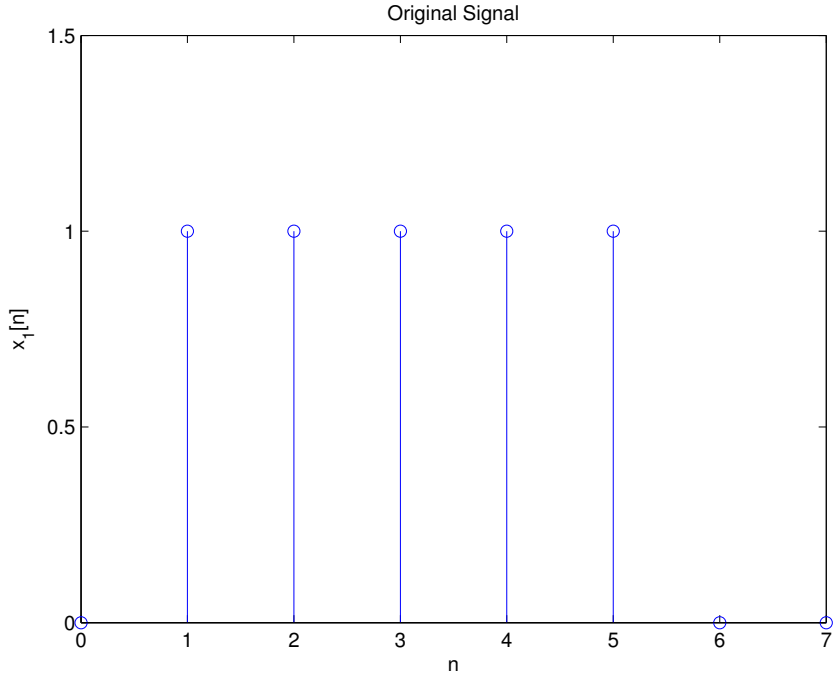
ECE 2713
Homework 6 Solution

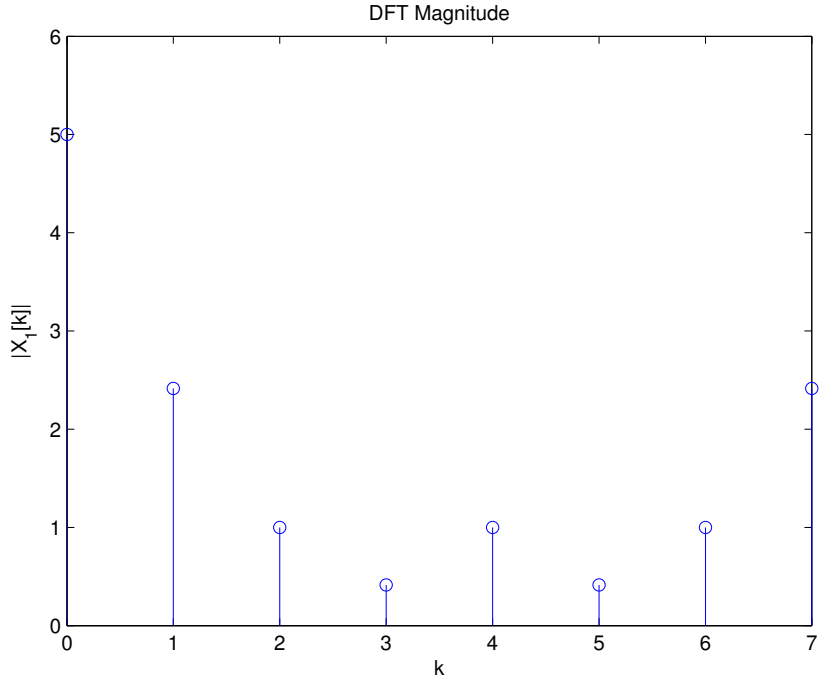
Spring 2024 Dr. Havlicek

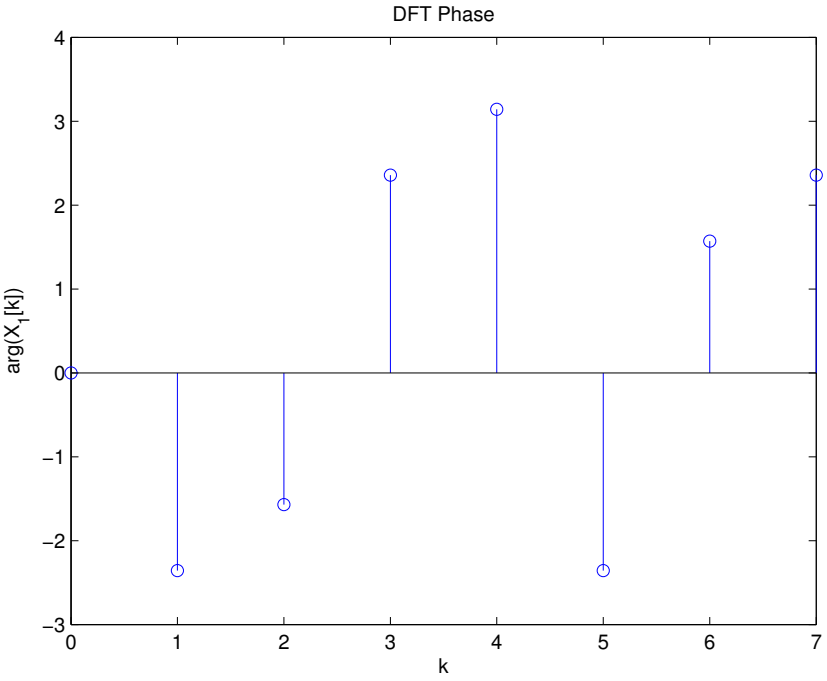
1. Matlab code:

