ECE 3793
Test 2

Thursday, April 26, 2007
7:00 PM - 10:00 PM

Spring 2007

Dr. Havlicek

Name: SOLUTION

Student Num: ____________

Directions: This test is closed book and closed notes. Calculators are not allowed. All work must be your own. You have 180 minutes to complete the test.

► You are allowed to use the **separate** formula sheet provided with this test.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________
2. (25) ________
3. (25) ________
4. (25) ________

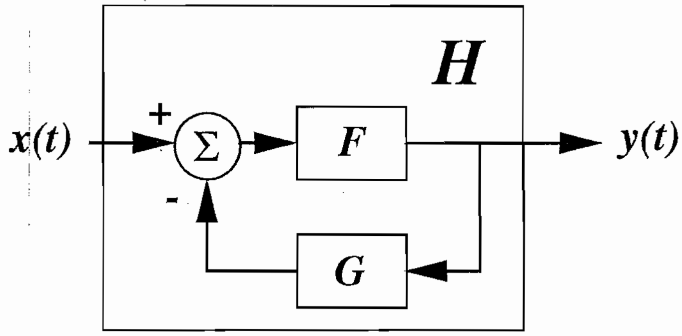
TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: ____________________ Date: ____________________

1. **25 pts**. Consider the continuous-time LTI system H shown below. This system is causal and stable.



The system input-ouptut relation is given by

$$y'(t) + 5y(t) = x(t).$$

The impulse response of system F is given by

$$f(t) = 3e^{-4t}u(t).$$

(a) **10 pts**. Find the system frequency response $H(j\omega)$.

$$y'(t) + 5y(t) = x(t)$$

$$j\omega Y(\omega) + 5Y(\omega) = X(\omega)$$

$$Y(\omega)\left[j\omega+5\right] = X(\omega)$$

$$\boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{j\omega+5}}$$

Problem 1, cont...

(b) **5 pts.** Find the system impulse response $h(t)$.

$$H(\omega) = \frac{1}{5+j\omega}$$

TABLE: $\boxed{h(t) = e^{-5t}u(t)}$

(c) **10 pts.** Find the impulse response $g(t)$ of system G.

$$H(\omega) = \frac{F(\omega)}{1+F(\omega)G(\omega)} = \frac{1}{5+j\omega} \quad (*)$$

$f(t) = 3e^{-4t}u(t)$

TABLE: $F(\omega) = \dfrac{3}{4+j\omega}$ (**)

PLUG (**) into (*):

$$\frac{1}{5+j\omega} = \frac{\frac{3}{4+j\omega}}{1+\frac{3}{4+j\omega}G(\omega)} \cdot \frac{4+j\omega}{4+j\omega}$$

$$\frac{1}{5+j\omega} = \frac{3}{4+j\omega+3G(\omega)}$$

$$5+j\omega = \frac{4}{3} + \frac{1}{3}j\omega + G(\omega)$$

$$G(\omega) = \frac{11}{3} + \frac{2}{3}j\omega$$

TABLE:

$$\boxed{g(t) = \frac{11}{3}\delta(t) + \frac{2}{3}\delta'(t)}$$

2. **20 pts**. The input $x(t)$ and output $y(t)$ of a continuous-time LTI system H are related by

$$\frac{d^2}{dt^2}y(t) + 4\frac{d}{dt}y(t) + 3y(t) = \frac{d}{dt}x(t) + 2x(t).$$

(a) **6 pts**. Find the system frequency response $H(\omega)$.

$$\left[(j\omega)^2 + 4j\omega + 3\right] Y(\omega) = \left[j\omega + 2\right] X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{j\omega+2}{(j\omega)^2 + 4j\omega + 3}$$

$$\boxed{H(\omega) = \frac{j\omega+2}{(j\omega+1)(j\omega+3)}}$$

(b) **9 pts**. Find the system impulse response $h(t)$.

$$\left.\begin{aligned} &\frac{\theta+2}{(\theta+1)(\theta+3)} = \frac{A}{\theta+1} + \frac{B}{\theta+3} \\ &A = \left.\frac{\theta+2}{\theta+3}\right|_{\theta=-1} = \frac{1}{2} \\ &B = \left.\frac{\theta+2}{\theta+1}\right|_{\theta=-3} = \frac{-1}{-2} = \frac{1}{2} \end{aligned}\right\} \quad H(\omega) = \frac{1/2}{j\omega+1} + \frac{1/2}{j\omega+3}$$

$$\boxed{\text{TABLE: } h(t) = \tfrac{1}{2}e^{-t}u(t) + \tfrac{1}{2}e^{-3t}u(t)}$$

Problem 2, cont...

