EE 3793
Test 1

Wednesday, March 5, 1997
12:30 PM - 1:20 PM

Spring 1997

Dr. Havlicek

Name: SOLUTION

Student Num:____________________

Directions: You have 50 minutes to complete the test. All work must be your own. You may use one 8.5×11 inch two-sided note sheet as well as the summation formula sheet given out in class.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________

2. (25) ________

3. (15) ________

4. (20) ________

5. (20) ________

TOTAL (100):

1. **20 pts**. True or False. Mark the correct answer. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
___	X	(a) **3 pts. Any** linear system is completely characterized by its impulse response.
X	___	(b) **3 pts**. If H_1 is a linear shift invariant system and H_2 is a linear shift invariant system, then a system H formed by the cascade connection of H_1 and H_2 is also a linear shift-invariant system.
___	X	(c) **3 pts**. If H_1 is a linear shift invariant system and H_2 is a linear system that is **not** shift invariant, then a system H formed by the parallel connection of H_1 and H_2 is a linear shift-invariant system.
X	___	(d) **4 pts**. If $x[n]$ is an **odd** signal, then $\sum_{n=-\infty}^{\infty} x[n] = 0$.
___	X	(e) **3 pts**. If $h[n]$ is the unit-pulse response of a BIBO stable discrete-time linear shift invariant system, then $\sum_{n=-\infty}^{\infty} \lvert h[n] \rvert = \infty$.
X	___	(f) **4 pts**. If $h(t)$ is the impulse response of a linear shift invariant system and $h(t)$ is nonzero and $h(t)$ is periodic, then the system is unstable.

2. **25 pts**. Short answer.

(a) **6 pts**. The unit step function $u(t)$ has a derivative that is undefined at the point $t = 0$. What is the only sensible meaning of the expression $u'(t) = \delta(t)$? Answer in **two sentences or less.**

The derivative of the <u>distribution</u> $u(t)$ takes every testing function to the same number as does the distribution $\delta(t)$.

(b) **5 pts**. Is the signal $x[n] = 3\cos\left[\frac{3}{5}n + \frac{1}{2}\right]$ periodic ? If so, what is its fundamental period ?

$$\omega_0 = \frac{3}{5}$$

$$\frac{\omega_0}{2\pi} = \frac{3}{10\pi}$$

<u>NOT</u> <u>PERIODIC</u> because $\frac{\omega_0}{2\pi}$ <u>not</u> <u>rational</u>.

(c) **7 pts**. Is the signal $x(t) = \mathcal{E}v\{\sin(4\pi t)\}$ periodic ? If so, what is its fundamental period ?

$$\mathcal{E}v\{\sin(4\pi t)\} = \frac{1}{2}\{\sin(4\pi t) - \sin(-4\pi t)\}$$
$$= 0.$$

So $\mathcal{E}v\{\sin(4\pi t)\}$ is a constant.

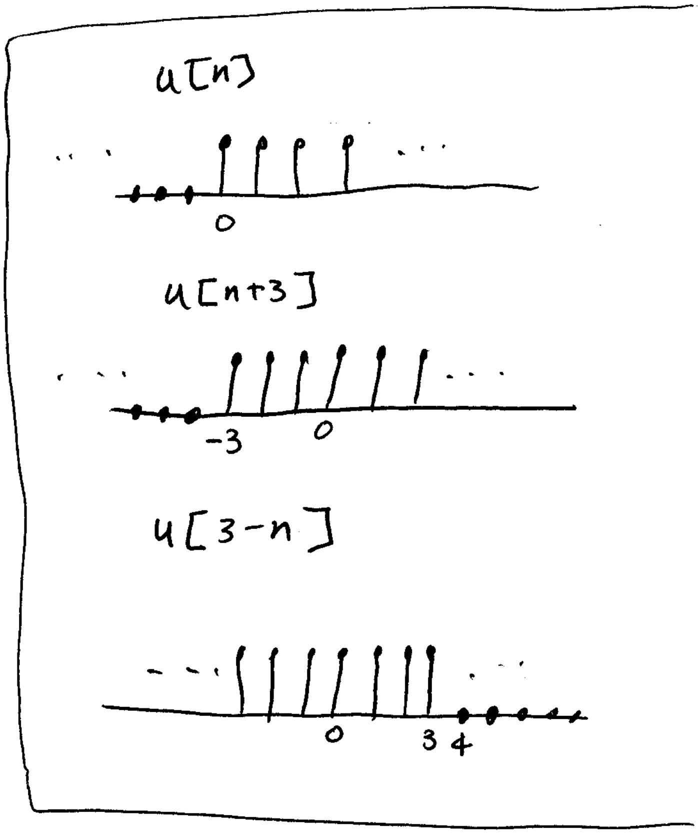
$\Rightarrow$ <u>Periodic</u>, but fundamental period is undefined.

