ECE 3793
Test 1

Tuesday, March 13, 2007
7:30 PM - 10:30 PM

Spring 2007

Dr. Havlicek

Name: SOLUTION

Student Num:________________

Directions: This test is closed book and closed notes. Calculators are not allowed. All work must be your own. You have 180 minutes to complete the test. There are **four** problems. Work **all four**. Formulas appear at the **end** of the test.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________
2. (25) ________
3. (25) ________
4. (25) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:________________ Date:________________

1. **25 pts.** A continuous-time system H has input $x(t)$ and output $y(t)$ related by

$$y(t) = \frac{1}{2}x(-t).$$

(a) **3 pts.** Is the system H memoryless? Justify your answer.

When $t = 1$, we have $y(1) = \frac{1}{2}x(-1)$, which depends on the value of the input signal from a previous time $t = -1$. Therefore, H is NOT memoryless.

(b) **3 pts.** Is the system H causal? Justify your answer.

When $t = -1$, we have $y(-1) = \frac{1}{2}x(1)$, which depends on the value of the input signal from the future time $t = +1$. Therefore, H is NOT causal.

(c) **5 pts.** Is the system H linear? Justify your answer.

Let $x_1(t)$ and $x_2(t)$ be two arbitrary input signals.

Then $y_1(t) = H\{x_1(t)\} = \frac{1}{2}x_1(-t)$ and

$y_2(t) = H\{x_2(t)\} = \frac{1}{2}x_2(-t)$.

Let $c_1, c_2 \in \mathbb{C}$ be arbitrary constants and let

$x_3(t) = c_1 x_1(t) + c_2 x_2(t)$.

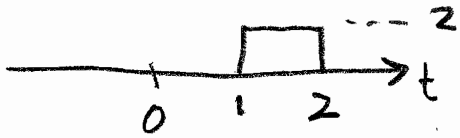
Then
$$\begin{aligned} y_3(t) = H\{x_3(t)\} &= \tfrac{1}{2}x_3(-t) \\ &= \tfrac{1}{2}[c_1 x_1(-t) + c_2 x_2(-t)\} \\ &= \tfrac{1}{2}c_1 x_1(-t) + \tfrac{1}{2}c_2 x_2(-t) \\ &= c_1 y_1(t) + c_2 y_2(t) \checkmark \end{aligned}$$

Therefore, H IS Linear.

Problem 1, cont...

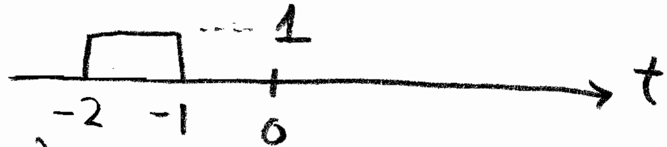
(d) **5 pts**. Is the system H time invariant? Justify your answer.

Let $x_1(t) =$



$= 2u(t-1) - 2u(t-2).$

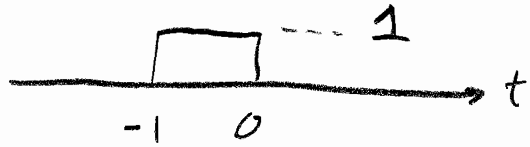
Then $y_1(t) = \frac{1}{2} x_1(-t) =$



$= u(-t-1) - u(-t-2)$

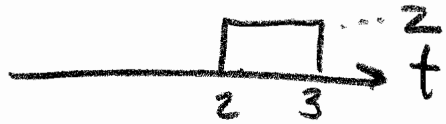
$= u(t+2) - u(t+1)$

Then $y_1(t-1) =$



$= u(t+1) - u(t).$

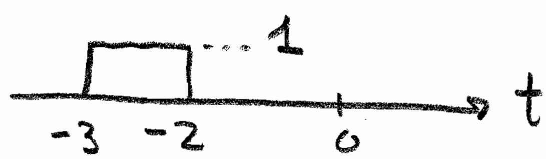
Now let $x_2(t) = x_1(t-1) =$



$= 2u(t-2) - 2u(t-3).$

Then $y_3(t) = \frac{1}{2} x_2(-t)$

$=$



$\neq y_1(t-1).$

Therefore, H is NOT Time Invariant.