(c) **10 pts.** Suppose that the system input is given by

$$x(t) = e^{-t}u(t).$$

Find the system output y(t). $X(\omega) = \frac{1}{j\omega+1}$

$$Y(\omega) = X(\omega)H(\omega) = \frac{j\omega+2}{(j\omega+1)^2(j\omega+3)}$$

$$\frac{\theta+2}{(\theta+1)^2(\theta+3)} = \frac{A}{\theta+3} + \frac{B}{(\theta+1)^2} + \frac{C}{\theta+1}$$

$$A = \left.\frac{\theta+2}{(\theta+1)^2}\right|_{\theta=-3} = \frac{-1}{(-2)^2} = -\frac{1}{4}$$

$$B = \left.\frac{\theta+2}{\theta+3}\right|_{\theta=-1} = \frac{1}{2}$$

$$\frac{d}{d\theta}\left[(\theta+2)(\theta+3)^{-1}\right]_{\theta=-1} = \frac{d}{d\theta}\left[(\theta+1)C\right]_{\theta=-1}$$

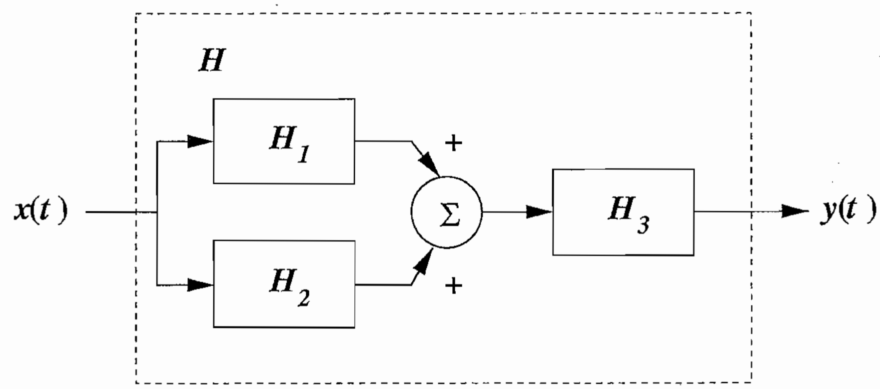
$$\left[(\theta+3)^{-1} + (\theta+2)(-1)(\theta+3)^{-2}\right]_{\theta=-1} = C$$

$$C = \frac{1}{2} - \frac{1}{(2)^2} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$Y(\omega) = \frac{-1/4}{j\omega+3} + \frac{1/2}{(j\omega+1)^2} + \frac{1/4}{j\omega+1}$$

$$\boxed{\text{TABLE: } y(t) = \tfrac{1}{4}e^{-t}u(t) + \tfrac{1}{2}te^{-t}u(t) - \tfrac{1}{4}e^{-3t}u(t)}$$

3. **25 pts.** Consider the continuous-time LTI system H shown below:



The impulse response of LTI system H_1 is given by $h_1(t) = -\frac{1}{2}e^{4t}u(-t)$.
LTI system H_2 is causal and has input $x_2(t)$ and output $y_2(t)$ related by

$$4y_2(t) - 2y_2'(t) = x_2(t).$$

LTI system H_3 is causal and stable and has impulse response $h_3(t) = \delta(t-2)$.

(a) **8 pts.** Find the transfer function $H(s)$ for the overall system. Be sure to specify the ROC.

$h_1(t) = -\frac{1}{2}e^{4t}u(-t)$

TABLE; $(\alpha = -4)$

$H_1(s) = \frac{1}{2}\frac{1}{s-4}$, $Re[s] < 4$

H_2 is causal → right-sided ROC

$4Y_2(s) - 2sY_2(s) = X_2(s)$

$[-2s+4]\,Y_2(s) = X_2(s)$

$H_2(s) = Y_2(s)/X_2(s)$

$= \frac{1}{-2s+4}$

$= \frac{-1/2}{s-2}$, $Re[s] > 2$

TABLE:

$H_3(s) = 1 \cdot e^{-2s}$

$= e^{-2s}$,

All s.

$H(s) = [H_1(s) + H_2(s)]\,H_3(s)$, $\{Re[s] < 4\} \cap \{Re[s] > 2\} \cap \{\text{all } s\}$

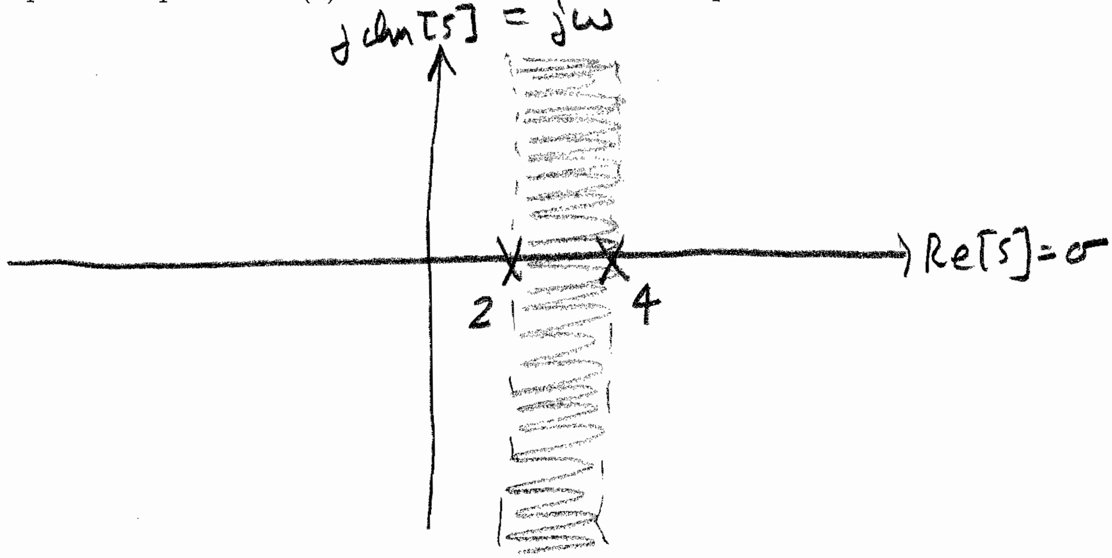
$= \left[\frac{1/2}{s-4} - \frac{1/2}{s-2}\right]e^{-2s}$, $2 < Re[s] < 4$

