + \phi = 0$ ⟶

- From the graph, we see that $x(t) = 9$ when $t = -3$ msec $= -3 \times 10^{-3}$ sec.

$\boxed{P\text{-}2.9\text{-}2}$

- Plugging this in to the last equation on the previous page, we get

$$250\pi t + \phi = 250\pi(-3 \times 10^{-3}) + \phi = 0$$

$$-750\pi \times 10^{-3} + \phi = 0$$

$$\phi = +750\pi \times 10^{-3} = 0.75\pi = 3\pi/4$$

$$\Rightarrow \quad \boxed{X = Ae^{j\phi} = 9e^{j3\pi/4}}$$

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P-2.10 a)

P-2.10-1

$$x(t) = 3\cos\left(\omega_0 t - \frac{2\pi}{3}\right) + \cos(\omega_0 t)$$

For $3\cos\left(\omega_0 t - \frac{2\pi}{3}\right)$, we have

$A = 3$, frequency $= \omega_0$, $\phi = -\frac{2\pi}{3}$

→ The phasor for $3\cos(\omega_0 t - 2\pi/3)$ is

$$X_1 = Ae^{j\phi} = 3e^{-j2\pi/3}$$

For $\cos(\omega_0 t)$, we have

$A = 1$, frequency $= \omega_0$, $\phi = 0$

→ The phasor for $\cos(\omega_0 t)$ is

$$X_2 = Ae^{j\phi} = 1e^{j0} = 1$$

For $x(t)$, the phasor is

$$X = X_1 + X_2 = 3e^{-j2\pi/3} + 1$$
$$= 3\cos(-2\pi/3) + j3\sin(-2\pi/3) + 1$$
$$= 1 + 3\cos(2\pi/3) - j3\sin(2\pi/3)$$
$$= -\frac{1}{2} - j2.59808$$

⟶

$\boxed{\text{P-2.10-2}}$

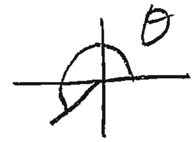
$$A = |X| = \sqrt{\left(-\tfrac{1}{2}\right)^2 + (-2.59808)^2}$$

$$= \sqrt{7} = 2.64575$$

$$\theta = \arctan\left(\frac{-2.59808}{-0.5}\right) - \pi$$

$$= 1.38067 - \pi \text{ rad}$$

$$= -1.76092 \text{ rad}$$



→ 3^{rd} quadrant

→ atan will give the wrong angle

→ must add or subtract π

So the phasor for $x(t)$

is $X = \sqrt{7}\, e^{-j1.76092}$

$A = \sqrt{7}$, frequency $= \omega_0$, $\phi = -1.76092$ rad

$$\boxed{x(t) = \sqrt{7}\cos(\omega_0 t - 1.76092)}$$

⑥ 2.11)

P-2.11-1

$$x(t) = \cos(\omega t - \pi) + \cos(\omega t + \tfrac{\pi}{3}) + 2\cos(\omega t - \tfrac{\pi}{3})$$

Phasor for $\cos(\omega t - \pi)$: $X_1 = 1e^{-j\pi} = -1$

Phasor for $\cos(\omega t + \frac{\pi}{3})$: $X_2 = 1e^{j\pi/3}$

Phasor for $2\cos(\omega t - \pi/3)$: $X_3 = 2e^{-j\pi/3}$

Phasor for $x(t)$:

$$X = X_1 + X_2 + X_3$$

$$= -1 + e^{j\pi/3} + 2e^{-j\pi/3}$$

$$= -1 + \cos(\pi/3) + j\sin(\pi/3) + 2\cos(-\pi/3) + j2\sin(-\pi/3)$$

$$= -1 + \tfrac{1}{2} + 2(\tfrac{1}{2}) + j\left[\sin(\pi/3) - 2\sin(\pi/3)\right]$$

$$= (1-1) + \tfrac{1}{2} + j\left[-\sin(\pi/3)\right]$$

$$= \tfrac{1}{2} - j\sin(\pi/3) = \tfrac{1}{2} + j\sin(-\pi/3)$$

$$= \cos(-\pi/3) + j\sin(-\pi/3) = 1e^{-j\pi/3}$$

$$\left.\begin{array}{l} A = 1 \\ \text{frequency} = \omega \\ \phi = -\pi/3 \end{array}\right\} \quad \boxed{x(t) = \cos(\omega t - \pi/3)}$$