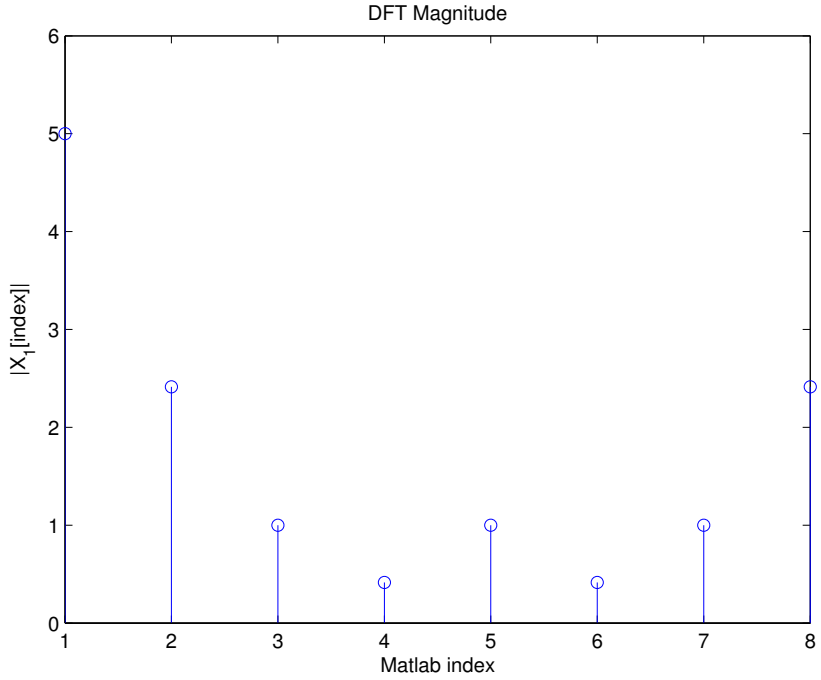
```
%-------------------------------------------------------
% P1
%
% - Create and plot the signal x_1[n] as a function of n.
% - Compute the DFT X_1[k].  Plot the magnitude and phase
%   as functions of k.
% - Plot the DFT magnitude as a function of the matlab
%   array index.
% - Plot the DFT magnitude as a function of the discrete
%   radian frequency w.
% - Compute and plot the IDFT.
%
n = 0:7;                          % time variable
x1n = [0 1 1 1 1 1 0 0];          % our 8-point signal
X1k = fft(x1n);                   % compute the DFT
X1kmag = abs(X1k);                % magnitude of the DFT
X1karg = angle(X1k);              % phase of the DFT

% plot the signal
figure(1);
stem(n,x1n);
axis([0 7 0 1.5]);
title('Original Signal');
xlabel('n');
ylabel('x_1[n]');

% plot DFT magnitude and phase as functions of k
k = 0:7;                          % frequency index
figure(2);
stem(k,X1kmag); ylim([0 6]);
title('DFT Magnitude');
xlabel('k');
ylabel('|X_1[k]|');
figure(3);
stem(k,X1karg);
title('DFT Phase');
xlabel('k');
ylabel('arg(X_1[k])');
```