Problem 1, cont...

(e) **5 pts.** Is the system H BIBO stable? Justify your answer.

Let $x(t)$ be a bounded input. Then $\exists B \in \mathbb{R}$ s.t. $B > 0$ and $|x(t)| \leq B \quad \forall t \in \mathbb{R}$.

Now, $y(t) = H\{x(t)\} = \frac{1}{2}x(-t)$,

so $|y(t)| = \frac{1}{2}|x(-t)|$. But $|x(t)|$ is always $\leq B$,

so $|y(t)| \leq \frac{1}{2}B$. Therefore $y(t)$ is bounded.

$\Rightarrow$ H is BIBO stable, because every bounded input produces a bounded output.

(f) **4 pts.** Is the system H invertible? Justify your answer.

- If you answer *yes*, then give the input-output relation for the inverse system.
- If you answer *no*, then give two distinct input signals that both result in the same output signal.

The system H does two things to the input signal: it "flips" it around the "y-axis" and it scales by $\frac{1}{2}$.

$\Rightarrow$ These can both be "undone". So I guess that H is invertible... which means I should try to solve the I/O relation for $x(t)$:

$$y(t) = H\{x(t)\} = \frac{1}{2}x(-t)$$

$$2y(t) = x(-t)$$

$\boxed{\text{let } \theta = -t}$

$$2y(-\theta) = x(\theta)$$

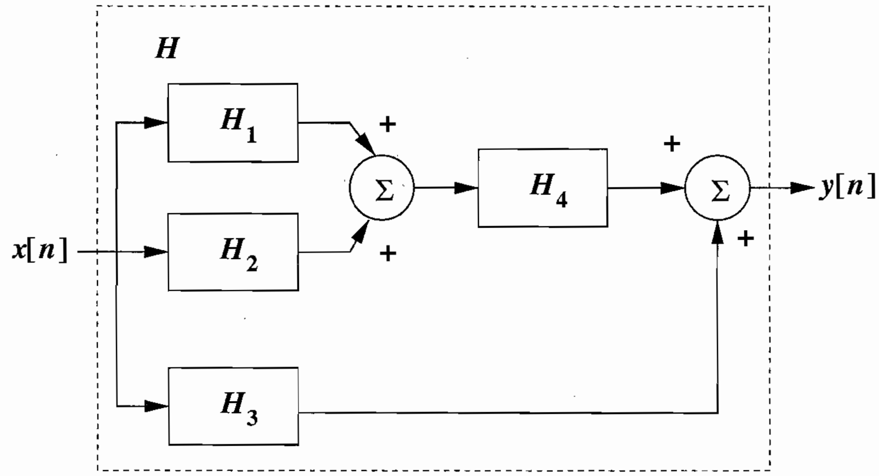
$\boxed{\text{write } t \text{ instead of } \theta}$

$$x(t) = 2y(-t).$$

Success! So the system H is invertible. To get the I/O relation for the inverse system, switch the names of input and output:

$\boxed{\text{INVERSE: } y(t) = 2x(-t)}$

2. **25 pts**. The discrete-time LTI system H is formed by connecting four LTI systems H_1, H_2, H_3, and H_4 as shown in the figure below.



It follows immediately from results proven in class that the overall system H is LTI. The impulse responses of the four LTI systems H_1 through H_4 are given by

$$\begin{aligned} h_1[n] &= 2^n u[-n], \\ h_2[n] &= \left(\frac{1}{2}\right)^n u[n-1], \\ h_3[n] &= \left(\frac{1}{3}\right)^n u[n], \\ h_4[n] &= \frac{1}{2}\delta[n-1]. \end{aligned}$$

(a) **12 pts.** Find the impulse response $h[n]$.

