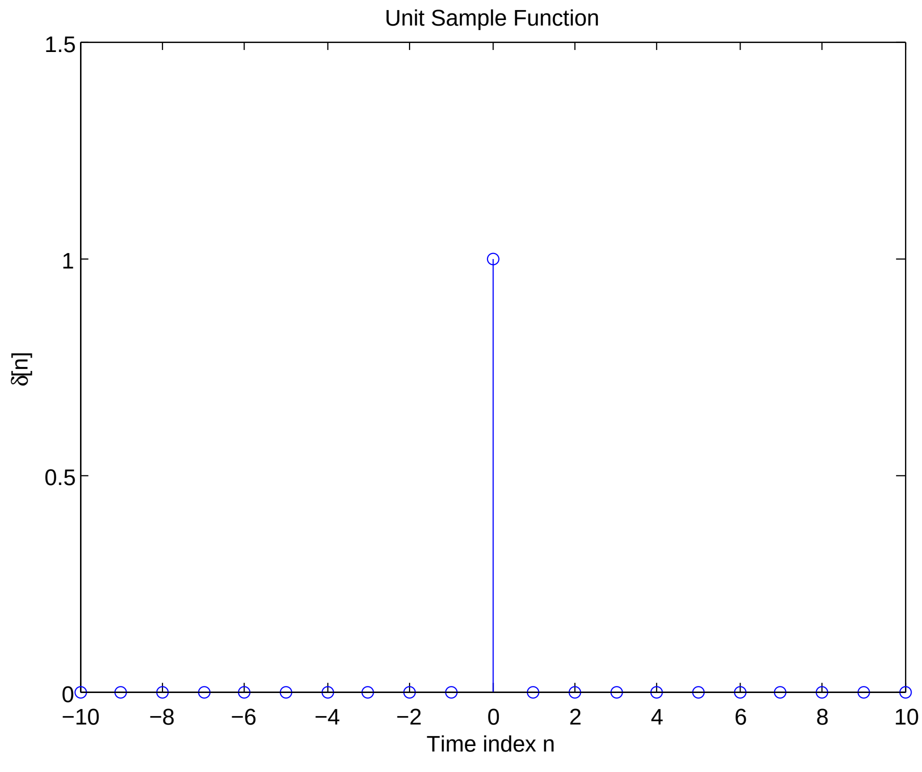
# ECE 2713
# Homework 3 Solution

Spring 2024 Dr. Havlicek

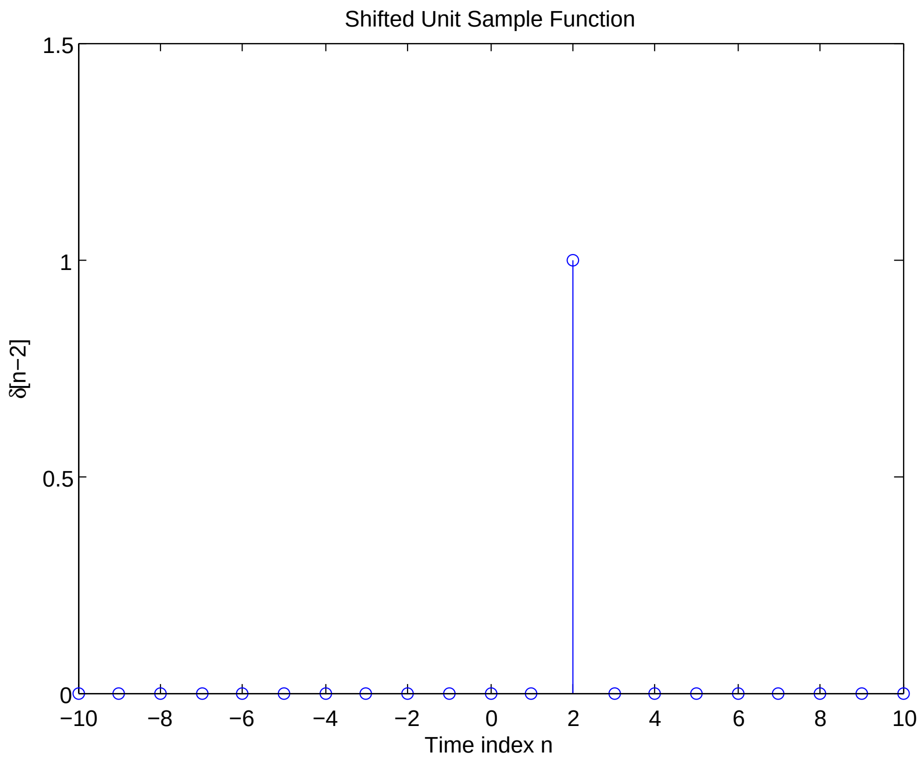
1. (a) Matlab code:

```
%----------------------------------------------------------
% P1a
%
% generate the signal \delta[n] and plot it
%
n = -10:10;          % values of the time variable
delta_n = [zeros(1,10) 1 zeros(1,10)];
stem(n,delta_n);
axis([-10 10 0 1.5]);
title('Unit Sample Function');
xlabel('Time index n');
ylabel('\delta[n]');
```



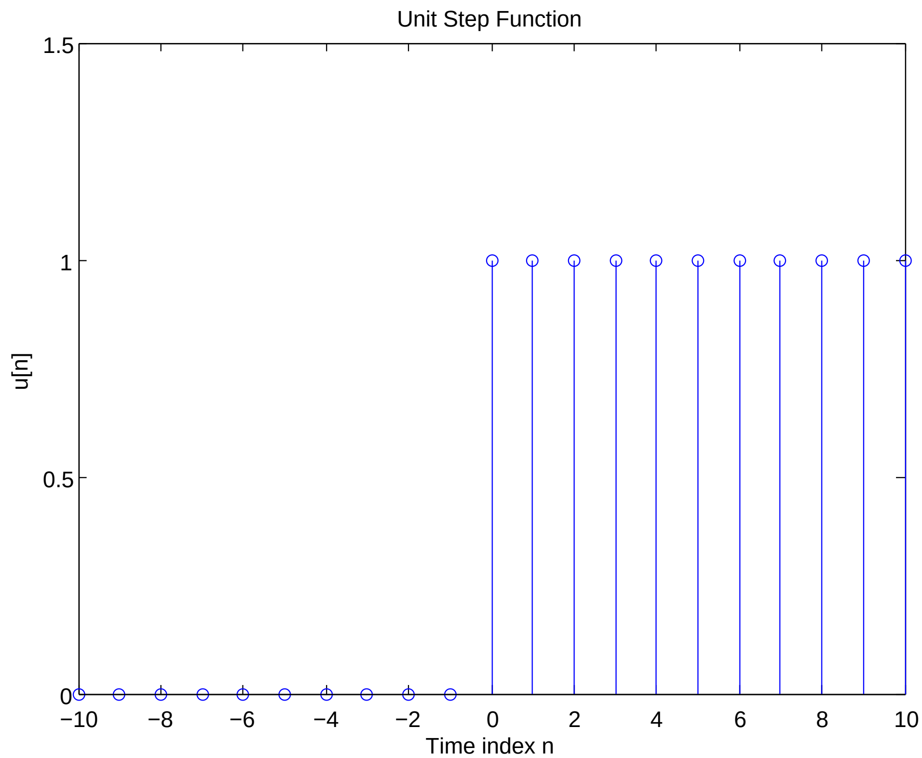
(b)

```
%-------------------------------------------------------
% P1b
%
% generate the signal \delta[n-2] and plot it
%
n = -10:10;           % values of the time variable
delta_nm2 = [zeros(1,12) 1 zeros(1,8)];
stem(n,delta_nm2);
axis([-10 10 0 1.5]);
title('Shifted Unit Sample Function');
xlabel('Time index n');
ylabel('\delta[n-2]');
```