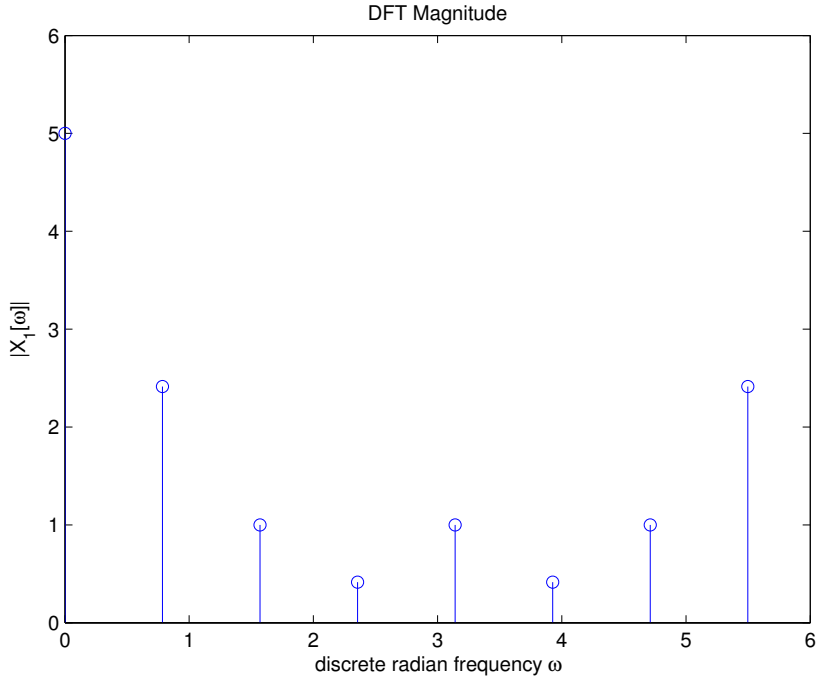
```
% plot DFT magnitude as a function of Matlab index
Matlab_idx = [1:8];               % Matlab index
figure(4);
stem(Matlab_idx,X1kmag); ylim([0 6]);
title('DFT Magnitude');
xlabel('Matlab index');
ylabel('|X_1[index]|');

% plot DFT magnitude as a function of discrete frequency
%  (radians per sample)
w = [0:2*pi/8:7*2*pi/8];          % discrete frequency
figure(5);
stem(w,X1kmag); ylim([0 6]);
title('DFT Magnitude'); ylim([0 6]);
xlabel('discrete radian frequency \omega');
ylabel('|X_1[\omega]|');

% Compute and plot the IDFT
x2n = ifft(X1k);
figure(6);
stem(n,x2n);
axis([0 7 0 1.5]);
title('IDFT');
xlabel('n');
ylabel('IDFT');
```