$$h[n] = \big(h_1[n] + h_2[n]\big) * h_4[n] + h_3[n]$$

$$= h_1[n] * h_4[n] + h_2[n] * h_4[n] + h_3[n]$$

$$= \tfrac{1}{2} h_1[n-1] + \tfrac{1}{2} h_2[n-1] + h_3[n]$$

$$= \tfrac{1}{2}(2)^{n-1} u[-n+1] + \tfrac{1}{2}\left(\tfrac{1}{2}\right)^{n-1} u[n-2] + \left(\tfrac{1}{3}\right)^n u[n]$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2}\right)^{-(n-1)} u[-n+1] + \tfrac{1}{2}\left(\tfrac{1}{2}\right)^{n-1} u[n-2] + \left(\tfrac{1}{3}\right)^n u[n] \quad (*)$$

$$= \tfrac{1}{2}\left\{ \left(\tfrac{1}{2}\right)^{-(n-1)} \underbrace{u[-n+1]}_{\text{"on" for } n \le 1} + 5\left(\tfrac{1}{2}\right)^{n-1} \underbrace{u[n-2]}_{\text{on for } \longrightarrow} \right\} + \left(\tfrac{1}{3}\right)^n u[n]$$

Problem 2, cont. . .

$$\ldots \boxed{h[n] = \frac{1}{2}\left(\frac{1}{2}\right)^{|n-1|} + \left(\frac{1}{3}\right)^n u[n]}$$

Note: from (*) on page 5, $h[n]$ can also be written as

$$\ldots h[n] = \frac{1}{2}\left(\frac{1}{2}\right)^{-n}\left(\frac{1}{2}\right)u[-n+1] + \frac{1}{2}\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^{-1}u[n-2] + \left(\frac{1}{3}\right)^n u[n]$$

$$= \frac{1}{4}2^n \underbrace{u[-n+1]}_{\text{"on" for } n \leq 1} + \left(\frac{1}{2}\right)^n \underbrace{u[n-2]}_{\text{"on" for } n \geq 2} + \left(\frac{1}{3}\right)^n \underbrace{u[n]}_{\text{"on" for } n \geq 0}$$

$$= \begin{cases} \frac{1}{4}\cdot 2^n, & n < 0 \\ \frac{1}{4}\cdot 2^n + \left(\frac{1}{3}\right)^n, & 0 \leq n < 2 \\ \left(\frac{1}{2}\right)^n + \left(\frac{1}{3}\right)^n, & n \geq 2 \end{cases}$$

(b) **3 pts**. Is the system H memoryless? Justify your answer.

A discrete-time LTI system is memoryless iff the impulse response is a constant multiple of $\delta[n]$. As shown in (a), that's <u>not</u> the case here. So H is <u>not</u> memoryless.

(c) **3 pts**. Is the system H causal? Justify your answer.

A discrete-time LTI system H is causal iff $h[n] = 0 \ \forall \ n < 0$.

Here, we have $h[-1] = \frac{1}{4}\cdot 2^{-1} = \frac{1}{8} \neq 0$.

Therefore H is <u>NOT</u> causal.

Problem 2, cont...

(d) **7 pts.** Is the system H BIBO stable? Justify your answer.

A discrete LTI system H is BIBO stable iff $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$. Here, we have;

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{-1} \left|\left(\tfrac{1}{4}\right)2^n\right| + \sum_{n=0}^{1} \left|\left(\tfrac{1}{4}\right)2^n + \left(\tfrac{1}{3}\right)^n\right| + \sum_{n=2}^{\infty}\left|\left(\tfrac{1}{2}\right)^n + \left(\tfrac{1}{3}\right)^n\right|$$

$$= \sum_{n=-\infty}^{-1} \left(\tfrac{1}{4}\right)2^n + \sum_{n=0}^{1} \left(\tfrac{1}{4}\right)2^n + \left(\tfrac{1}{3}\right)^n + \sum_{n=2}^{\infty} \left(\tfrac{1}{2}\right)^n + \left(\tfrac{1}{3}\right)^n$$

$$= \frac{1}{4}\sum_{n=-\infty}^{-1} 2^n + \frac{1}{4}2^0 + \left(\tfrac{1}{3}\right)^0 + \frac{1}{4}2^1 + \left(\tfrac{1}{3}\right)^1 + \sum_{n=2}^{\infty}\left(\tfrac{1}{2}\right)^n + \sum_{n=2}^{\infty}\left(\tfrac{1}{3}\right)^n$$