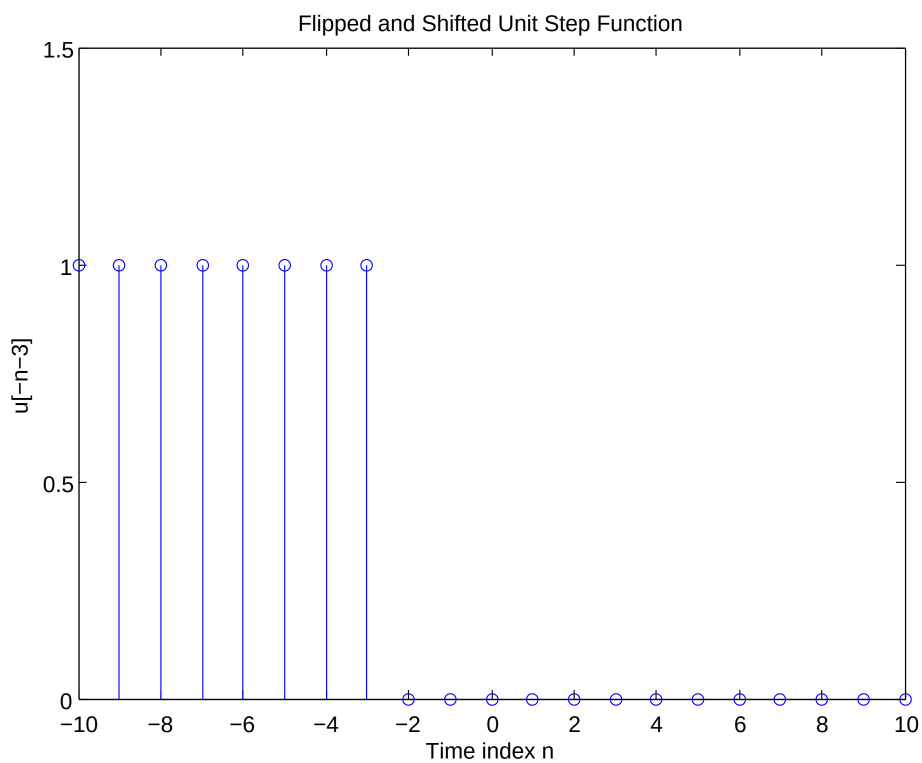
(c)

```
%-------------------------------------------------------
% P1c
%
% generate the signal u[n] and plot it
%
n = -10:10;           % values of the time variable
un = [zeros(1,10) ones(1,11)];
stem(n,un);
axis([-10 10 0 1.5]);
title('Unit Step Function');
xlabel('Time index n');
ylabel('u[n]');
```



(d)

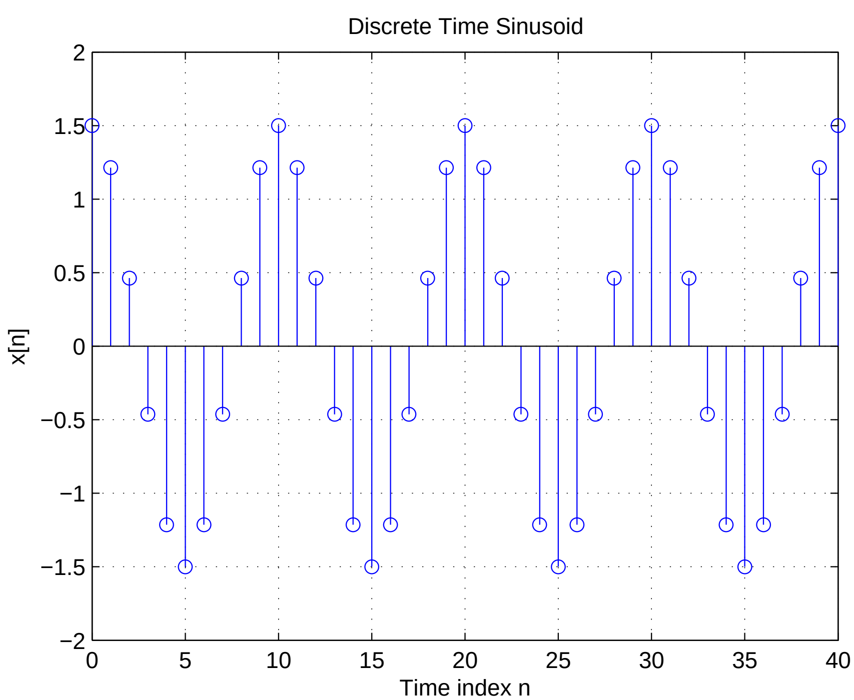
```
%---------------------------------------------------------
% P1d
%
% generate the signal u[-n-3] and plot it
%
n = -10:10;           % values of the time variable
u_mnm3 = [ones(1,8) zeros(1,13)];
stem(n,u_mnm3);
axis([-10 10 0 1.5]);
title('Flipped and Shifted Unit Step Function');
xlabel('Time index n');
ylabel('u[-n-3]');
```



2.

(a)

```
%---------------------------------------------------------------
% P2a
%
% generate and plot a discrete-time cosine signal
%
n = 0:40;                    % values of the time variable
w = 0.1*2*pi;                % frequency of the sinusoid.
phi = 0;                     % initial phase offset.
A = 1.5;                     % amplitude.
xn = A * cos(w*n + phi);
stem(n,xn);
axis([0 40 -2 2]);
grid;
title('Discrete Time Sinusoid');
xlabel('Time index n');
ylabel('x[n]');
```




(b) The signal has values from $n = 0$ to $n = 40$, so the length is 41.