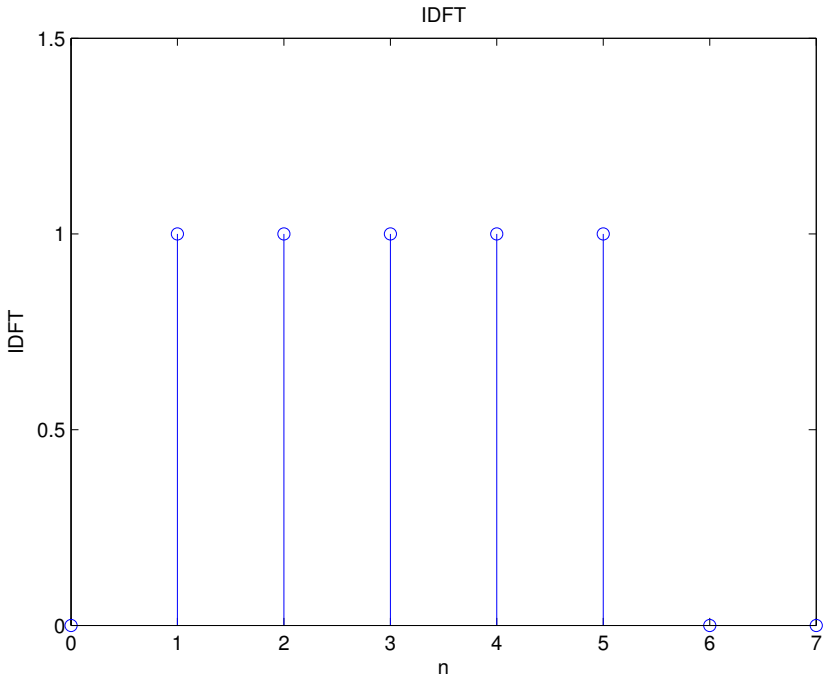



DFT Magnitude
|X1[k]|
k
0
1
2
3
4
5
6
7


DFT Phase
arg(X1[k])
k
−3
−2
−1
0
1
2
3
4
0
1
2
3
4
5
6
7


DFT Magnitude
|X1[index]|
Matlab index
0
1
2
3
4
5
6
7
8

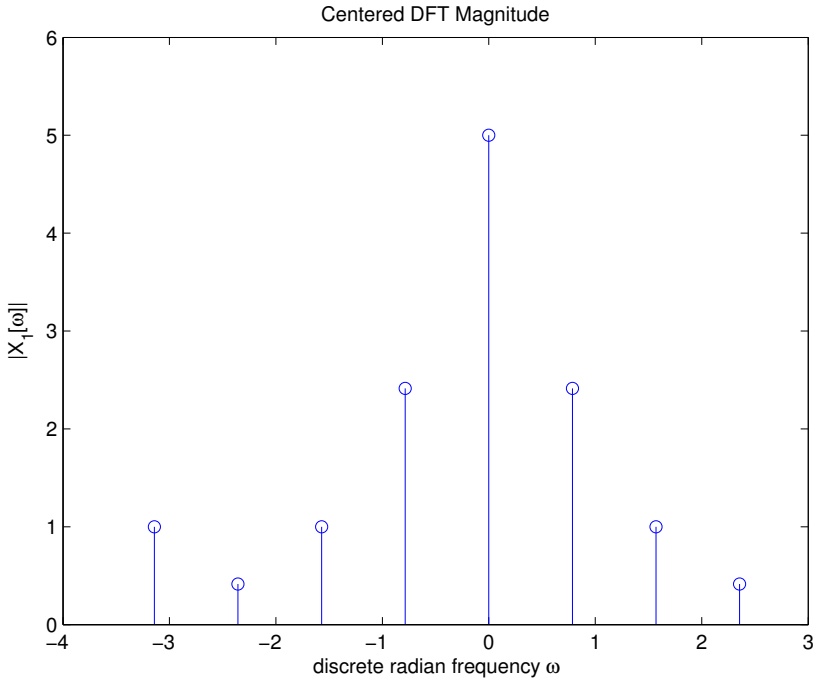

DFT Magnitude
|X1[ω]|
discrete radian frequency ω
0
1
2
3
4
5
6

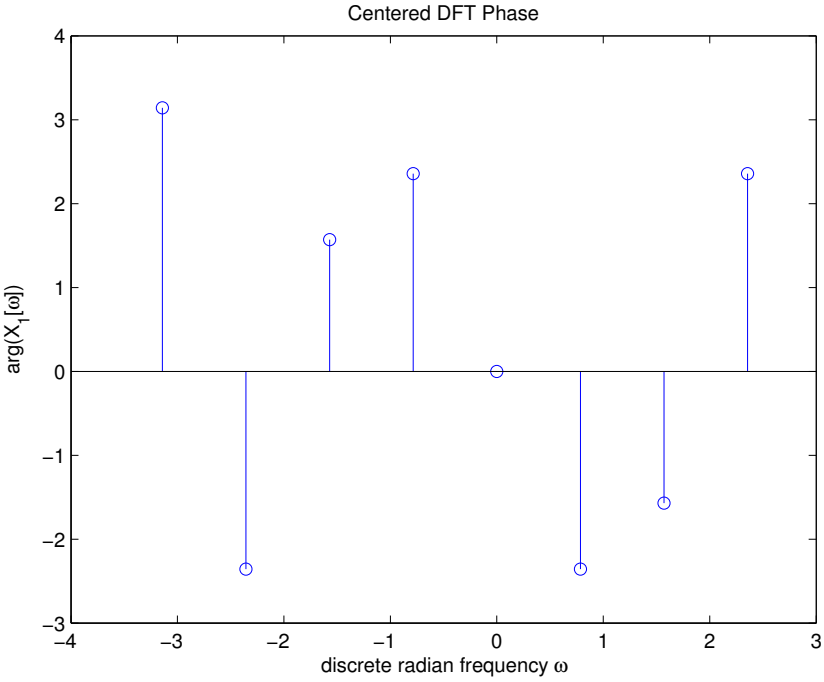


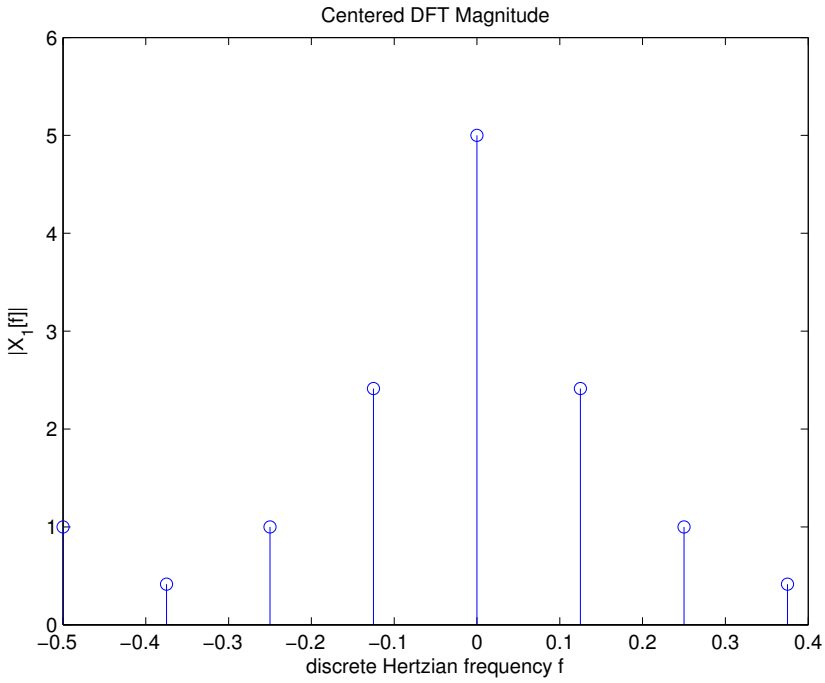
2. Matlab code:

