ECE 3793
Test 2

Thursday, April 25, 2002
7:00 PM - 10:00 PM

Spring 2002

Dr. Havlicek

Name: SOLUTION

Student Num: ____________

Directions: This test is closed book and closed notes. You have 180 minutes to complete the test. All work must be your own. There are **four** problems. Work all **four**.

► Formulas appear **after** problem four.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________

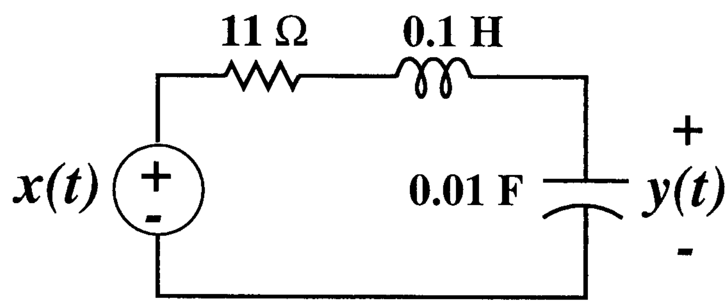
2. (25) ________

3. (25) ________

4. (25) ________

TOTAL (100):

1. **25 pts.** Consider the causal, stable LTI system H shown below. Voltage $x(t)$ is the system input and voltage $y(t)$ is the system output.



(a) **15 pts.** Find the system frequency response $H(\omega)$.

RESISTOR: $V_R(t) = 11\, i(t)$; INDUCTOR: $V_L(t) = 0.1\, i'(t)$

CAPACITOR: $i(t) = 0.01\, y'(t)$.

$\rightarrow\ i'(t) = 0.01\, y''(t) \longrightarrow V_L(t) = (0.1)(0.01)\, y''(t)$
$= (0.001)\, y''(t)$.

KVL: $x(t) = V_R(t) + V_L(t) + y(t)$
$= (11)(.01)\, y'(t) + (0.001)\, y''(t) + y(t)$
$= (0.11)\, y'(t) + (0.001)\, y''(t) + y(t)$

$1000\, x(t) = y''(t) + 110\, y'(t) + 1000\, y(t)$

$1000\, X(\omega) = (j\omega)^2 Y(\omega) + 110 (j\omega) Y(\omega) + 1000\, Y(\omega)$

$1000\, X(\omega) = \left[(j\omega)^2 + 110 j\omega + 1000 \right] Y(\omega)$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1000}{(j\omega)^2 + 110 j\omega + 1000}$$

Problem 1, cont...

$$H(\omega) = \frac{1000}{(j\omega)^2 + 110j\omega + 1000} = \frac{1000}{(j\omega+10)(j\omega+100)}$$

$$= \frac{A}{j\omega+10} + \frac{B}{j\omega+100}$$

$$A = \left.\frac{1000}{\theta+100}\right|_{\theta=-10} = \frac{1000}{90} = \frac{100}{9}$$

$$B = \left.\frac{1000}{\theta+10}\right|_{\theta=-100} = \frac{1000}{-90} = -\frac{100}{9}$$

(b) **10 pts**. Find the system impulse response $h(t)$.

$$H(\omega) = \frac{100}{9}\frac{1}{j\omega+10} - \frac{100}{9}\frac{1}{j\omega+100}$$

$$= \frac{100}{9}\left[\frac{1}{j\omega+10} - \frac{1}{j\omega+100}\right]$$

$$h(t) = \frac{100}{9}\left[e^{-10t} - e^{-100t}\right]u(t)$$

2. **25 pts**. Consider an LTI system G with input $x(t)$ and output $y(t)$ related by

$$y''(t) + 3y'(t) + 2y(t) = x''(t) + 6x'(t) + 9x(t).$$

(a) **8 pts**. Find the system frequency response $G(\omega)$.

$$\left[(j\omega)^2 + 3j\omega + 2\right] Y(\omega) = \left[(j\omega)^2 + 6j\omega + 9\right] X(\omega)$$

$$G(\omega) = \frac{Y(\omega)}{X(\omega)} = \underline{\underline{\frac{(j\omega)^2 + 6j\omega + 9}{(j\omega)^2 + 3j\omega + 2}}}$$

(b) **5 pts**. Let H be the *inverse system* of G. Then H is also an LTI system. Find the frequency response $H(\omega)$ of the inverse system.

$$H(\omega) = \frac{1}{G(\omega)} = \underline{\underline{\frac{(j\omega)^2 + 3j\omega + 2}{(j\omega)^2 + 6j\omega + 9}}}$$

Problem 2, cont...