(c) We have:

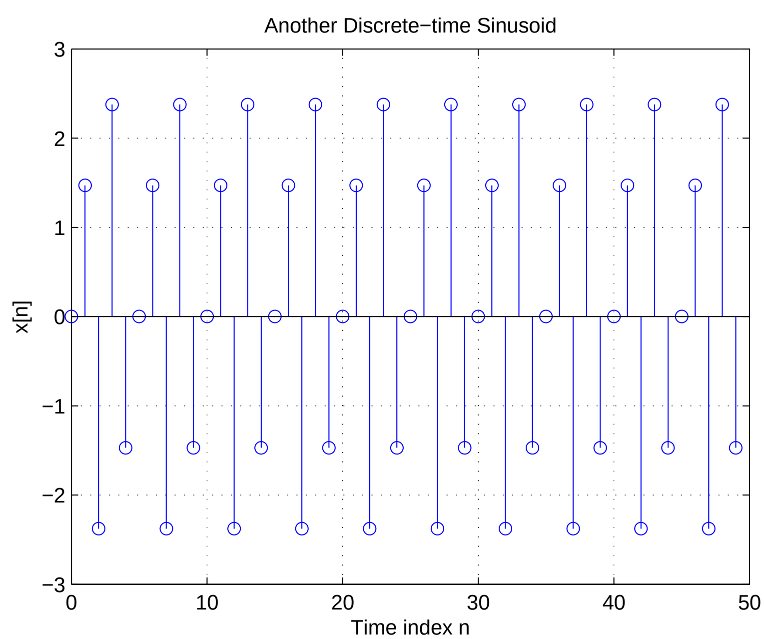
$$\frac{\omega}{2\pi} = \frac{0.2\pi}{2\pi} = \frac{1}{10} = \frac{m}{N}.$$

So the fundamental period is $N = 10$ and each period of the discrete-time signal looks like $m = 1$ period of the continuous-time function.

(d) The `grid` command draws grid lines on the graph.

(e)

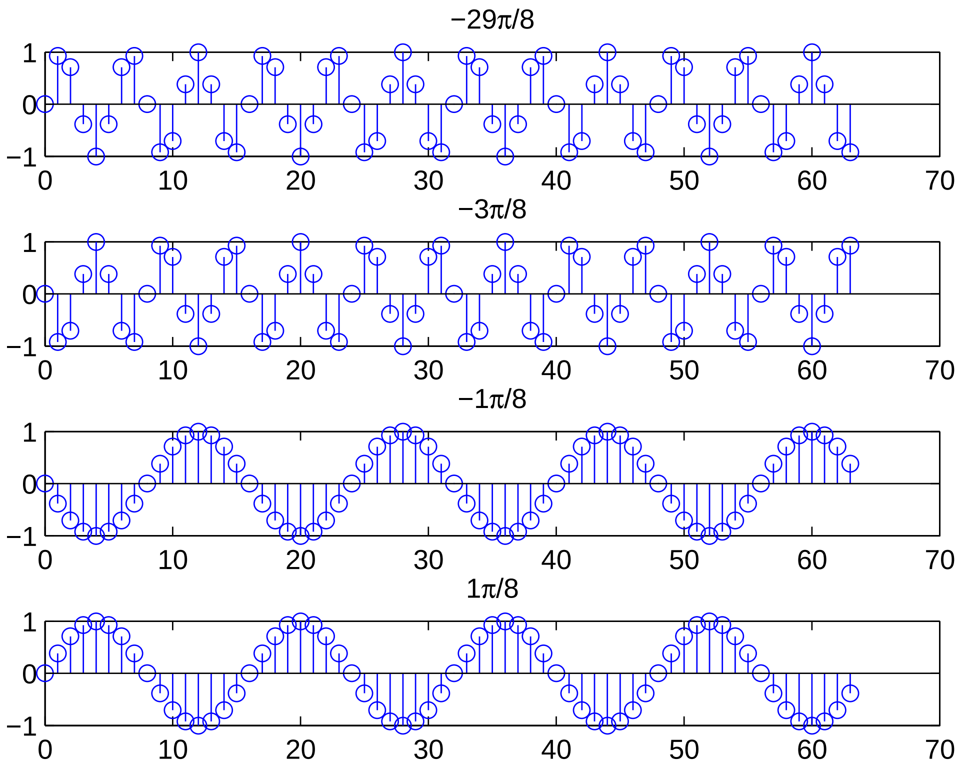
```
%----------------------------------------------------------
% P2e
%
% generate and plot another discrete-time cosine signal
%
n = 0:49;                       % values of the time variable
w = 0.4*2*pi;                   % frequency of the sinusoid.
A = 2.5;                        % amplitude.
phi = -pi/2;                    % phase offset.
xn = A * cos(w*n + phi);
stem(n,xn);
axis([0 50 -3 3]);
grid;
title('Another Discrete-time Sinusoid');
xlabel('Time index n');
ylabel('x[n]');
```