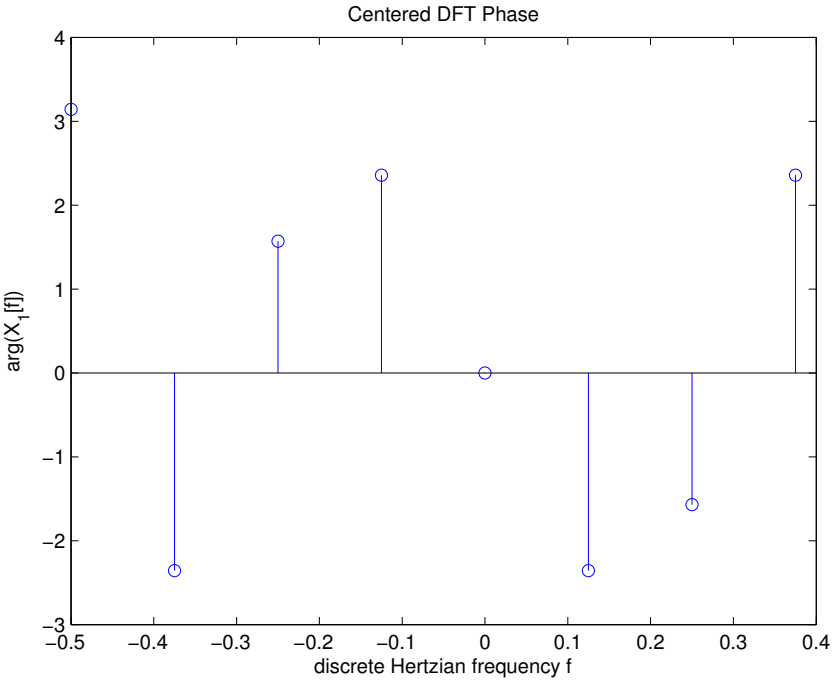
```
%-------------------------------------------------------
% P2
%
% - Compute the centered DFT of x_1[n].
% - Plot the centered magnitude and phase as function of
%   - the discrete radian frequency w,
%   - the discrete hertzian frequency f.
% - Invert the DFT and plot.
%
n = 0:7;                          % time variable
x1n = [0 1 1 1 1 1 0 0];          % our 8-point signal
X1k = fftshift(fft(x1n));         % compute the centered DFT
X1kmag = abs(X1k);                % magnitude of the centered DFT
X1karg = angle(X1k);              % phase of the centered DFT

% plot centered DFT magnitude & phase as functions of radian freq
w = [-4*2*pi/8:2*pi/8:3*2*pi/8];  % radian discrete freq
figure(1);
stem(w,X1kmag); ylim([0 6]);
title('Centered DFT Magnitude');
xlabel('discrete radian frequency \omega');
ylabel('|X_1[\omega]|');
figure(2);
stem(w,X1karg);
```

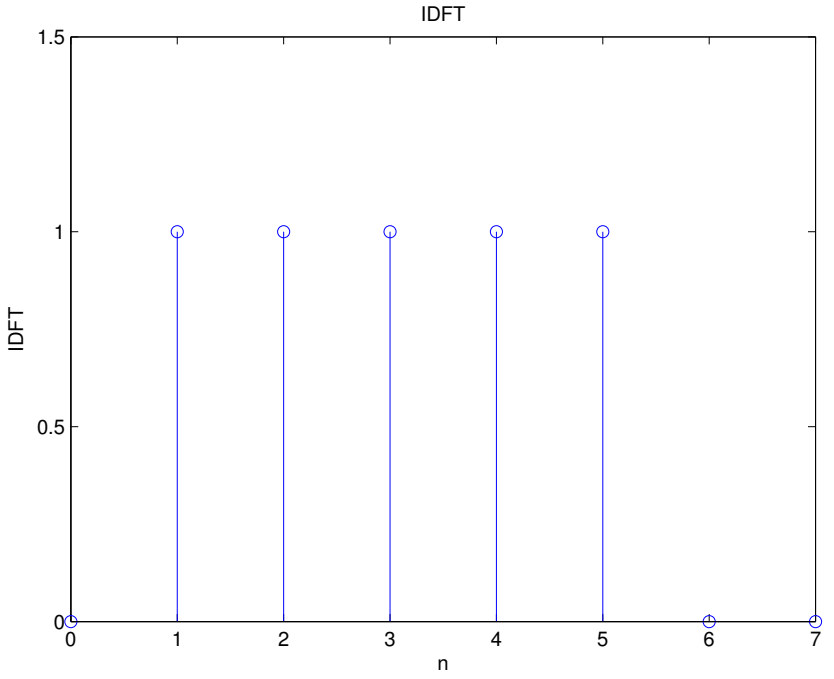
```
title('Centered DFT Phase');
xlabel('discrete radian frequency \omega');
ylabel('arg(X_1[\omega])');

% plot centered DFT magnitude & phase as functions of
%   Hertzian freq
f = [-0.5:1/8:3/8];               % Hertzian discrete freq
figure(3);
stem(f,X1kmag); ylim([0 6]);
title('Centered DFT Magnitude');
xlabel('discrete Hertzian frequency f');
ylabel('|X_1[f]|');
figure(4);
stem(f,X1karg);
title('Centered DFT Phase');
xlabel('discrete Hertzian frequency f');
ylabel('arg(X_1[f])');

% Compute and plot the IDFT
x2n = ifft(ifftshift(X1k));
figure(5);
stem(n,x2n);
axis([0 7 0 1.5]);
title('IDFT');
xlabel('n');
ylabel('IDFT');
```




Centered DFT Phase
arg(X1[ω])
discrete radian frequency ω
4
3
2
1
0
−1
−2
−3
−4
−3
−2
−1
0
1
2
3