(c) **12 pts**. Find the impulse response $h(t)$ of the inverse system.

H(ω) is not a proper fraction, so we must do long division

$$
\begin{array}{r|l}
 & \quad 1 \\
(j\omega)^2+6j\omega+9 & (j\omega)^2+3j\omega+2 \\
 & \underline{(j\omega)^2+6j\omega+9} \\
 & \quad -3j\omega-7
\end{array}
\quad H(\omega) = 1 - \frac{3j\omega+7}{(j\omega)^2+6j\omega+9}
$$

$$\frac{3j\omega+7}{(j\omega)^2+6j\omega+9} = \frac{3j\omega+7}{(j\omega+3)^2} = \frac{A}{j\omega+3} + \frac{B}{(j\omega+3)^2}$$

$$B = 3\theta+7\Big|_{\theta=0-3} = -9+7 = \underline{-2}$$

$$\frac{d}{d\theta}(\theta+3)A\Big|_{\theta=-3} + \frac{d}{d\theta}B\Big|_{\theta=-3} = \frac{d}{d\theta}(3\theta+7)\Big|_{\theta=-3}$$

$$\underline{A=3}$$

$$\frac{3j\omega+7}{(j\omega)^2+6j\omega+9} = \frac{3}{j\omega+3} - \frac{2}{(j\omega+3)^2}$$

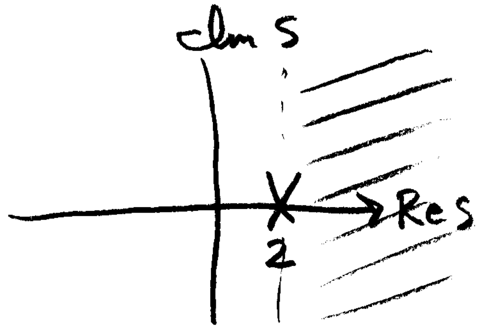
$$H(\omega) = 1 + \frac{2}{(j\omega+3)^2} - \frac{3}{j\omega+3}$$

$$\underline{\underline{h(t) = \delta(t) + 2te^{-3t}u(t) - 3e^{-3t}u(t)}}$$

3. **25 pts**. A **causal** LTI system F has transfer function

$$F(s) = \frac{1}{s-2}.$$

(a) **3 pts**. Give a pole zero plot for $F(s)$. What is the ROC of $F(s)$? Justify your answer.



Since F is causal, ROC is the right half-plane to the right of the rightmost pole.

ROC: $\mathrm{Re}[s] > 2$.

(b) **2 pts**. Is the system F BIBO stable? Justify your answer.

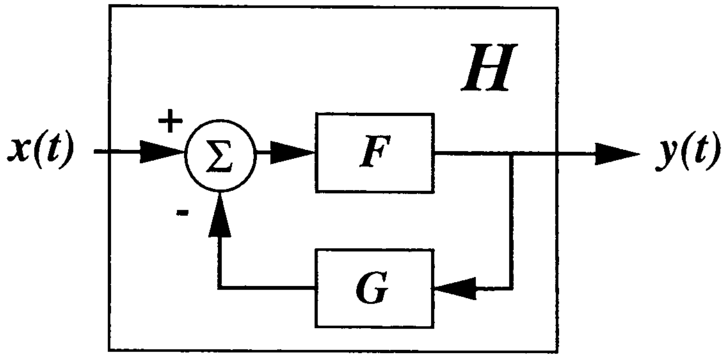
NOT STABLE because the pole at $s=2$ is in the RHP.