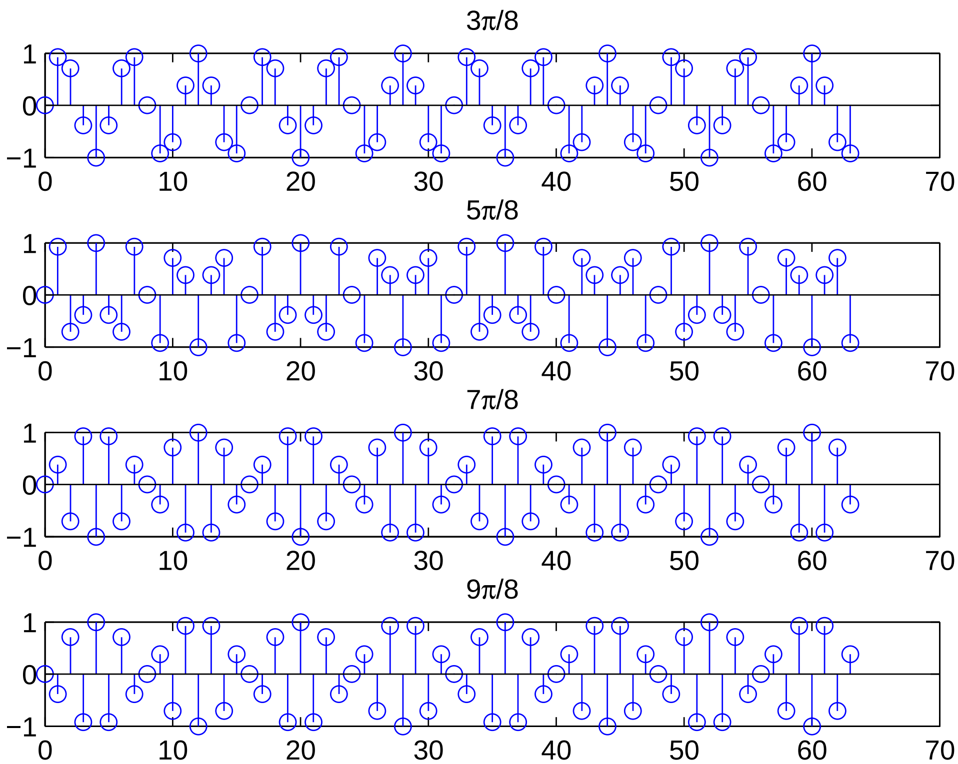


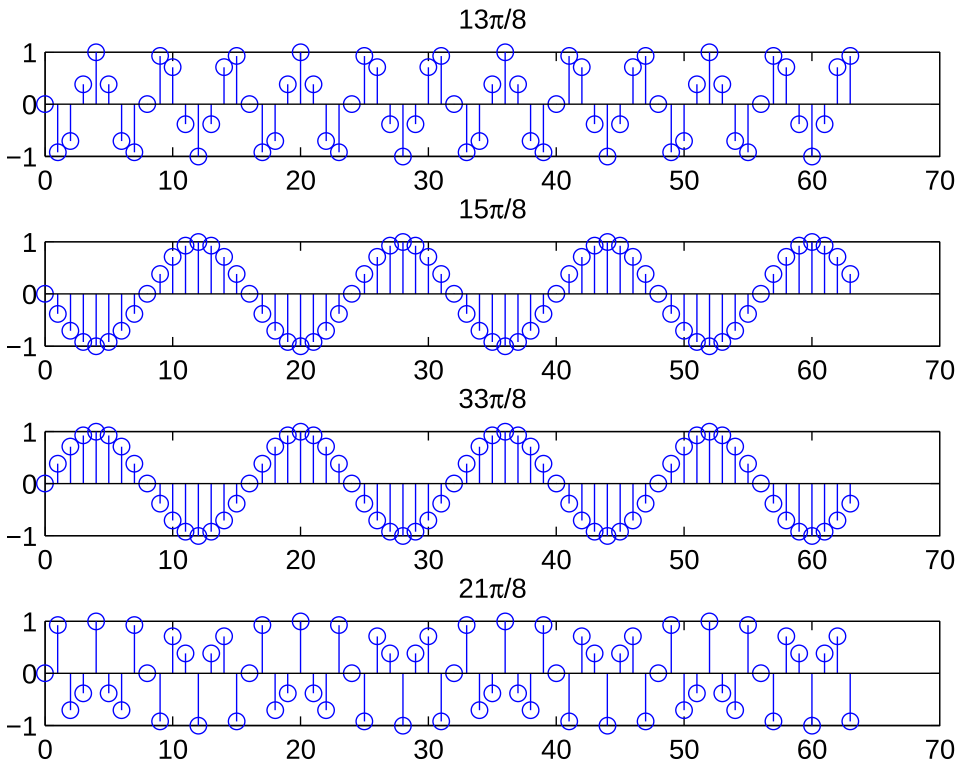
3.

(a)

```
%-----------------------------------------------------------
% P3a
%
% plot a bunch of discrete-time sine signals
%

% The frequency will be w = k*pi/8.
% Load up the k's into a vector:
%
kvals = [-29 -3 -1 1 3 5 7 9 13 15 33 21];

% make a counter to index the "next" k to use:
next_k = 1;

n = 0:63;    % the time variable

% There are 12 k values.  We will plot four per
% figure.  So we will need three figures all
% together.  Loop on figures.

for Fig_num=1:3
  figure(Fig_num);   % selects the "current" figure

  % each time through this loop, we are going to do
  % 4 of the k's.  Loop on Sub Figure number:

  for SubFig_num = 1:4
    k = kvals(next_k);
    next_k = next_k + 1;
    w = k * pi/8;          % the frequency
    xn = sin(w*n);
    subplot(4,1,SubFig_num);
    stem(n,xn);
    title(sprintf('%d%s',k,'\pi/8'));
  end   % for SubFig_num
end   % for Fig_num
```


−29π/8
−3π/8
−1π/8
1π/8
1
0
−1
0
10
20
30
40
50
60
70


3π/8
5π/8
7π/8
9π/8
1
0
−1
0
10
20
30
40
50
60
70




(b) For $\omega_0 = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}$, and $\frac{7\pi}{8}$, we have that $-\pi \leq \omega_0 \leq \pi$. So these discrete sinusoids are not aliased.

The rest of the frequencies $\omega_0$ are outside the range $[-\pi, \pi]$, so these signals will be aliased. Remember, when $\omega_0$ is outside the range $[-\pi, \pi]$, then the graph of $\sin(\omega_0 n)$ is the same as the graph of $\sin(\omega_1 n)$ for another frequency $\omega_1$ that is **inside** the range $[-\pi, \pi]$ and differs from $\omega_0$ by an integer multiple of $2\pi$.

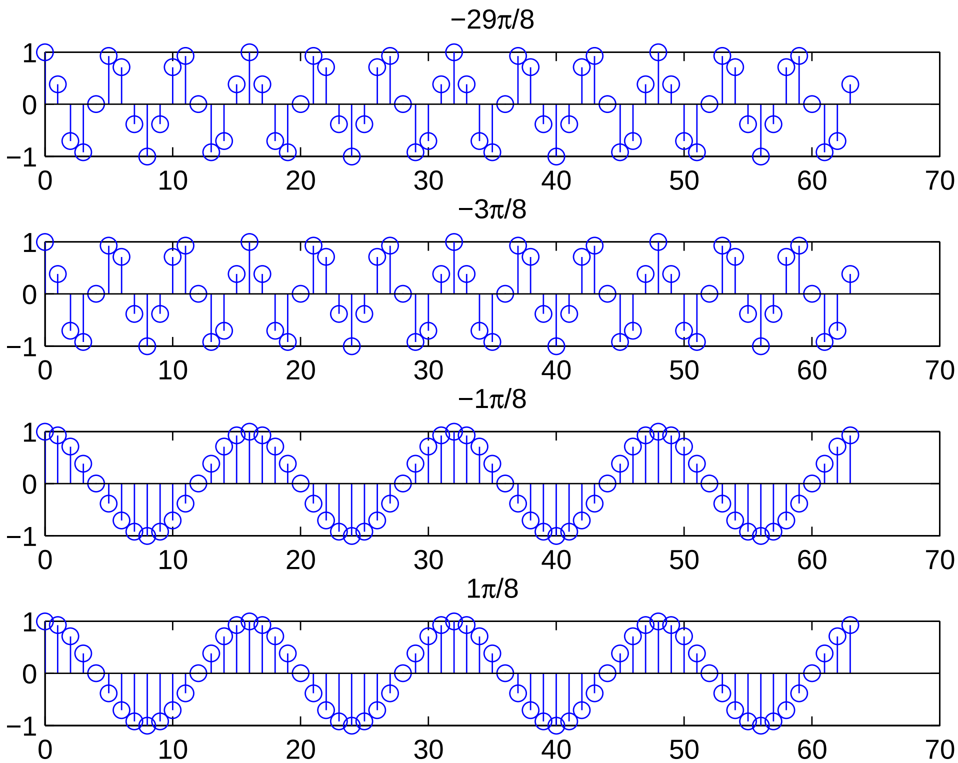
- Since $\sin(\omega_0 n) = -\sin(-\omega_0 n)$, the graph of $\sin\left(-\frac{3\pi}{8}n\right)$ is the negative of the graph of $\sin\left(\frac{3\pi}{8}n\right)$.
- Likewise, the graph of $\sin\left(-\frac{\pi}{8}n\right)$ is the the negative of the graph of $\sin\left(\frac{\pi}{8}n\right)$.
- For the rest of the frequencies, $|\omega_0| > \pi$, so each of these signals is just a *different name* for a discrete sinusoid whose frequency *is* in the range $[-\pi, \pi]$.
- $\frac{13\pi}{8} - 2\pi = -\frac{3\pi}{8}$, so the graph of $\sin\left(\frac{13\pi}{8}n\right)$ is the same as the graph of $\sin\left(-\frac{3\pi}{8}n\right)$.
- Likewise, $\frac{15\pi}{8} - 2\pi = -\frac{\pi}{8}$, so the graph of $\sin\left(\frac{15\pi}{8}n\right)$ is the same as the graph of $\sin\left(-\frac{\pi}{8}n\right)$.
- $\frac{33\pi}{8} - 2\pi = \frac{17\pi}{8}$ and $\frac{17\pi}{8} - 2\pi = \frac{\pi}{8}$. So $\sin\left(\frac{33\pi}{8}n\right)$ and $\sin\left(\frac{\pi}{8}n\right)$ are the same signal. Their graphs are the same.
- $-\frac{29\pi}{8} + 4\pi = \frac{3\pi}{8}$, so $\sin\left(-\frac{29\pi}{8}n\right)$ and $\sin\left(\frac{3\pi}{8}n\right)$ are the same signal; they have the same graph.
- $\frac{21\pi}{8} - 2\pi = \frac{5\pi}{8}$, so $\sin\left(\frac{21\pi}{8}n\right)$ and $\sin\left(\frac{5\pi}{8}n\right)$ have the same graph.