$$= \frac{1}{4}\lim_{A\to\infty}\frac{2^{-A} - 2^0}{1-2} + \frac{1}{4} + 1 + \frac{1}{2} + \frac{1}{3} + \lim_{A\to\infty}\frac{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^{A+1}}{1-\frac{1}{2}} + \lim_{A\to\infty}\frac{\left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^{A+1}}{1-1/3}$$

$$= \frac{1}{4}\,\frac{-1}{-1} + \frac{3+12+6+4}{12} + \frac{1/4}{1-1/2} + \frac{1/9}{1-1/3} = \frac{1}{4} + \frac{25}{12} + \frac{\frac{1}{4}}{1/2} + \frac{1/9}{2/3}$$

$$= \frac{1}{4} + \frac{25}{12} + \frac{1}{2} + \frac{3}{18} = \frac{3+25+6}{12} + \frac{1}{6} = \frac{34}{12} + \frac{2}{12}$$

$$= \frac{36}{12} = 3 < \underline{\underline{\infty}}.$$

Therefore, H <u>is</u> BIBO Stable

3. **25 pts.** A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \begin{cases} -1, & -3 \le n \le -1 \\ 0, & \text{otherwise} \end{cases} = -u[n+3] + u[n].$$

The system input is given by

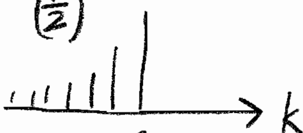
$$x[n] = 2^{-n}u[-n].$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

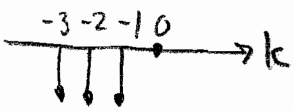
Find the system output $y[n]$.

$x[k]$

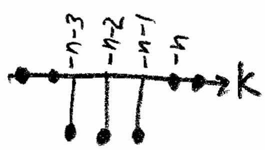
$\left(\frac{1}{2}\right)^k$



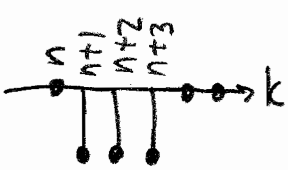
$h[k]$



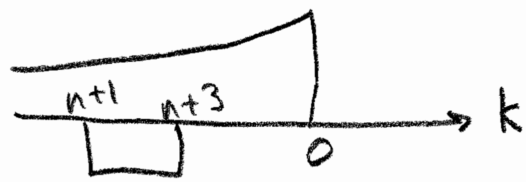
$h[k - -n] = h[n+k]$



$h[n-k]$



I) $n+3<0 \; : \; n<-3$



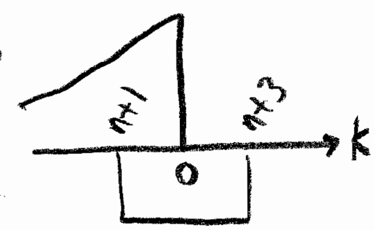
$$y[n] = \sum_{k=n+1}^{n+3} \left(\tfrac{1}{2}\right)^k(-1) = -\sum_{k=n+1}^{n+3}\left(\tfrac{1}{2}\right)^k = -\frac{\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{2}\right)^{n+3+1}}{1-\frac{1}{2}}$$

$$= -\frac{\frac{1}{2}\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^n}{1/2} = -\left(\tfrac{1}{2}\right)^n + \left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right)^n$$

$$= \left[\tfrac{1}{8} - 1\right]\left(\tfrac{1}{2}\right)^n = -\tfrac{7}{8}\left(\tfrac{1}{2}\right)^n$$

II) $n+3 \geq 0$ and $n+1<1 \; : \; n \geq -3$ and $n<0$

$-3 \le n < 0$;