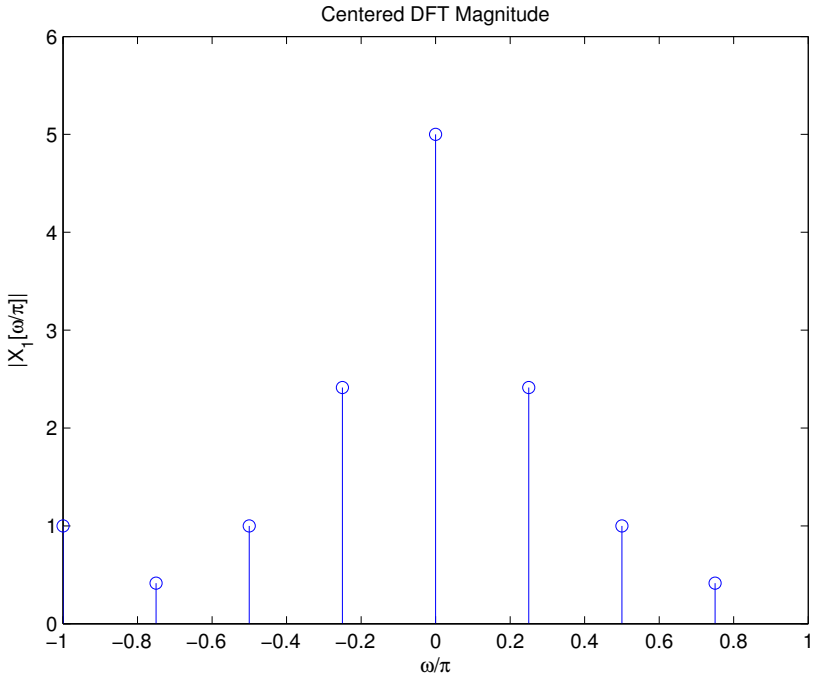

Centered DFT Magnitude
|X1[f]|
discrete Hertzian frequency f
6
5
4
3
2
1
0
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4


Centered DFT Phase
arg(X1[f])
discrete Hertzian frequency f
4
3
2
1
0
−1
−2
−3
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4


IDFT
IDFT
n
1.5
1
0.5
0
0
1
2
3
4
5
6
7

3. Matlab code:

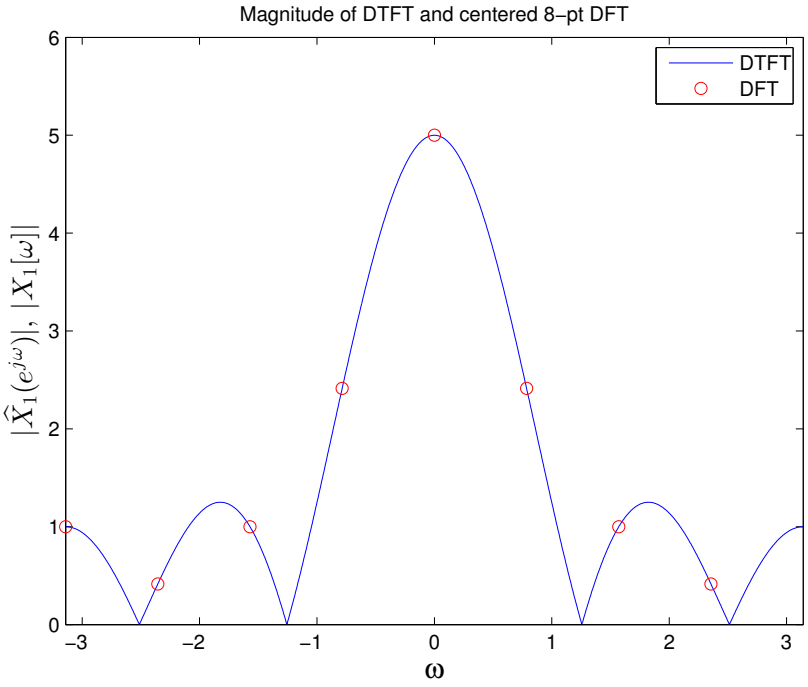
```
%-------------------------------------------------------
% P3
%
% Plot the centered DFT magnitude of x_1[n] using
% normalized radian frequency.
%
x1n = [0 1 1 1 1 1 0 0];          % our 8-point signal
X1k = fftshift(fft(x1n));         % compute the centered DFT
X1kmag = abs(X1k);                % magnitude of the centered DFT
X1karg = angle(X1k);              % phase of the centered DFT

w = [-4*2*pi/8:2*pi/8:3*2*pi/8];  % radian discrete freq
figure(1);
stem(w/pi,X1kmag);
axis([-1 1 0 6]);
title('Centered DFT Magnitude');
xlabel('\omega/\pi');
ylabel('|X_1[\omega/\pi]|');
```