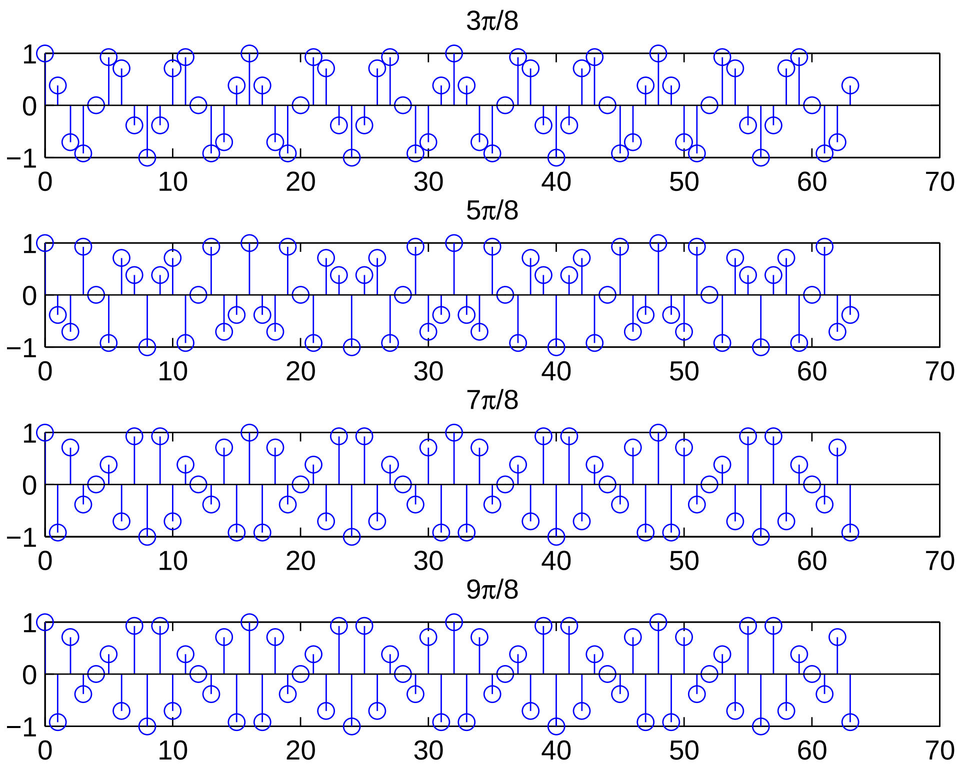
(c) $\frac{9\pi}{8} - 2\pi = -\frac{7\pi}{8}$, so $\sin\left(\frac{9\pi}{8}n\right) = -\sin\left(\frac{7\pi}{8}n\right)$. The graphs of $\sin\left(\frac{9\pi}{8}n\right)$ and $\sin\left(\frac{7\pi}{8}n\right)$ are therefore negatives of one another.

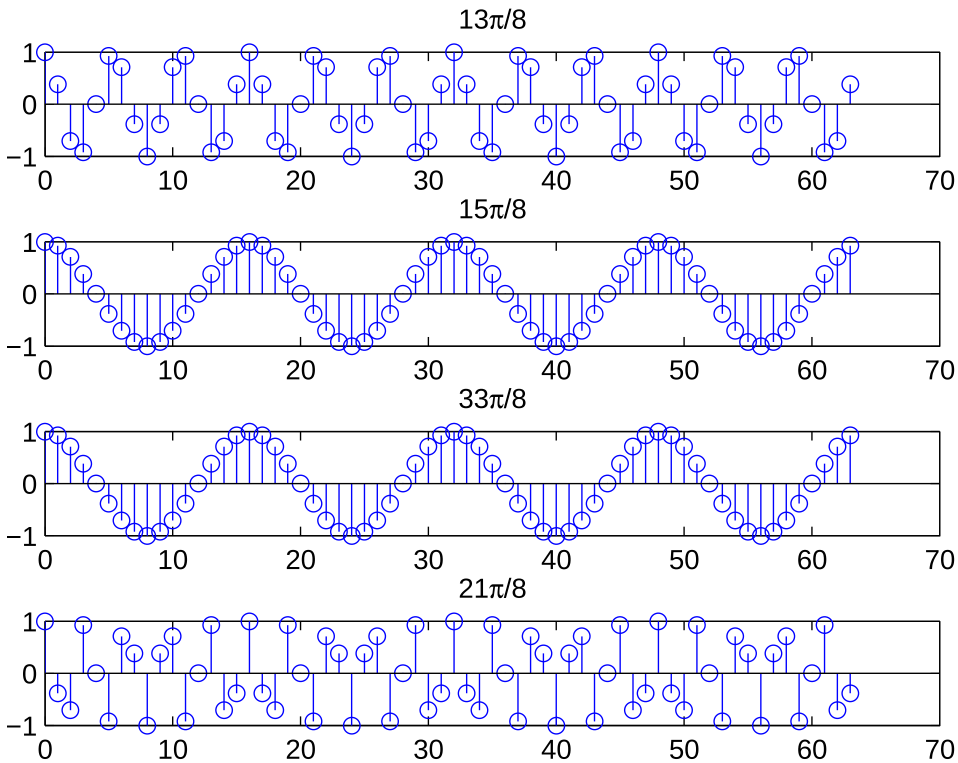
4.