⑦ P-2.15)

$$x(t) = 2\cos(\omega t+5) + 8\cos(\omega t+9) + 4\cos(\omega t)$$

Phasor for $2\cos(\omega t+5)$: $A=2$, $\varphi=5$: $X_1 = 2e^{j5}$

Phasor for $8\cos(\omega t+9)$: $A=8$, $\varphi=9$: $X_2 = 8e^{j9}$

Phasor for $4\cos(\omega t)$: $A=4$, $\varphi=0$: $X_3 = 4e^{j0} = 4$

Phasor for $x(t)$:

$$X = X_1 + X_2 + X_3 = 2e^{j5} + 8e^{j9} + 4$$

$$= 2\cos(5) + j2\sin(5) + 8\cos(9) + j8\sin(9) + 4$$

$$= [4 + 2\cos(5) + 8\cos(9)] + j[2\sin(5) + 8\sin(9)]$$

$$= [4 + 0.567324 - 7.28904] + j[-1.91785 + 3.29695]$$

$$= -2.72172 + j1.37910$$

$$A = |X| = \sqrt{(-2.72172)^2 + (1.37910)^2}$$

$$= \sqrt{9.30966} = 3.05117$$

$\longrightarrow$

P-2.15-2

$$\theta = \arctan\left(\frac{1.37910}{-2.72172}\right) + \pi$$

$$= -0.468995 + \pi$$

$$= 2.67260 \text{ rad}$$

θ is 2nd quadrant. So atan will give the wrong angle. Must add or subtract π.

$$\boxed{X = 3.05117\, e^{j2.67260}}$$

$$\boxed{x(t) = 3.05117 \cos(\omega t + 2.67260)}$$

⑧ P-2.16

$\boxed{\text{P-2.16-1}}$

$$T_0 = 8 = \frac{2\pi}{\omega_0}$$

$$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$$

NOTE $2\pi f_0 = \omega_0 = \pi/4$

So $x(t) = A\cos(2\pi f_0(t - t_1))$

$$= A\cos(\omega_0(t - t_1))$$

$$= A\cos(\omega_0 t - \omega_0 t_1)$$

$$= A\cos(\pi/4\, t - \pi/4\, t_1)$$

Compare to $x(t) = A\cos(\omega_0 t + \phi)$

$$\Rightarrow \phi = -\frac{\pi}{4} t_1$$

a) if $t_1 = -2$, then $\phi = -\frac{\pi}{4}(-2) = \frac{\pi}{2}$

→ So the statement is <u>true</u>.

b) if $t_1 = 3$, then $\phi = \frac{-\pi}{4} \cdot 3 = \frac{-3\pi}{4}$

- So the statement appears to be false.
- But adding an integer multiple of 2π to ϕ doesn't change the graph of $x(t)$. ⟶

P-2.16-2

So we need to check if adding $2\pi k$ to ϕ can give us $+3\pi/4$, because that would make the statement True.

→ We would need $-\frac{3\pi}{4} + 2\pi k = \frac{3\pi}{4}$ for some $k \in \mathbb{Z}$.

→ This would mean $2\pi k = 2\left(\frac{3\pi}{4}\right) = \frac{3\pi}{2}$

$$2k = \frac{3}{2}$$

$$k = \frac{3}{4} \quad \times$$

→ k can't be an integer

→ so this can't happen

→ Therefore, the statement is <u>FALSE</u>.