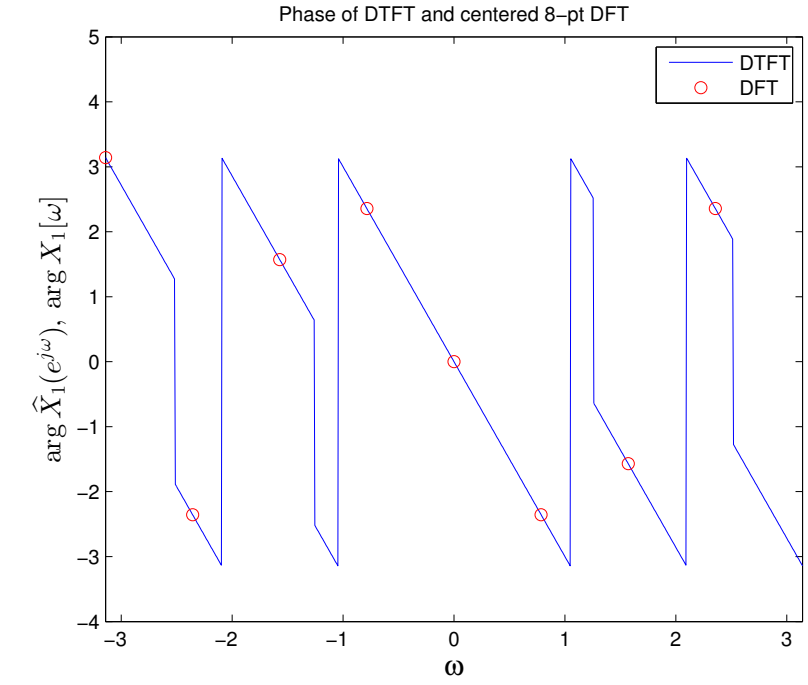


4. (a) Matlab code:

```
%-------------------------------------------------------
% P4a
%
% Show that the DFT of the finite-length signal is given
% by samples of the DTFT of the zero-extended signal.
% - plot the DTFT magnitude of x1hat from -pi to pi.
%   - plot the centered DFT magnitude of x_1[n] on the
%     same graph.
% -plot the DTFT phase of x1hat from -pi to pi.
%   - plot the centered DFT phase of x_1[n] on the same
%     graph.
%

% Frequency vector for plotting the DTFT.  Use 1000 points.
w = linspace(-pi,pi,1000);

% The DTFT was computed analytically (i.e., with paper and
% pencil - not by computer)
X1hat = sin(2.5*w)./sin(w/2) .* exp(-3*j*w);
X1hatmag = abs(X1hat);
X1hatarg = angle(X1hat);

% Now compute the 8-point DFT of the finite-length signal
x1n = [0 1 1 1 1 1 0 0];  % our 8-point signal
k =  -4:3;           % frequency index for the centered DFT
X1k = fftshift(fft(x1n));
X1kmag = abs(X1k);
X1karg = angle(X1k);

figure(1);
plot(w,X1hatmag,'-b');    % plot the DTFT magnitude
axis([-pi pi 0 6]);
hold on;                  % makes the next plot come out on the
                          %   same graph
plot(k*2*pi/8,X1kmag,'ro');  % plot the centered DFT magnitude
hold off;                    %   using a symbol, but no line
                             %   and no stem.
title('Magnitude of DTFT and centered 8-pt DFT');
xlabel('\omega','FontSize',14);
ylabel('$|\widehat X_1(e^{j\omega})|$, $|X_1[\omega]|$',...
       'Interpreter','latex','FontSize',14);
legend('DTFT','DFT');

figure(2);
plot(w,X1hatarg,'-b');    % plot the DTFT phase
axis([-pi pi -4 5]);
```

```
hold on;
plot(k*2*pi/8,X1karg,'ro');  % plot the centered DFT phase
hold off;
title('Phase of DTFT and centered 8-pt DFT');
xlabel('\omega','FontSize',14);
ylabel('$\arg\widehat X_1(e^{j\omega})$, $\arg X_1[\omega]$',...
       'Interpreter','latex','FontSize',14);
legend('DTFT','DFT');
```

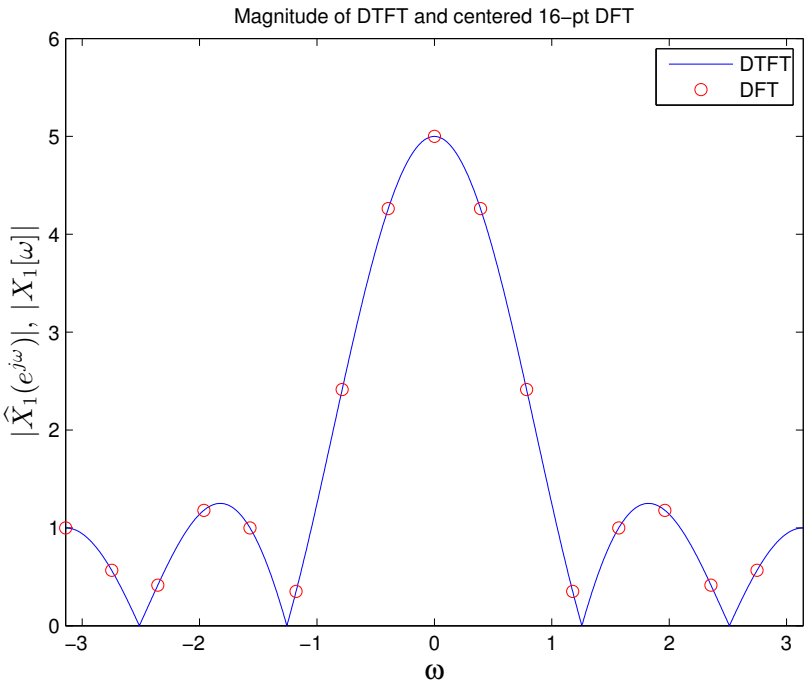

Magnitude of DTFT and centered 8−pt DFT
DTFT
DFT
$|\widehat{X}_1(e^{j\omega})|,\ |X_1[\omega]|$
ω

