EE 3793
Test 1

Spring 1997
Dr. Havlicek

Wednesday, March 5, 1997
12:30 PM - 1:20 PM

Name: SOLUTION
Student Num:

Directions: You have 50 minutes to complete the test. All work must be your own. You may use one 8.5 × 11 inch two-sided note sheet as well as the summation formula sheet given out in class.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:
1. (20) _______
2. (25) _______
3. (15) _______
4. (20) _______
5. (20) _______

TOTAL (100):

1
1. 20 pts. True or False. Mark the correct answer. Mark True only if the statement is always true.

**TRUE**  **FALSE**

___  ___ (a) 3 pts. Any linear system is completely characterized by its impulse response.

___  ___ (b) 3 pts. If $H_1$ is a linear shift invariant system and $H_2$ is a linear shift invariant system, then a system $H$ formed by the cascade connection of $H_1$ and $H_2$ is also a linear shift-invariant system.

___  ___ (c) 3 pts. If $H_1$ is a linear shift invariant system and $H_2$ is a linear system that is not shift invariant, then a system $H$ formed by the parallel connection of $H_1$ and $H_2$ is a linear shift-invariant system.

___  ___ (d) 4 pts. If $x[n]$ is an odd signal, then $\sum_{n=-\infty}^{\infty} x[n] = 0$.

___  ___ (e) 3 pts. If $h[n]$ is the unit-pulse response of a BIBO stable discrete-time linear shift invariant system, then $\sum_{n=-\infty}^{\infty} |h[n]| = \infty$.

___  ___ (f) 4 pts. If $h(t)$ is the impulse response of a linear shift invariant system and $h(t)$ is nonzero and $h(t)$ is periodic, then the system is unstable.
2. 25 pts. Short answer.

(a) 6 pts. The unit step function \( u(t) \) has a derivative that is undefined at the point \( t = 0 \). What is the only sensible meaning of the expression \( u'(t) = \delta(t) \)? Answer in two sentences or less.

The derivative of the distribution \( u(t) \) takes every testing function to the same number as does the distribution \( \delta(t) \).

(b) 5 pts. Is the signal \( x[n] = 3 \cos \left[ \frac{2}{5} n + \frac{1}{2} \right] \) periodic? If so, what is its fundamental period?

\[ \omega_0 = \frac{3}{5} \]

\[ \frac{\omega_0}{2\pi} = \frac{3}{10\pi} \]

NOT PERIODIC because \( \frac{\omega_0}{2\pi} \) not rational.

(c) 7 pts. Is the signal \( x(t) = \mathcal{E}[\sin(4\pi t)] \) periodic? If so, what is its fundamental period?

\[ \mathcal{E} \left\{ \sin(4\pi t) \right\} = \frac{1}{2} \left\{ \sin(4\pi t) - \sin(-4\pi t) \right\} \]

= 0.

So \( \mathcal{E} \left\{ \sin(4\pi t) \right\} \) is a constant.

\( \Rightarrow \) Periodic, but fundamental period is undefined.
(d) 7 pts. Is the system with unit pulse response \( h[n] = (5)^n u[3 - n] \) stable? Why?

\[
\|h\|_\infty = \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |5^n u[3-n]| = \sum_{n=-\infty}^{\infty} 5^n u[3-n]
\]

\[
= \sum_{n=-3}^{3} 5^n + \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n
\]

\[
= \sum_{n=1}^{3} 5^n + \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n
\]

\[
= \sum_{n=0}^{3} 5^n - 1 + \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n
\]

\[
= \frac{1 - \frac{5^4}{1-5}}{1-\frac{1}{5}} - 1 + \frac{1}{1-\frac{5}{5}} = \frac{-624}{-4} - 1 + \frac{5}{5-1}
\]

\[
= 156 - 1 + \frac{5}{4} = 155 + \frac{5}{4} = \frac{625}{4}
\]

The system is stable because \( \|h\|_\infty < \infty \).
3. **15 pts.** Consider a discrete-time linear shift invariant system with unit pulse response

\[ h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]. \]

Find the system response \( y[n] \) when the input is

\[ x[n] = \delta[n + 1] - 2\delta[n] + \delta[n - 1]. \]

\[
y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = h[n+1] - 2h[n] + h[n-1]
\]

\[
= \underbrace{\delta[n+1]}_{\delta[n+1]} + 2\underbrace{\delta[n]}_{\delta[n]} + 3\underbrace{\delta[n-1]}_{\delta[n-1]}
- 2\underbrace{\delta[n]}_{\delta[n]} - 4\underbrace{\delta[n-1]}_{\delta[n-1]} - 6\underbrace{\delta[n-2]}_{\delta[n-2]}
+ \underbrace{\delta[n-1]}_{\delta[n-1]} + 2\underbrace{\delta[n-2]}_{\delta[n-2]} + 3\underbrace{\delta[n-3]}_{\delta[n-3]}
\]

\[
y[n] = \delta[n+1] - 4\delta[n-2] + 3\delta[n-3]
\]
4. 20 pts. Consider a linear shift invariant system with input $x(t)$ and output $y(t)$ related by

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2) d\tau.$$ 

(a) 16 pts. What is the impulse response $h(t)$ of the system?

**Method 1:**

$$y(t) = h(t) * x(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2) d\tau$$

$$= \int_{-\infty}^{t-2} e^{-(t-u-2)} x(u) du$$

$$= \int_{-\infty}^{2} e^{-(v-2)} x(t-v) dv$$

$$= \int_{t-2}^{\infty} e^{-(\tau-2)} x(\tau-2) d\tau = \int_{-\infty}^{\infty} e^{-(\tau-2)} u(\tau-2) x(t-\tau) d\tau$$

$$h(t) = e^{-(t-2)} u(t-2)$$

**Method 2:**

Since $h(t)$ is what comes out when the input is $\delta(t)$,

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2) d\tau = \int_{-\infty}^{\infty} e^{-(t-\tau)} u(t-\tau) \delta(\tau-2) d\tau$$

(b) 4 pts. Is the system causal?

The system is causal because $h(t) = 0$ for $t < 0$. 

$$= \langle \delta(t-2), e^{-(t-2)} u(t-2) \rangle$$

$$= e^{-(t-2)} u(t-2)$$
5. **20 pts.** Consider two signals \( x_1(t) \) and \( x_2(t) \) defined by

\[
\begin{align*}
  x_1(t) &= \begin{cases} 
    1, & -1 < t < 1 \\
    0, & \text{otherwise,}
  \end{cases} \\
  x_2(t) &= \begin{cases} 
    t, & -1 < t < 1 \\
    0, & \text{otherwise.}
  \end{cases}
\end{align*}
\]

Each of these two signals is a vector in the signal space \( L^1(\mathbb{R}) \). Are they orthogonal vectors?

\[
\langle x_1(t), x_2(t) \rangle = \int_{\mathbb{R}} x_1(t) x_2(t) \, dt
\]

\[
= \int_{-1}^{1} t \, dt
\]

\[
= 0 \quad (\text{since integrand is odd}).
\]

\( x_1(t) \) and \( x_2(t) \) are orthogonal because their inner product is zero.