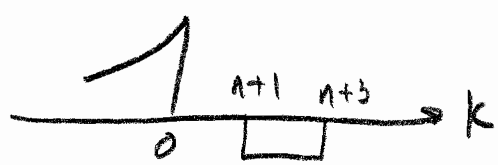


$$y[n] = \sum_{k=n+1}^{0} \left(\tfrac{1}{2}\right)^k(-1)$$

$$= -\sum_{k=n+1}^{0}\left(\tfrac{1}{2}\right)^k = -\frac{\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{2}\right)^1}{1-\frac{1}{2}}$$

$$= -\frac{\frac{1}{2}\left[\left(\frac{1}{2}\right)^n - 1\right]}{1/2} = 1-\left(\tfrac{1}{2}\right)^n$$

III) $n \geq 0$;



$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0.$$

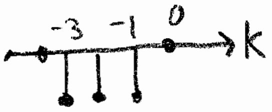
all Together: $y[n] = \begin{cases} -\frac{7}{8}\left(\frac{1}{2}\right)^n, & n<-3 \\ 1-\left(\frac{1}{2}\right)^n, & -3 \le n < 0 \\ 0, & n \geq 0 \end{cases}$

More Workspace for Problem 3...

OTHER WAY:

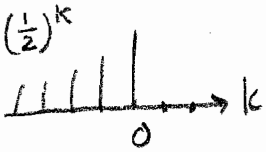
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$h[k]$



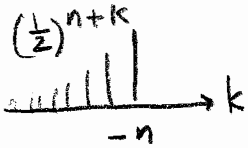
$x[k]$

$\left(\frac{1}{2}\right)^k$

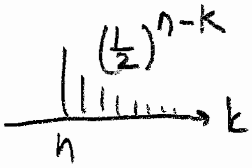


$x[k - -n] = x[n+k]$

$\left(\frac{1}{2}\right)^{n+k}$



$x[n-k]$



I) $n < -3$:

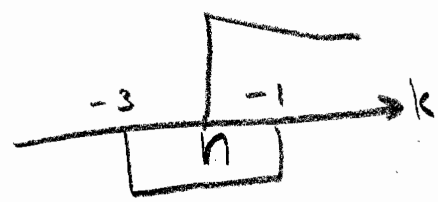
$$y[n] = \sum_{k=-3}^{-1} \left(\tfrac{1}{2}\right)^{n-k}(-1) = -\sum_{k=-3}^{-1} \left(\tfrac{1}{2}\right)^{n}\left(\tfrac{1}{2}\right)^{-k}$$

$$= -\left(\tfrac{1}{2}\right)^n \sum_{k=-3}^{-1} 2^k = -\left(\tfrac{1}{2}\right)^n \frac{2^{-3} - 2^{-1+1}}{1-2}$$

$$= -\left(\tfrac{1}{2}\right)^n \frac{2^{-3} - 1}{-1} = -\left(\tfrac{1}{2}\right)^n \left[1 - \tfrac{1}{8}\right]$$

$$= -\tfrac{7}{8}\left(\tfrac{1}{2}\right)^n$$

II) $-3 \leq n$ and $n < 0$: $-3 \leq n < 0$:

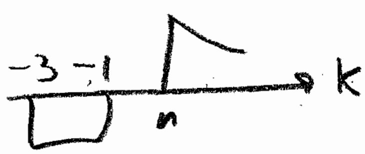


$$y[n] = \sum_{k=n}^{-1} \left(\tfrac{1}{2}\right)^{n-k}(-1)$$

$$= -\sum_{k=n}^{-1} \left(\tfrac{1}{2}\right)^{n}\left(\tfrac{1}{2}\right)^{-k} = -\left(\tfrac{1}{2}\right)^n \sum_{k=n}^{-1} 2^k$$

$$= -\left(\tfrac{1}{2}\right)^n \frac{2^n - 2^{-1+1}}{1-2} = -\left(\tfrac{1}{2}\right)^n \frac{2^n - 1}{-1} = 1 - \left(\tfrac{1}{2}\right)^n$$

III) $n \geq 0$:



$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0.$$

All Together:

$$y[n] = \begin{cases} -\tfrac{7}{8}\left(\tfrac{1}{2}\right)^n, & n < -3 \\ 1 - \left(\tfrac{1}{2}\right)^n, & -3 \leq n < 0 \\ 0, & n \geq 0 \end{cases}$$