(a)

```
%-----------------------------------------------------------
% P4a
%
% plot a bunch of discrete-time cosine signals
%

% The frequency will be w = k*pi/8.
% Load up the k's into a vector:
%
kvals = [-29 -3 -1 1 3 5 7 9 13 15 33 21];

% make a counter to index the "next" k to use:
next_k = 1;

n = 0:63;    % the time variable

% There are 12 k values.  We will plot four per
% figure.  So we will need three figures all
% together.  Loop on figures.

for Fig_num=1:3
  figure(Fig_num);   % selects the "current" figure

  % each time through this loop, we are going to do
  % 4 of the k's.  Loop on Sub Figure number:

  for SubFig_num = 1:4
    k = kvals(next_k);
    next_k = next_k + 1;
    w = k * pi/8;          % the frequency
    xn = cos(w*n);
    subplot(4,1,SubFig_num);
    stem(n,xn);
    title(sprintf('%d%s',k,'\pi/8'));
  end   % for SubFig_num
end   % for Fig_num
```


−29π/8
−3π/8
−1π/8
1π/8
1
0
−1
0
10
20
30
40
50
60
70


3π/8
5π/8
7π/8
9π/8
1
0
−1
0
10
20
30
40
50
60
70




(b) For $\omega_0 = -\frac{3\pi}{8}$, $-\frac{\pi}{8}$, $\frac{\pi}{8}$, $\frac{3\pi}{8}$, $\frac{5\pi}{8}$, and $\frac{7\pi}{8}$, we have that $-\pi \leq \omega_0 \leq \pi$. So, as in the last problem, these discrete sinusoids are not aliased.

The rest of the frequencies $\omega_0$ are outside the range $[-\pi, \pi]$, so these signals will be aliased. Remember, when $\omega_0$ is outside the range $[-\pi, \pi]$, then the graph of $\cos(\omega_0 n)$ is the same as the graph of $\cos(\omega_1 n)$ for another frequency $\omega_1$ that is **inside** the range $[-\pi, \pi]$ and differs from $\omega_0$ by an integer multiple of $2\pi$.

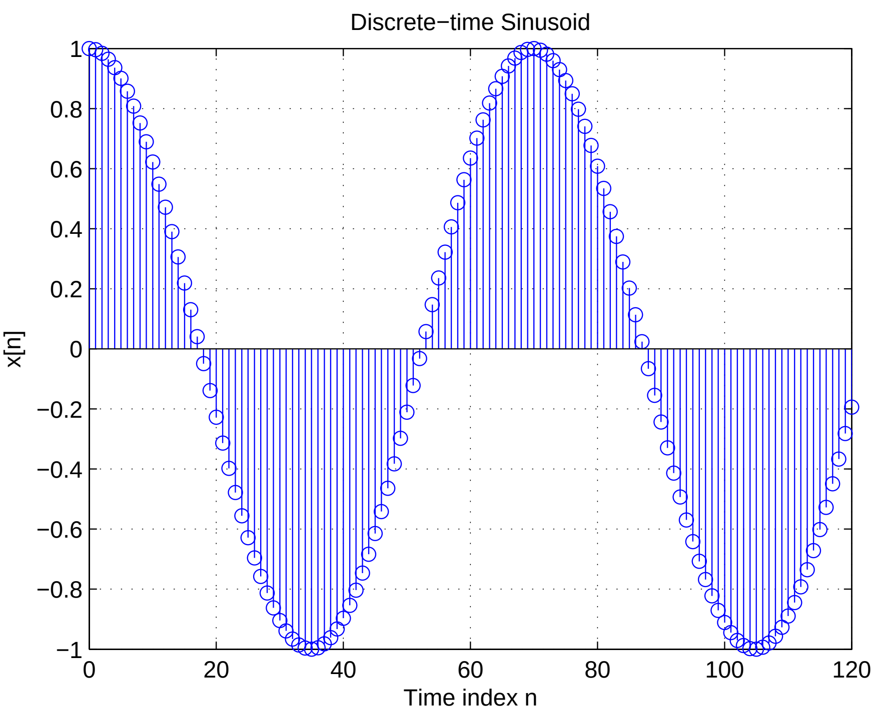
- Since $\cos(\omega_0 n) = \cos(-\omega_0 n)$, the graph of $\cos\left(-\frac{3\pi}{8}n\right)$ is the same as the graph of $\cos\left(\frac{3\pi}{8}n\right)$. They are two different ways of writing the same signal.
- Likewise, the graph of $\cos\left(-\frac{\pi}{8}n\right)$ is the the sams as the graph of $\cos\left(\frac{\pi}{8}n\right)$.
- For the rest of the frequencies, $|\omega_0| > \pi$, so each of these signals is just a *different name* for a discrete sinusoid whose frequency *is* in the range $[-\pi, \pi]$.
- $\frac{9\pi}{8} - 2\pi = -\frac{7\pi}{8}$. So, since cosine is even, the graphs of $\cos\left(\frac{9\pi}{8}n\right)$ and $\cos\left(\frac{7\pi}{8}n\right)$ are the same. They are two different ways of writing the same signal.
- $\frac{13\pi}{8} - 2\pi = -\frac{3\pi}{8}$, so the graph of $\cos\left(\frac{13\pi}{8}n\right)$ is the same as the graph of $\cos\left(-\frac{3\pi}{8}n\right)$ and the same as the graph of $\cos\left(\frac{3\pi}{8}n\right)$.
- Likewise, $\frac{15\pi}{8} - 2\pi = -\frac{\pi}{8}$, so the graph of $\cos\left(\frac{15\pi}{8}n\right)$ is the same as the graph of $\cos\left(-\frac{\pi}{8}n\right)$ and the same as the graph of $\cos\left(\frac{\pi}{8}n\right)$.
- $\frac{33\pi}{8} - 4\pi = \frac{\pi}{8}$. So $\cos\left(\frac{33\pi}{8}n\right)$ and $\cos\left(\frac{\pi}{8}n\right)$ are the same signal. Their graphs are the same and are also the same as the graph of $\cos\left(-\frac{\pi}{8}n\right)$ (and are also the same as the graph of $\cos\left(\frac{15\pi}{8}n\right)$ as just discussed).