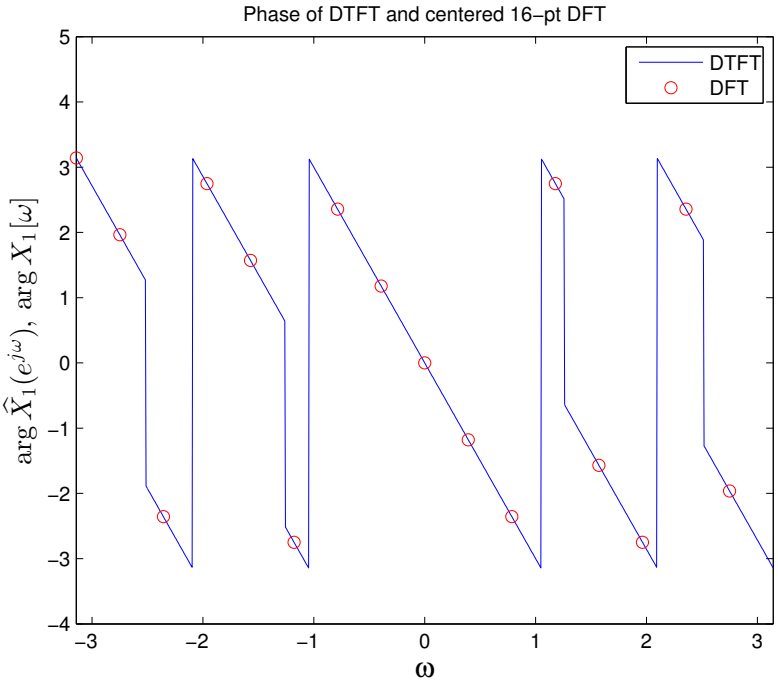

Phase of DTFT and centered 8−pt DFT
DTFT
DFT
$\arg \widehat{X}_1(e^{j\omega}),\ \arg X_1[\omega]$
ω

(b) Matlab code:

```
%-------------------------------------------------------
% P4b
%
% Change the length of x_1[n] to N=16.
%
% - plot the DTFT magnitude of x1hat from -pi to pi.
%   - plot the centered DFT magnitude of x_1[n] on the
%     same graph.
% -plot the DTFT phase of x1hat from -pi to pi.
%   - plot the centered DFT phase of x_1[n] on the same
%     graph.
%

% Frequency vector for plotting the DTFT.  Use 1000 points.
w = linspace(-pi,pi,1000);

% The DTFT was computed analytically
X1hat = sin(2.5*w)./sin(w/2) .* exp(-3*j*w);
X1hatmag = abs(X1hat);
X1hatarg = angle(X1hat);

% Now compute the 16-point DFT
x1n = [0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0];  % 16-point signal
k =  -8:7;           % frequency index for the centered DFT
X1k = fftshift(fft(x1n));
X1kmag = abs(X1k);
X1karg = angle(X1k);

figure(1);
plot(w,X1hatmag,'-b');    % plot the DTFT magnitude
axis([-pi pi 0 6]);
hold on;                  % makes the next plot come out on the
                          %   same graph
plot(k*2*pi/16,X1kmag,'ro');  % plot the centered DFT magnitude
hold off;                    %   using a symbol, but no line
                             %   and no stem.
title('Magnitude of DTFT and centered 16-pt DFT');
xlabel('\omega','FontSize',14);
ylabel('$|\widehat X_1(e^{j\omega})|$, $|X_1[\omega]|$',...
       'Interpreter','latex','FontSize',14);
legend('DTFT','DFT');

figure(2);
plot(w,X1hatarg,'-b');    % plot the DTFT phase
axis([-pi pi -4 5]);
hold on;
```

```
plot(k*2*pi/16,X1karg,'ro');  % plot the centered DFT phase
hold off;
title('Phase of DTFT and centered 16-pt DFT');
xlabel('\omega','FontSize',14);
ylabel('$\arg\widehat X_1(e^{j\omega})$, $\arg X_1[\omega]$',...
       'Interpreter','latex','FontSize',14);
legend('DTFT','DFT');
```




Phase of DTFT and centered 16−pt DFT
DTFT
DFT
$\arg \widehat{X}_1(e^{j\omega})$, $\arg X_1[\omega]$
ω
5
4
3
2
1
0
−1
−2
−3
−4
−3
−2
−1
0
1
2
3