ECE 3793
Test 2

Thursday, November 19, 1998
12:00 PM - 1:15 PM

Fall 1998

Dr. Havlicek

Name: SOLUTION

Student Num:____________

Directions: You have 75 minutes to complete the test. All work must be your own. You may use two 8.5 × 11 inch two-sided note sheets as well as the summation formula sheet handed out in class.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________

2. (20) ________

3. (20) ________

4. (20) ________

5. (20) ________

TOTAL (100):

20

1. ~~26~~ **pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
	X	(a) **2 pts.** If $h(t)$ is the impulse response of an LSI system an $h(t)$ is nonzero for all $t > 0$, then the system is not BIBO stable.
	X	(b) **2 pts.** The Riemann-Lebesgue lemma states that $\lim_{\omega \to \infty} e^{j\omega t} = \delta(t)$.
X		(c) **2 pts.** If $x(t) \in L^2(\mathbb{R})$, then the Fourier transform of $x(t)$ exists as an ordinary integral or as a Cauchy principal value.
X		(d) **2 pts.** For any constant $z \in \mathbb{C}$, the signal $x[n] = z^n$ is an eigenfunction of any discrete-time LSI system.
X		(e) **2 pts.** If $x(t) \leftrightarrow X(\omega)$, then $\mathcal{F}^{-1}\{X(\omega)\}$ converges to the midpoints of discontinuities in $x(t)$.
X		(f) **2 pts.** If $x(t)$ is real and even, then $X(\omega)$ is real and even.
X		(g) **2 pts.** Along the line $\mathrm{Re}\,[s] = 0$, the Laplace transform $X(s) = \mathcal{L}\{x(t)\}$ is equal to the Fourier transform of $x(t)$.
X		(h) **2 pts.** If $X(s)$ is rational in s and the numerator and denominator polynomials have the same order, then $X(s)$ has no poles at infinity and no zeros at infinity.
	X	(i) **2 pts.** If $x(t) = 0 \; \forall \; t > t_0$, then the ROC of $X(s)$ is a right half-plane.
X		(j) **2 pts.** There is no such thing as Santa Claus.

2. **20 pts.** Consider an LSI system whose response to the input

$$x(t) = \left[e^{-t} + e^{-3t}\right] u(t)$$

is

$$y(t) = \left[2e^{-t} - 2e^{-4t}\right] u(t)$$

(a) **10 pts** Find the frequency response of this system.

$$e^{-t}u(t) \longleftrightarrow \frac{1}{1+j\omega} \quad ; \quad e^{-3t}u(t) \longleftrightarrow \frac{1}{3+j\omega}$$

$$X(\omega) = \frac{1}{1+j\omega} + \frac{1}{3+j\omega} = \frac{3+j\omega+1+j\omega}{(1+j\omega)(3+j\omega)} = \frac{2(2+j\omega)}{(1+j\omega)(3+j\omega)}$$

$$2e^{-t}u(t) \longleftrightarrow \frac{2}{1+j\omega} \quad ; \quad -2e^{-4t}u(t) \longleftrightarrow \frac{-2}{4+j\omega}$$

$$Y(\omega) = \frac{2}{1+j\omega} - \frac{2}{4+j\omega} = \frac{2(4+j\omega-1-j\omega)}{(1+j\omega)(4+j\omega)} = \frac{6}{(1+j\omega)(4+j\omega)}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{6}{(1+j\omega)(4+j\omega)} \frac{(1+j\omega)(3+j\omega)}{2(2+j\omega)}$$

$$= \frac{3(3+j\omega)}{(2+j\omega)(4+j\omega)}$$

Problem 2, cont...

(b) **6 pts** Determine the system's impulse response.

$$H(\omega) = \frac{3(3+j\omega)}{(2+j\omega)(4+j\omega)} = \frac{A}{2+j\omega} + \frac{B}{4+j\omega}$$

$$A = \left.\frac{3(3+j\omega)}{4+j\omega}\right|_{\omega=j2} = \frac{3(3-2)}{4-2} = \frac{3}{2}$$

$$B = \left.\frac{3(3+j\omega)}{2+j\omega}\right|_{\omega=j4} = \frac{3(3-4)}{2-4} = \frac{3}{2}$$

$$H(\omega) = \frac{3}{2}\frac{1}{2+j\omega} + \frac{3}{2}\frac{1}{4+j\omega}$$

$$h(t) = \mathcal{F}^{-1}[H(\omega)] = \frac{3}{2}e^{-2t}u(t) + \frac{3}{2}e^{-4t}u(t)$$

(c) **4 pts** Find the differential equation relating the input and the output of this system.

$$\frac{Y(\omega)}{X(\omega)} = H(\omega) = \frac{3(3+j\omega)}{(2+j\omega)(4+j\omega)} = \frac{3(3+j\omega)}{8+6j\omega-\omega^2}$$

$$\left[8+6j\omega+(j\omega)^2\right]Y(\omega) = 3(3+j\omega)X(\omega)$$

$$8Y(\omega) + 6j\omega Y(\omega) + (j\omega)^2 Y(\omega) = 9X(\omega) + 3j\omega X(\omega)$$

$$\mathcal{F}^{-1}: \quad 8y(t) + 6\frac{d}{dt}y(t) + \frac{d^2}{dt^2}y(t) = 9x(t) + 3\frac{d}{dt}x(t)$$

