ECE 3793
Test 1

Thursday, October 14, 1999
12:00 PM - 1:15 PM

Fall 1999

Dr. Havlicek

Name: SOLUTION

Student Num: ____________________

Directions: There are **five** problems on this test. Work any **four** of them. Only **four** problems will be graded. You have 75 minutes to complete the test. All work must be your own. You may use one 8.5 $\times$ 11 inch two-sided note sheet as well as the summation formula sheet handed out in class.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

Circle the numbers of the **four** problems you wish to have graded:

1. **2.** **3.** **4.** **5.**

SCORE:

1. (25) ________
2. (25) ________
3. (25) ________
4. (25) ________
5. (25) ________

TOTAL (100):

1. **25 pts.** The input $x(t)$ and output $y(t)$ of a system are related by

$$y(t) = \begin{cases} 0, & t < 0, \\ x(t) + x(t-2), & t \geq 0. \end{cases}$$

(a) **5 pts.** Is the system memoryless? Justify your answer.

The system is not memoryless, because $y(t)$ depends on the past input $x(t-2)$ when $t \geq 0$.

(b) **5 pts.** Is the system causal? Justify your answer.

The output $y(t)$ never depends on future values of the input.

Therefore, the system is causal.

(c) **5 pts.** Is the system stable? Justify your answer.

Suppose $x(t)$ is a bounded input. Then $\exists B \in \mathbb{R}, B > 0$, such that $|x(t)| \leq B \ \forall t$.

Then
$$\begin{aligned} |y(t)| &\leq |x(t) + x(t-2)| \\ &\leq |x(t)| + |x(t-2)| \\ &\leq B + B \\ &= 2B \\ &< \infty. \end{aligned}$$

$\Rightarrow$ The system is stable.

Problem 1, cont...

(d) **5 pts.** Is the system linear? Justify your answer.

Let $y_1(t) = \begin{cases} 0, & t<0 \\ x_1(t)+x_1(t-2), & t\geq 0 \end{cases}$. Let $y_2(t) = \begin{cases} 0, & t<0 \\ x_2(t)+x_2(t-2), & t\geq 0. \end{cases}$

Let $x_3(t) = ax_1(t) + bx_2(t)$, $a, b \in \mathbb{R}$.

Then $y_3(t) = \begin{cases} 0, & t<0 \\ x_3(t)+x_3(t-2), & t\geq 0 \end{cases} = \begin{cases} 0+0, & t<0 \\ ax_1(t)+bx_2(t)+ax_1(t-2)+bx_2(t-2), & t\geq 0. \end{cases}$

$= \begin{cases} 0+0, & t<0 \\ a[x_1(t)+x_1(t-2)] + b[x_2(t)+x_2(t-2)], & t\geq 0 \end{cases}$

$= ay_1(t) + by_2(t)$ ✓

The system <u>is</u> linear.

(e) **5 pts.** Is the system time invariant? Justify your answer.

Let $x_1(t) = 1 \ \forall t$. Then $x_1(t) + x_1(t-2) = 1+1 = 2$.

So $y_1(t) = \begin{cases} 0, & t<0 \\ 2, & t\geq 0 \end{cases} = 2u(t)$.

Now let $x_2(t) = x_1(t-t_0) = 1 = x_1(t)$.

NOTE: $y_1(t-t_0) = 2u(t-t_0)$.

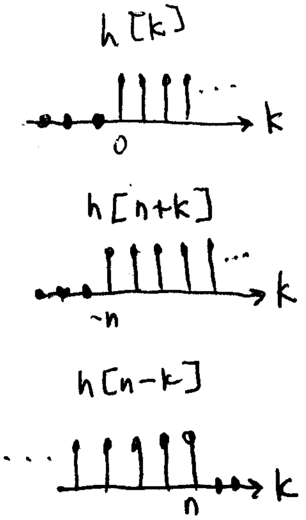
But, $y_2(t) = \begin{cases} 0, & t<0 \\ 2, & t\geq 0 \end{cases} = 2u(t) \neq y_1(t-t_0)$.

Therefore, the system is <u>not</u> time invariant.

2. **25 pts.** An invertible discrete-time LTI system H has unit pulse response

$$h[n] = u[n].$$

Find the unit pulse response $g[n]$ of the inverse system G.



For H, the output is

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]u[n-k]$$
$$= \sum_{k=-\infty}^{n} x[k]$$

So $y[n-1] = \sum_{k=-\infty}^{n-1} x[k]$

So $y[n] - y[n-1] = \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k] = x[n].$

Then $x[n] = y[n] - y[n-1]$.

Then the input/output relation for the inverse system G is

$$y[n] = x[n] - x[n-1].$$

In other words, for the system G,

$$y[n] = x[n] * \delta[n] - x[n] * \delta[n-1]$$
$$= x[n] * \{\delta[n] - \delta[n-1]\}.$$

Therefore, the unit pulse response of G is

$$g[n] = \delta[n] - \delta[n-1]$$

3. **25 pts.** The unit pulse response of a discrete-time LTI system is given by

$$h[n] = u[n].$$

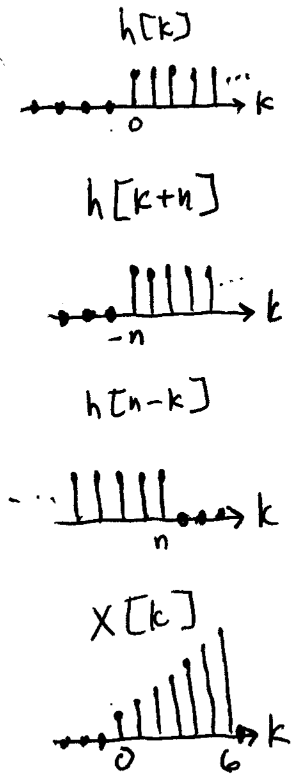
(a) **5 pts.** Is the system stable? Justify your answer.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} 1 \rightarrow \infty.$$

Therefore, the system is <u>not</u> stable.