$= \frac{[\frac{1}{2}(s-2) - \frac{1}{2}(s-4)]\,e^{-2s}}{(s-4)(s-2)} = \frac{[\frac{1}{2}s - 1 - \frac{1}{2}s + 2]\,e^{-2s}}{(s-4)(s-2)}$, $2 < Re[s] < 4$

$$\boxed{H(s) = \frac{e^{-2s}}{(s-4)(s-2)},\ 2 < Re[s] < 4}$$

Problem 3, cont...

(b) **3 pts**. Give a pole-zero plot for $H(s)$. Indicate the ROC on the plot.



(c) **3 pts**. Is the system H causal? (Justify your answer)

- For a causal system, the ROC can be a right half-plane or the entire s-plane.
- Here we have a ROC that doesn't meet that requirement... it is a strip.
- So this system is not causal.

(d) **3 pts**. Is the system H BIBO stable? (Justify your answer)

The ROC of H(s) does not include the jω-axis, so the system is

NOT STABLE.

(e) **8 pts.** Find the system output $y(t)$ when the input is given by $x(t) = \delta'(t) - 2\delta(t)$.

TABLE: $X(s) = s-2$, all s.

$$Y(s) = X(s)H(s) = \frac{[s-2]e^{-2s}}{(s-4)(s-2)}, \quad 2 < Re[s] < 4$$

$$= \frac{e^{-2s}}{s-4}, \quad Re[s] < 4$$

TABLE:

$$-e^{4t}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-4}, \quad Re[s] < 4$$

TIME SHIFT PROPERTY:

$$-e^{4(t-2)}u(-(t-2)) \overset{\mathcal{L}}{\longleftrightarrow} \frac{e^{-2s}}{s-4}, \quad Re[s] < 4$$

$$= -e^{4(t-2)}u(2-t)$$

$$\boxed{y(t) = -e^{4(t-2)}u(2-t)}$$

4. **25 pts.** The input $x(t)$ and output $y(t)$ of LTI system H are related by the differential equation

$$y''(t) + 3y'(t) + 2y(t) = x(t).$$

The system input is given by $x(t) = 2u(t)$ for $t > 0$. Given the initial conditions $y(0^-) = 3$ and $y'(0^-) = -5$, find the system response $y(t)$ for $t > 0$.

$$\mathcal{UL}: \quad s^2 Y(s) - s y(0^-) - y'(0^-) + 3\left[s Y(s) - y(0^-) \right] + 2Y(s) = X(s)$$

$$s^2 Y(s) - 3s + 5 + 3s Y(s) - 3(3) + 2Y(s) = X(s)$$

$$\left[s^2 + 3s + 2\right] Y(s) - 3s + 5 - 9 = X(s)$$

$$\left[s^2 + 3s + 2\right] Y(s) = X(s) + 3s + 4$$

$$\text{TABLE}: \quad X(s) = \frac{2}{s}$$

$$\left[s^2 + 3s + 2\right] Y(s) = \frac{2}{s} + 3s + 4 = \frac{3s^2 + 4s + 2}{s}$$

$$Y(s) = \frac{3s^2 + 4s + 2}{s(s^2 + 3s + 2)} = \frac{3s^2 + 4s + 2}{s(s+1)(s+2)}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$A = \left.\frac{3s^2 + 4s + 2}{(s+1)(s+2)}\right|_{s=0} = \frac{2}{2} = 1$$

$$B = \left.\frac{3s^2 + 4s + 2}{s(s+2)}\right|_{s=-1} = \frac{3 - 4 + 2}{(-1)(1)} = \frac{1}{-1} = -1$$

$$C = \left.\frac{3s^2 + 4s + 2}{s(s+1)}\right|_{s=-2} = \frac{12 - 8 + 2}{(-2)(-1)} = \frac{6}{2} = 3$$

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} + \frac{3}{s+2}$$

$$\boxed{\text{TABLE}: \quad y(t) = u(t) - e^{-t} u(t) + 3e^{-2t} u(t)}$$