- $-\frac{29\pi}{8} + 4\pi = \frac{3\pi}{8}$, so $\cos\left(-\frac{29\pi}{8}n\right)$ and $\cos\left(\frac{3\pi}{8}n\right)$ are the same signal; they have the same graph. Because cosine is even, it is also the same as the graph of $\cos\left(-\frac{3\pi}{8}n\right)$.
- $\frac{21\pi}{8} - 2\pi = \frac{5\pi}{8}$, so $\cos\left(\frac{21\pi}{8}n\right)$ and $\cos\left(\frac{5\pi}{8}n\right)$ have the same graph.

5.

(a)

```
%--------------------------------------------------------------
% P5a
%
% generate and plot a discrete-time cosine signal
%
n = 0:120;                      % values of the time variable
w = 0.09;                       % frequency of the sinusoid.
xn = cos(w*n);
stem(n,xn);
axis([0 120 -1.0 1.0]);
grid;
title('Discrete-time Sinusoid');
xlabel('Time index n');
ylabel('x[n]');
```



(b) We have:

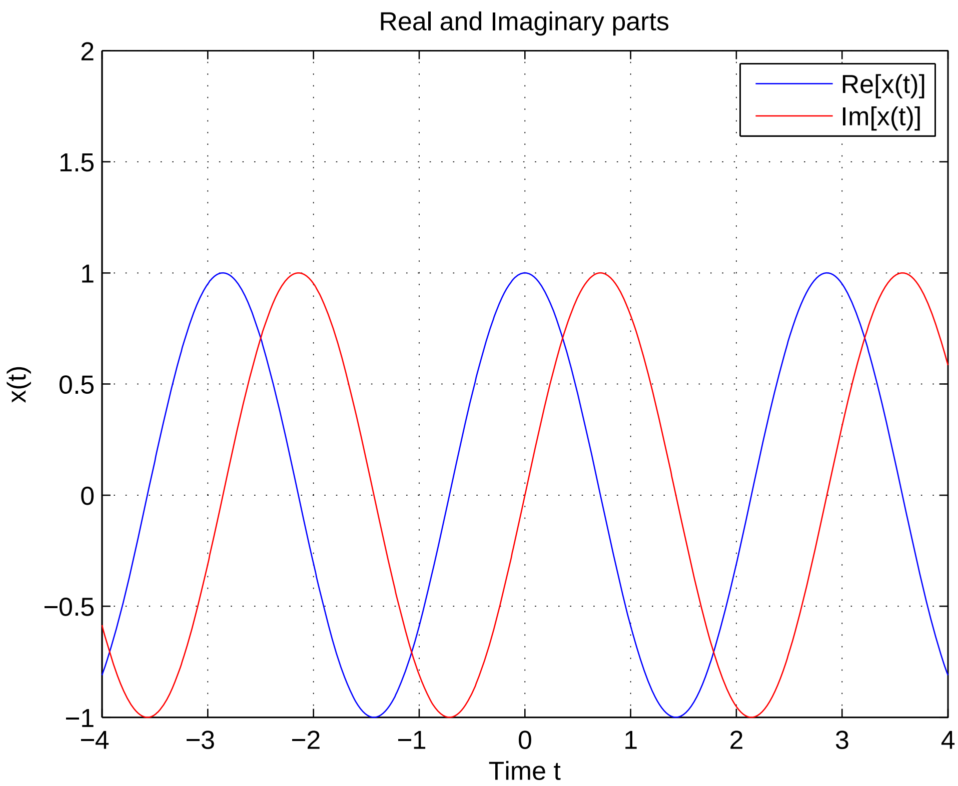
$$\frac{\omega}{2\pi} = \frac{0.09}{2\pi} \notin \mathbb{Q}.$$

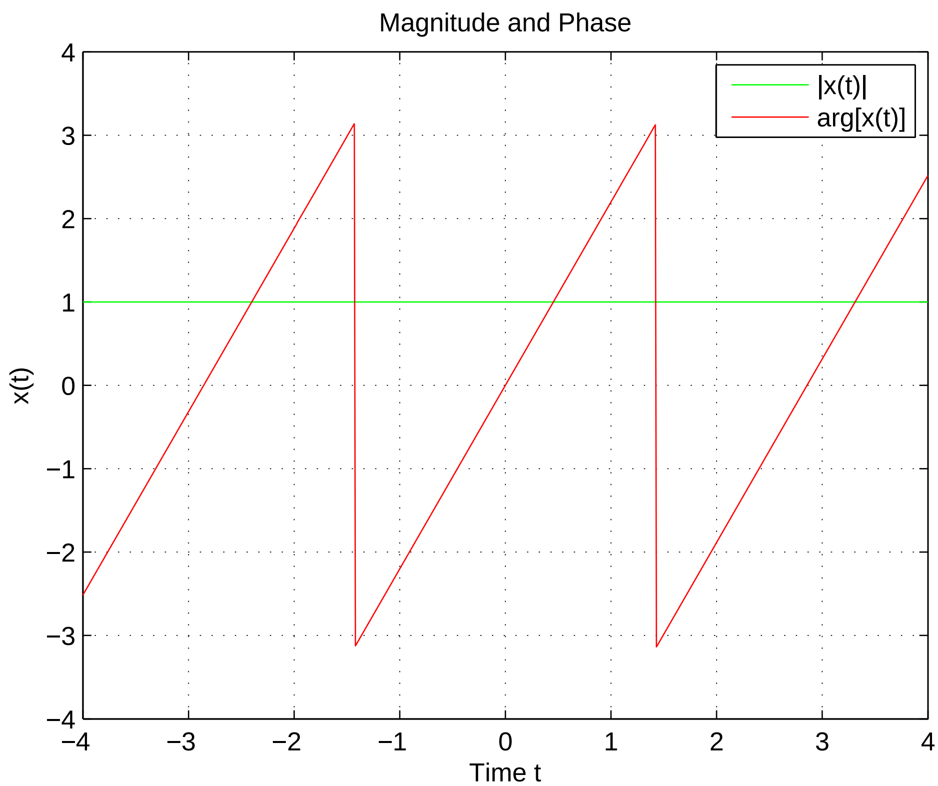
The signal is therefore **not periodic**.

6.