(c) **2 pts**. Consider the system impulse response $f(t)$. Does $f(t)$ have a Fourier transform? Justify your answer.

NO. $F(\omega)$ does not converge because the $j\omega$-axis is not in the ROC of $F(s)$.

Problem 3, cont...

(d) **11 pts**. A new system H is formed by adding negative feedback to F as shown in the figure below. H is both LTI and causal.



The transfer function of causal LTI system G is given by $G(s) = 4$.

Find the transfer function $H(s)$ of the overall system H.

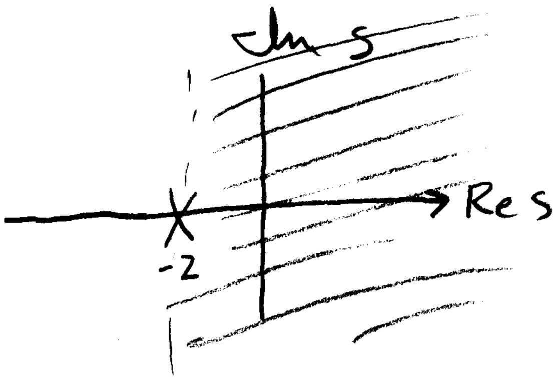
$$H(s) = \frac{F(s)}{1+F(s)G(s)} = \frac{\frac{1}{s-2}}{1+4\frac{1}{s-2}}$$

$$= \frac{1/(s-2)}{1+4/(s-2)} = \frac{1}{s-2+4}$$

$$= \underline{\underline{\frac{1}{s+2}}}$$

Problem 3, cont...

(e) **3 pts.** Give a pole zero plot for $H(s)$. What is the ROC of $H(s)$? Justify your answer.



Since H is causal, ROC is the right half-plane to the right of the rightmost pole.

ROC: $Re[s] > -2$

(f) **2 pts.** Is the system H BIBO stable? Justify your answer.

Yes. H is causal and all poles are in the LHP.

(g) **2 pts.** Consider the system impulse response $h(t)$. Does $h(t)$ have a Fourier transform? Justify your answer.

Yes - because the $j\omega$-axis is in the ROC of $H(s)$.

4. **25 pts.** The input $x(t)$ and output $y(t)$ of an LTI system H are related by

$$y''(t) + 5y'(t) + 6y(t) = 2x'(t) + 2x(t).$$

The initial conditions on the output are $y(0^-) = 1$ and $y'(0^-) = 2$. Use the unilateral Laplace transform to find the system output for $t > 0$ when the input is $x(t) = e^{-t}u(t)$.

$$s^2Y(s) - sy(0^-) - y'(0^-) + 5\left[sY(s) - y(0^-)\right] + 6Y(s)$$

$$= 2\left[sX(s) - x(0^-)\right] + 2X(s)$$

$\rightarrow x(0^-) = 0$:

$$s^2Y(s) - s - 2 + 5sY(s) - 5 + 6Y(s) = 2sX(s) - 2\cdot 0 + 2X(s)$$

$$\left[s^2 + 5s + 6\right]Y(s) - s - 7 = \left[2s + 2\right]X(s)$$

$\rightarrow X(s) = \dfrac{1}{s+1}$

$$\left[s^2 + 5s + 6\right]Y(s) - s - 7 = \frac{2s+2}{s+1} = \frac{2[s+1]}{s+1} = 2$$

$$\left[s^2 + 5s + 6\right]Y(s) = 9 + s$$

$$Y(s) = \frac{s+9}{s^2+5s+6} = \frac{s+9}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = \left.\frac{s+9}{s+3}\right|_{s=-2} = \frac{7}{1} = 7; \quad B = \left.\frac{s+9}{s+2}\right|_{s=-3} = \frac{6}{-1} = -6$$

$$Y(s) = \frac{7}{s+2} - \frac{6}{s+3}$$

$$\boxed{y(t) = 7e^{-2t}u(t) - 6e^{-3t}u(t)}$$

More Work Space for Problem 4...