4. **25 pts**. A continuous-time LTI system H has impulse response $h(t)$ given by

$$h(t) = e^{-3t}u(t).$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

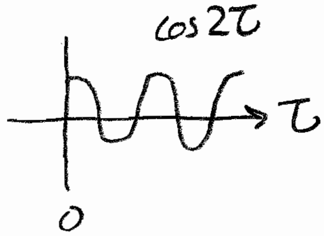
The system input is given by

$$x(t) = \cos(2t)\,u(t).$$

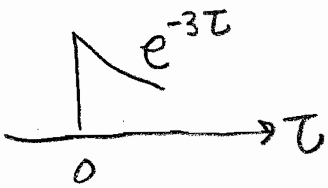
Find the system output $y(t)$.

Hint: This problem is **easier** to work if you put the "τ" on $x(t)$ and the "$t-\tau$" on $h(t)$; *i.e.*, if you work it as $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$ instead of the other way.

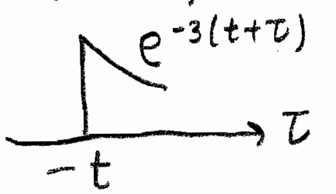
$x(\tau)$



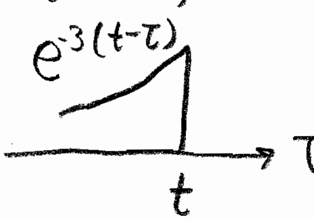
$h(\tau)$



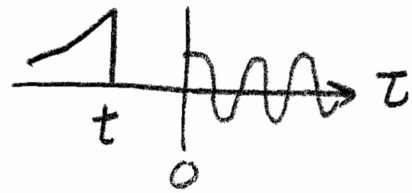
$h(\tau - -t) = h(t+\tau)$



$h(t-\tau)$

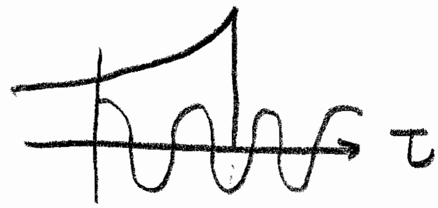


case I) $t < 0$:



$$y(t) = \int_{-\infty}^{\infty} 0\,d\tau = 0$$

case II) $t \geq 0$:



$$y(t) = \int_0^t \cos(2\tau)e^{-3(t-\tau)}\,d\tau \qquad a=3 \quad b=2$$

$$= \int_0^t \cos(2\tau)\,e^{-3t}e^{3\tau}\,d\tau = e^{-3t}\int_0^t e^{3\tau}\cos(2\tau)\,d\tau$$

$$= e^{-3t}\left\{\frac{e^{3\tau}\left[3\cos(2\tau)+2\sin(2\tau)\right]}{3^2+2^2}\right\}_{\tau=0}^{t}$$

$$= \frac{e^{-3t}}{13}\left\{e^{3t}\left[3\cos(2t)+2\sin(2t)\right] - 1\left[3\cdot 1 + 2\cdot 0\right]\right\}$$

$$= \frac{1}{13}\left[3\cos(2t)+2\sin(2t) - 3e^{-3t}\right]$$

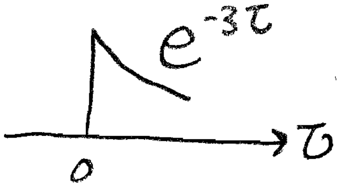
$$= \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t) - \frac{3}{13}e^{-3t}$$

All Together:

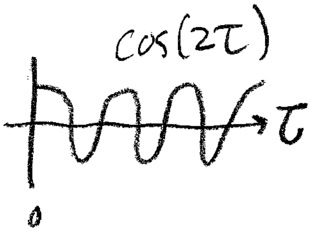
$$\boxed{y(t) = \left[\frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t) - \frac{3}{13}e^{-3t}\right]u(t)}$$