(a)

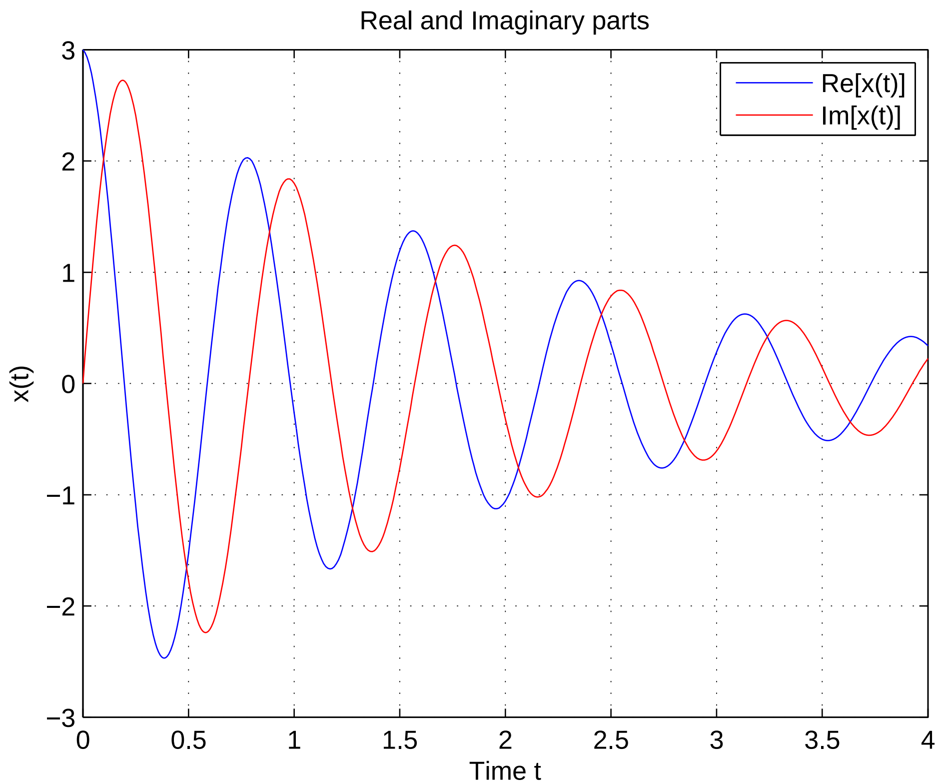
```
%-------------------------------------------------------------
% P6a
%
% generate and plot a continous-time complex sinusoid
%
t = -4:0.01:4;        % values of the time variable
w = 2.2;              % frequency of the sinusoid.
xt = exp(j*w*t);
xtR = real(xt);
xtI = imag(xt);
figure(1);             % make Fig 1 active
plot(t,xtR,'-b');      % '-b' means 'solid blue line'
axis([-4 4 -1.0 2.0]);
grid;
hold on;               % add more curves to the same graph
plot(t,xtI,'-r');                    % 'r' = red
title('Real and Imaginary parts');
xlabel('Time t');
ylabel('x(t)');
legend('Re[x(t)]','Im[x(t)]');
hold off;

mag = abs(xt);
phase = angle(xt);
figure(2);             % make Fig 2 active
plot(t,mag,'-g');      % '-' = solid line; 'g' = green
grid;
hold on;               % add more curves to the graph
plot(t,phase,'-r');              % 'r' = red
title('Magnitude and Phase');
legend('|x(t)|','arg[x(t)]');
xlabel('Time t');
ylabel('x(t)');
hold off;
```

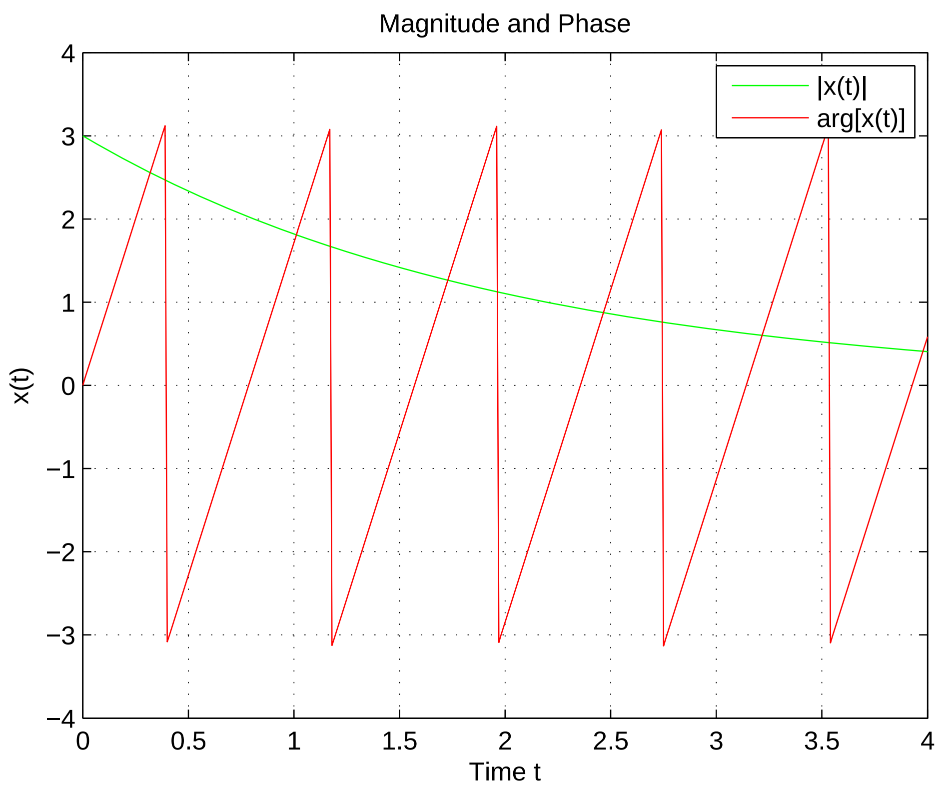

Real and Imaginary parts
Re[x(t)]
Im[x(t)]
x(t)
Time t


Magnitude and Phase
|x(t)|
arg[x(t)]
x(t)
Time t

(b)

```
%----------------------------------------------------------------
% P6b
%
% generate and plot a continous-time damped complex exponential
%
t = 0:0.01:4;                       % values of the time variable
w = 8.0;                            % frequency of the sinusoid.
xt = 3.0*exp(-t/2).*exp(j*w*t);
xtR = real(xt);
xtI = imag(xt);
figure(1);              % make Fig 1 active
plot(t,xtR,'-b');       % '-b' means 'solid blue line'
axis([0 4 -3 3]);
grid;
hold on;                % add more curves to the same graph
plot(t,xtI,'-r');                    % 'r' = red
title('Real and Imaginary parts');
xlabel('Time t');
ylabel('x(t)');
legend('Re[x(t)]','Im[x(t)]');
hold off;

mag = abs(xt);
phase = angle(xt);
figure(2);              % make Fig 2 active
plot(t,mag,'-g');       % '-' = solid line; 'g' = green
grid;
hold on;                % add more curves to the graph
plot(t,phase,'-r');                % 'r' = red
title('Magnitude and Phase');
legend('|x(t)|','arg[x(t)]');
xlabel('Time t');
ylabel('x(t)');
hold off;
```


Real and Imaginary parts
Re[x(t)]
Im[x(t)]
x(t)
Time t
3
2
1
0
−1
−2
−3
0
0.5
1
1.5
2
2.5
3
3.5
4


Magnitude and Phase
|x(t)|
arg[x(t)]
x(t)
Time t
4
3
2
1
0
−1
−2
−3
−4
0
0.5
1
1.5
2
2.5
3
3.5
4