2. cont...

(d) **7 pts.** Is the system with unit pulse response $h[n] = (5)^n u[3-n]$ stable ? Why ?

$$\|h\|_{\ell^1} = \sum_{n=-\infty}^{\infty} |h[k]| = \sum_{n=-\infty}^{\infty} |5^n u[3-n]|$$

$$= \sum_{n=-\infty}^{3} 5^n$$



$$= \sum_{n=-3}^{\infty} \left(\frac{1}{5}\right)^n$$

$$= \sum_{n=-3}^{-1} \left(\frac{1}{5}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n$$

$$= \sum_{n=1}^{3} 5^n + \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n$$

$$= \sum_{n=0}^{3} 5^n - 1 + \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n$$

$$= \frac{1-5^4}{1-5} - 1 + \frac{1}{1-\frac{1}{5}} = \frac{-624}{-4} - 1 + \frac{5}{5-1}$$

$$= 156 - 1 + \frac{5}{4} = 155 + \frac{5}{4} = \frac{625}{4}$$

$$\boxed{\text{The system } \underline{\text{is}} \text{ stable because } \|h\|_{\ell^1} < \infty.}$$

3. **15 pts**. Consider a discrete-time linear shift invariant system with unit pulse response

$$h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2].$$

Find the system response $y[n]$ when the input is

$$x[n] = \delta[n+1] - 2\delta[n] + \delta[n-1].$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = h[n+1] - 2h[n] + h[n-1]$$

$$= \underline{\delta[n+1]} + \underline{2\delta[n]} + \underline{3\delta[n-1]}$$

$$- \underline{2\delta[n]} - \underline{4\delta[n-1]} - \underline{6\delta[n-2]}$$

$$+ \underline{\delta[n-1]} + \underline{2\delta[n-2]} + 3\delta[n-3]$$

$$\boxed{y[n] = \delta[n+1] - 4\delta[n-2] + 3\delta[n-3]}$$

4. **20 pts.** Consider a linear shift invariant system with input $x(t)$ and output $y(t)$ related by

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau.$$

(a) **16 pts.** What is the impulse response $h(t)$ of the system ?

$\boxed{\text{Method 1:}}$

$$y(t) = h(t) * x(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)\, d\tau$$

$u = \tau - 2$
$du = d\tau$
$\tau = u + 2$
$d\tau = du$

$$= \int_{-\infty}^{t-2} e^{-(t-u-2)} x(u)\, du$$

$v = -u + t$
$dv = -du$
$u = -v + t$
$du = -dv$

$$= -\int_{\infty}^{2} e^{-(v-2)} x(t-v)\, dv$$

$$= \int_{2}^{\infty} e^{-(\tau-2)} x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \underbrace{e^{-(\tau-2)} u(\tau-2)}_{h(\tau)} x(t-\tau)\, d\tau$$

$$\boxed{h(t) = e^{-(t-2)} u(t-2)}$$

$\boxed{\text{Method 2:}}$

Since $h(t)$ is what comes out when the input is $\delta(t)$,

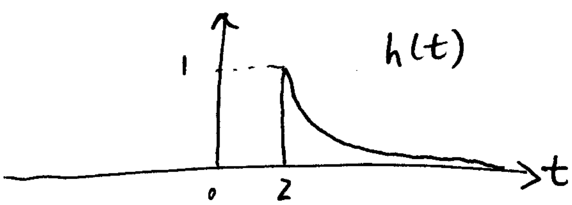
symbolic integrals

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)\, d\tau = \int_{-\infty}^{\infty} e^{-(t-\tau)} u(t-\tau)\, \delta(\tau-2)\, dt$$

$$= \langle \delta(\tau-2), e^{-(t-\tau)} u(t-\tau) \rangle$$

$$= e^{-(t-2)} u(t-2)$$

(b) **4 pts.** Is the system causal ?



The system <u>is</u> causal because $h(t) = 0$ for $t < 0$.

5. **20 pts**. Consider two signals $x_1(t)$ and $x_2(t)$ defined by

$$x_1(t) = \begin{cases} 1, & -1 < t < 1 \\ 0 & \text{otherwise,} \end{cases}$$

$$x_2(t) = \begin{cases} t, & -1 < t < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Each of these two signals is a vector in the signal space $L^1(\mathbb{R})$. Are they orthogonal vectors ?

$$\langle x_1(t), x_2(t) \rangle = \int_{\mathbb{R}} x_1(t)\, x_2(t)\, dt$$

$$= \int_{-1}^{1} t \, dt$$

$$= 0 \quad (\text{since integrand is } \underline{\text{odd}}).$$

$x_1(t)$ and $x_2(t)$ are orthogonal because their inner product is zero.