OTHER WAY: $y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$

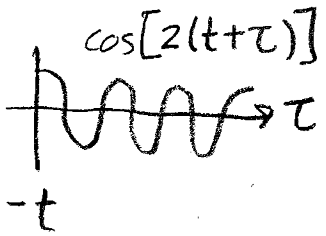
$h(\tau)$



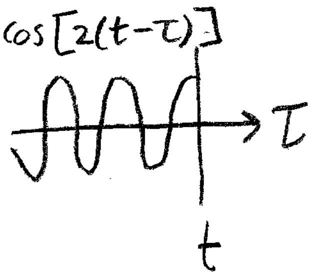
$x(\tau)$



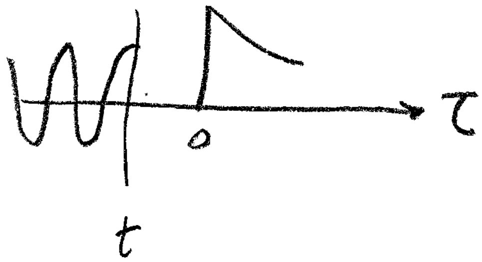
$x(\tau - -t) = x(t+\tau)$



$x(t-\tau)$

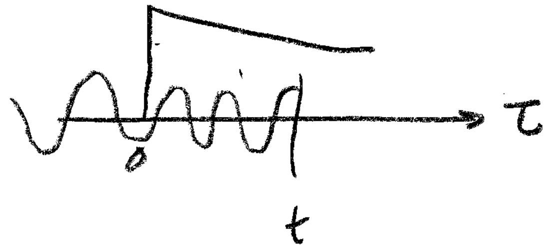


I) $t<0$:



$$y(t) = \int_{-\infty}^{\infty} 0\,d\tau = 0$$

II) $t \geq 0$:



$$y(t) = \int_0^t e^{-3\tau}\cos[2(t-\tau)]\,d\tau \quad (*)$$

$\Rightarrow$ There are 3 feasible ways to work this integral : (A) change of variable, (B) Trig substitution, (C) Euler formula.

$\Rightarrow$ ALL THREE Are shown here.

Method (A): change of variable

Let $u = t-\tau$ $\quad \tau = t-u$

$du = -d\tau \quad d\tau = -du$

when $\tau \to 0$, $u \to t$

when $\tau \to t$, $u \to 0$

So $(*)$ becomes:

$$y(t) = \int_t^0 e^{-3(t-u)}\cos(2u)\,(-du)$$

$$= \int_0^t e^{-3(t-u)}\cos(2u)\,du$$

This is the same integral as in the "first way" and the solution is as shown there

Method (B): Trig Substitution:

Use $\cos(A-B) = \cos A \cos B + \sin A \sin B$ to obtain

$$\cos(2t-2\tau) = \cos(2t)\cos(2\tau) + \sin(2t)\sin(2\tau).$$

$$(*) = \int_0^t e^{-3\tau} \cos(2t-2\tau)\, d\tau$$

$$= \int_0^t e^{-3\tau}\left[\cos(2t)\cos(2\tau) + \sin(2t)\sin(2\tau)\right] d\tau$$

$$= \cos(2t)\int_0^t \overset{a=-3}{e^{-3\tau}} \overset{b=2}{\cos(2\tau)}\, d\tau + \sin(2t)\int_0^t \overset{a=-3}{e^{-3\tau}} \overset{b=2}{\sin(2\tau)}\, d\tau$$

$$= \cos(2t)\left\{\frac{e^{-3\tau}\left[-3\cos(2\tau)+2\sin(2\tau)\right]}{(-3)^2+2^2}\right\}_{\tau=0}^{t}$$

$$+ \sin(2t)\left\{\frac{e^{-3\tau}\left[-3\sin(2\tau)-2\cos(2\tau)\right]}{(-3)^2+2^2}\right\}_{\tau=0}^{t}$$

$$= \tfrac{1}{13}\cos(2t)\left\{e^{-3t}\left[-3\cos(2t)+2\sin(2t)\right] - 1\left[-3\cdot 1 + 2\cdot 0\right]\right\}$$

$$+ \tfrac{1}{13}\sin(2t)\left\{e^{-3t}\left[-3\sin(2t) - 2\cos(2t)\right] - 1\left[-3\cdot 0 - 2\cdot 1\right]\right\}$$