(b) **20 pts.** Find the system output when the input is

$$x[n] = 2^n\{u[n] - u[n-7]\}. = \begin{cases} 2^n, & 0 \le n \le 6 \\ 0, & \text{otherwise.} \end{cases}$$



case I : $n < 0$: $y[n] = 0$

case II : $0 \le n < 6$:

$$y[n] = \sum_{k=0}^{n} x[k]\, h[n-k]$$

$$= \sum_{k=0}^{n} 2^k \cdot 1$$

$$= \frac{2^0 - 2^{n+1}}{1-2} = -(1-2^{n+1}) = 2^{n+1} - 1.$$

$\longrightarrow$

More Workspace for Problem 3...

case III : $6 \leq n$:

$$y[n] = \sum_{k=0}^{6} x[k]\, h[n-k]$$

$$= \sum_{k=0}^{6} 2^k \cdot 1$$

$$= \frac{2^0 - 2^7}{1-2} = -(1-2^7) = 2^7 - 1 = 128 - 1 = 127.$$

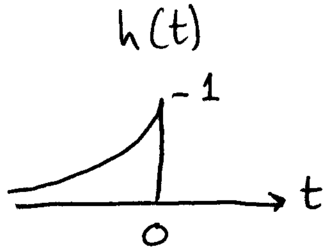
All together:

$$y[n] = \begin{cases} 0, & n < 0 \\ 2^{n+1} - 1, & 0 \leq n < 6 \\ 127, & 6 \leq n \end{cases} = \left(2^{n+1} - 1\right)\left(u[n] - u[n-7]\right)$$

4. **25 pts.** The impulse response of a continuous-time LTI system is given by

$$h(t) = e^{2t}u(-t).$$

(a) **5 pts.** Is the system causal? Justify your answer.

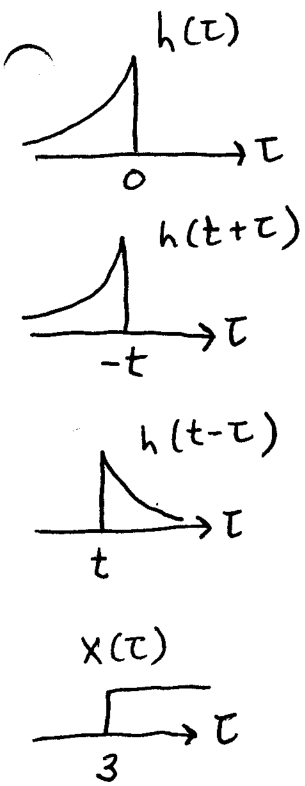


The system is <u>not</u> causal

because $h(-1) = e^{-2} \neq 0$.

(b) **20 pts.** Find the system output when the input is

$$x(t) = u(t-3).$$



Case I: $t \leq 3$:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{3}^{\infty} 1 \cdot e^{2(t-\tau)}\, d\tau$$

$$= e^{2t} \int_{3}^{\infty} e^{-2\tau}\, d\tau$$

$$= e^{2t}\left(-\tfrac{1}{2}\right)\left[e^{-2\tau}\right]_{3}^{\infty}$$

$$= -\frac{e^{2t}}{2} \lim_{A\to\infty}\left[e^{-2A} - e^{-6}\right]$$

$$= \frac{e^{2t}}{2} e^{-(2\cdot 3)} = \frac{1}{2} e^{+2(t-3)}.$$

→

Case II : $t > 3$:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{t}^{\infty} 1 \cdot e^{2(t-\tau)}\, d\tau$$

$$= e^{2t} \int_{t}^{\infty} e^{-2\tau}\, d\tau$$

$$= -\frac{e^{2t}}{2} \left[e^{-2\tau} \right]_{t}^{\infty}$$

$$= -\frac{e^{2t}}{2} \lim_{A \to \infty} \left[e^{-2A} - e^{-2t} \right]$$

$$= +\frac{e^{2t}}{2} e^{-2t} = \frac{1}{2}$$

All Together:

$$y(t) = \begin{cases} \frac{1}{2} e^{2(t-3)}, & t \leq 3 \\ \frac{1}{2}, & t > 3 \end{cases}$$

5. **25 pts.** A discrete-time system H is formed by the parallel connection of two discrete-time LTI systems H_1 and H_2. The unit pulse response $h_1[n]$ of H_1 is given by

$$h_1[n] = u[n] - u[n-47].$$

The unit pulse response $h_2[n]$ of H_2 is given by

$$h_2[n] = -u[n-1] + u[n-47].$$

Find the output $y[n]$ of the system H when the input is

$$x[n] = \sum_{m=-\infty}^{\infty} \{\delta[n-4m] - \delta[n-1-4m]\}.$$

Since H_1 and H_2 are LSI, so is H.

The impulse response of H is (for a parallel connection)

$$\begin{aligned} h[n] &= h_1[n] + h_2[n] \\ &= u[n] - u[n-47] - u[n-1] + u[n-47] \\ &= u[n] - u[n-1] \\ &= \delta[n]. \end{aligned}$$

$$\begin{aligned} y[n] &= x[n] * \delta[n] \\ &= x[n] \\ &= \sum_{m=-\infty}^{\infty} \{\delta[n-4m] - \delta[n-1-4m]\} \end{aligned}$$