3. **20 pts.** Consider an LSI system with input $x(t) = e^{-t+1}u(t-1)$ and frequency response

$$H(s) = \frac{1}{s+2}, \ \mathrm{Re}[s] > -2.$$

Find the system output $y(t)$.

$$x(t) = e^{-t+1}u(t-1) = e^{-(t-1)}u(t-1) \longleftrightarrow \frac{e^{-s}}{1+s}, \ \mathrm{Re}[s] > -1.$$

$$Y(s) = X(s)H(s) = \frac{e^{-s}}{(s+1)(s+2)}, \ \mathrm{Re}[s] > -1.$$

$$\text{Let } \Psi(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}, \ \mathrm{Re}[s] > -1.$$

$$A = \left.\frac{1}{s+2}\right|_{s=-1} = 1 \ ; \ B = \left.\frac{1}{s+1}\right|_{s=-2} = -1$$

$$\Psi(s) = \frac{1}{s+1} - \frac{1}{s+2}, \ \mathrm{Re}[s] > -1.$$

$$\psi(t) = \mathcal{L}^{-1}[\Psi(s)] = e^{-t}u(t) - e^{-2t}u(t)$$

$$Y(s) = e^{-s}\Psi(s) \Longrightarrow y(t) = \psi(t-1).$$

$$\underline{\underline{y(t) = e^{-(t-1)}u(t-1) - e^{-2(t-1)}u(t-1)}}$$

4. **20 pts.** Consider a discrete-time LSI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

Use Fourier transforms to determine the system response $y[n]$ when the input is

$$x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n].$$

(Table 5.2): $H(e^{j\omega}) = \dfrac{1}{1-\frac{1}{2}e^{-j\omega}}$

$$X(e^{j\omega}) = \frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})^2}$$

Let $Y(\theta) = \dfrac{1}{(1-\frac{1}{2}\theta)(1-\frac{1}{4}\theta)^2} = \dfrac{A}{1-\frac{1}{2}\theta} + \dfrac{B}{1-\frac{1}{4}\theta} + \dfrac{C}{(1-\frac{1}{4}\theta)^2}$

$$A = \left.\frac{1}{(1-\frac{1}{4}\theta)^2}\right|_{\theta=2} = \frac{1}{(1-\frac{1}{2})^2} = \frac{1}{(\frac{1}{2})^2} = \frac{1}{\frac{1}{4}} = 4$$

$$C = \left.\frac{1}{1-\frac{1}{2}\theta}\right|_{\theta=4} = \frac{1}{1-2} = -1$$

As in HW6 solution (page 10),

$$\left[\frac{d}{d\theta}\frac{1}{1-\frac{1}{2}\theta}\right]_{\theta=4} = \left[\frac{d}{d\theta}(1-\tfrac{1}{4}\theta)B\right]_{\theta=4}$$

$$\left.\frac{1/2}{(1-\frac{1}{2}\theta)^2}\right|_{\theta=4} = -\frac{1}{4}B \Longrightarrow \frac{1}{2} = -\frac{1}{4}B \Longrightarrow B = -2$$

$$Y(\theta) = \frac{4}{1-\frac{1}{2}\theta} - \frac{2}{1-\frac{1}{4}\theta} - \frac{1}{(1-\frac{1}{4}\theta)^2}$$

$$Y(e^{j\omega}) = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}} - \frac{1}{(1-\frac{1}{4}e^{-j\omega})^2}$$

$$\underline{\underline{y(t) = 4(\tfrac{1}{2})^n u[n] - 2(\tfrac{1}{4})^n u[n] - (n+1)(\tfrac{1}{4})^n u[n]}}$$

5. **20 pts.** A discrete-time signal $x[n]$ has z-transform

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1 - z^{-1})(1 + z^{-1})}, \; |z| > 2.$$

SHOULD HAVE BEEN $|z| > 1$.

Find $x[n]$.

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1 - z^{-1})(1 + z^{-1})} = \frac{A}{1 - z^{-1}} + \frac{B}{1 + z^{-1}}$$

$$A = \left. \frac{1 - \frac{1}{3}z^{-1}}{1 + z^{-1}} \right|_{z=1} = \frac{1 - \frac{1}{3}}{1 + 1} = \frac{\frac{2}{3}}{2} = \frac{1}{3}$$

$$B = \left. \frac{1 - \frac{1}{3}z^{-1}}{1 - z^{-1}} \right|_{z=-1} = \frac{1 + \frac{1}{3}}{1 + 1} = \frac{4/3}{2} = \frac{2}{3}$$

$$X(z) = \frac{1/3}{1 - z^{-1}} + \frac{2/3}{1 + z^{-1}}, \quad |z| > 1.$$

$$x[n] = \frac{1}{3}u[n] + \frac{2}{3}(-1)^n u[n]$$