$$= \tfrac{1}{13}\cos(2t)\left[-3e^{-3t}\cos(2t) + 2e^{-3t}\sin(2t) + 3\right]$$

$$+ \tfrac{1}{13}\sin(2t)\left[-3e^{-3t}\sin(2t) - 2e^{-3t}\cos(2t) + 2\right]$$

$$= -\tfrac{3}{13}e^{-3t}\cos^2(2t) + \tfrac{2}{13}e^{-3t}\cos(2t)\sin(2t) + \tfrac{3}{13}\cos(2t)$$

$$-\tfrac{3}{13}e^{-3t}\sin^2(2t) - \tfrac{2}{13}e^{-3t}\cos(2t)\sin(2t) + \tfrac{2}{13}\sin(2t)$$

$\longrightarrow$

(B) ... $= -\frac{3}{13}e^{-3t}\left[\underbrace{\cos^2(2t)+\sin^2(2t)}_{1}\right] + \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t)$

$$y(t) = \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t) - \frac{3}{13}e^{-3t}$$

Method (C): <u>Euler Formula</u>:

$$(*) = \int_0^t e^{-3\tau}\left(\tfrac{1}{2}\right)\left[e^{j2(t-\tau)} + e^{-j2(t-\tau)}\right]d\tau$$

$$= \int_0^t \tfrac{1}{2}e^{-3\tau}e^{j2t}e^{-j2\tau}\,d\tau + \int_0^t \tfrac{1}{2}e^{-3\tau}e^{-j2t}e^{j2\tau}\,d\tau$$

$$= \tfrac{1}{2}e^{j2t}\int_0^t e^{(-3-j2)\tau}\,d\tau + \tfrac{1}{2}e^{-j2t}\int_0^t e^{(-3+j2)\tau}\,d\tau$$

$$= \tfrac{1}{2}e^{j2t}\frac{1}{(-3-j2)}e^{(-3-j2)\tau}\Big|_{\tau=0}^{t} + \tfrac{1}{2}e^{-j2t}\frac{1}{(-3+j2)}e^{(-3+j2)\tau}\Big|_{\tau=0}^{t}$$

$$= \tfrac{1}{2}e^{j2t}\frac{-3+j2}{(-3-j2)(-3+j2)}\left[e^{(-3-j2)t} - 1\right]$$

$$+ \tfrac{1}{2}e^{-j2t}\frac{-3-j2}{(-3+j2)(-3-j2)}\left[e^{(-3+j2)t} - 1\right]$$

$\longrightarrow$

(c) ...

$$y(t) = \frac{1}{2}e^{j2t}\,\frac{-3+j2}{9+4}\left[e^{-3t}e^{-j2t} - 1\right]$$
$$+ \frac{1}{2}e^{-j2t}\,\frac{-3-j2}{9+4}\left[e^{-3t}e^{j2t} - 1\right]$$

$$= \frac{-3+j2}{13}\left[\frac{1}{2}e^{-3t} - \frac{1}{2}e^{j2t}\right] + \frac{-3-j2}{13}\left[\frac{1}{2}e^{-3t} - \frac{1}{2}e^{-j2t}\right]$$

$$= \left(\frac{1}{13}\right)\left(\frac{1}{2}\right)\Big\{-3e^{-3t} + j2e^{-3t} + 3e^{j2t} - j2e^{j2t}$$
$$-3e^{-3t} - j2e^{-3t} + 3e^{-j2t} + j2e^{-j2t}\Big\}$$

$$= \left(\frac{1}{13}\right)\left(\frac{1}{2}\right)\left\{-6e^{-3t} + 3\left[e^{j2t} + e^{-j2t}\right] - j2\left[e^{j2t} - e^{-j2t}\right]\right\}$$

$$= \frac{1}{13}\left\{-3e^{-3t} + 3\left(\frac{1}{2}\right)2\cos(2t) + 2\,\frac{e^{j2t} - e^{-j2t}}{2j}\right\}$$

$$= \frac{-3}{13}e^{-3t} + \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t)$$

All together:

$$y(t) = \left[\frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t) - \frac{3}{13}e^{-3t}\right]u(t)$$