① 7.1) $x(t)$ is uniquely recoverable from samples taken at a rate $\omega_s = 10{,}000\pi$. Then ω_s must equal or exceed the Nyquist rate.

Then $x(t)$ is bandlimited to some frequency ω_M with

$$\omega_M < \frac{\omega_s}{2} = 5{,}000\,\pi \text{ rad/sec.}$$

So $X(\omega) = 0$ for $|\omega| > 5{,}000\pi$ rad/sec

② 7.3)

a) $x(t) = 1 + \cos(2{,}000\pi t) + \sin(4{,}000\pi t)$

$$X(\omega) = \delta(\omega) + \pi\left[\delta(\omega - 2000\pi) + \delta(\omega + 2000\pi)\right] + \frac{\pi}{j}\left[\delta(\omega - 4000\pi) + \delta(\omega + 4000\pi)\right]$$

$X(\omega) = 0$ for $|\omega| > 4000\pi = \omega_M$

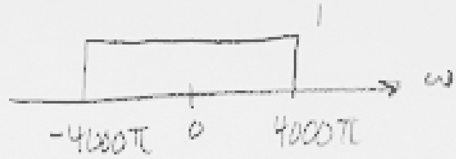
So the Nyquist rate is

$$\omega_N = 2\omega_M = 8000\pi \text{ rad/sec.}$$

2 7.3)...

b) $x(t) = \frac{\sin(4000\pi t)}{\pi t}$

$X(\omega) =$

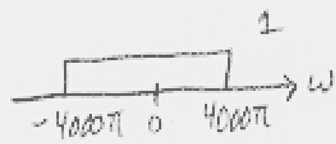
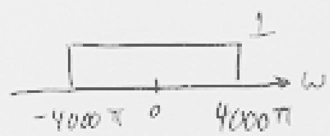


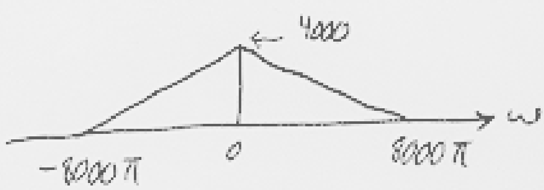
$X(\omega)$ is bandlimited to $\omega_M = 4000\pi$.

So the Nyquist rate is

$$\omega_N = 2\omega_M = 8000\pi \text{ rad/sec.}$$

c) $x(t) = \left(\frac{\sin 4000\pi t}{\pi t}\right)^2$

$X(\omega) = \frac{1}{2\pi}\Big[$  $*$  $\Big]$

$=$ 

$X(\omega)$ is bandlimited to $\omega_M = 8000\pi$.

So the Nyquist rate is

$$\omega_N = 2\omega_M = 16000\pi \text{ rad/sec}$$

3) 7.12)

$x_d[n] \overset{\mathcal{F}}{\longleftrightarrow} X_d(e^{j\omega})$ where

$$X_d(e^{j\omega}) = 0 \text{ for } \frac{3\pi}{4} < |\omega| < \pi.$$

$\Rightarrow$ So the fundamental period of $X_d(e^{j\omega})$ is bandlimited to $\omega_M = \frac{3\pi}{4}$.

$x_d[n]$ is then converted to a continuous-time signal $x_c(t)$ by ideal lowpass filtering as in Eq. (7.11).

Thus, we must switch to the notation of Section 7.4 and use "Ω" for discrete frequency and "ω" for continuous frequency.

So $X_d(e^{j\Omega})$ is bandlimited to $\Omega_M = \frac{3\pi}{4}$.

The reconstruction interval is $T = \frac{1}{1000}$.

The relationship between discrete and continuous frequency is $\Omega = \omega T$, or $\omega = \frac{\Omega}{T}$.

So $X_c(\omega)$ will be bandlimited to

$$\omega_m = \frac{\Omega_M}{T} = \frac{3000\pi}{4} \text{ rad/sec} = 750\pi \text{ rad/sec.}$$

So $X(\omega) = 0$ for $|\omega| > 750\pi$ rad/sec.

3 7.21) $x(t) \xleftrightarrow{\mathcal{F}} X(\omega)$

$$x_p(t) = x(t)p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)$$

$$T = 10^{-4} \text{ sec}$$

$$\omega_s = \frac{2\pi}{T} = 20{,}000\,\pi \text{ rad/sec}$$

To prevent aliasing, we need

$$\omega_s > 2\omega_M$$

$$\omega_M < \tfrac{1}{2}\omega_s = 10{,}000\,\pi \text{ rad/sec.}$$

a) $X(\omega) = 0$ for $|\omega| > 5000\pi$

$\omega_M \le 5000\pi < 10{,}000\pi$ ✓

$x(t)$ can be recovered.

b) $X(\omega) = 0$, $|\omega| > 15{,}000\pi$.

Since $\omega_M = 15{,}000\pi > \frac{1}{2}\omega_s$, $x(t)$ cannot be recovered.

c) $\mathrm{Re}[X(\omega)] = 0$, $|\omega| > 5000\pi$.

we don't know anything about $\mathrm{Im}\{X(\omega)\} = \mathcal{F}[\mathrm{Od}\{x(t)\}]$
so the odd part of the signal may contain
frequencies greater than $\frac{1}{2}\omega_s$.

$x(t)$ cannot be guaranteed recoverable.

d) $x(t) \in \mathbb{R}$ and $X(\omega) = 0$, $\omega > 5000\pi$.

For a real signal, $X(\omega)$ is conjugate symmetric.
So $X(\omega) = 0$, $\omega > 5000\pi$ implies
that $X(\omega) = 0$, $|\omega| > 5000\pi$.

So $\omega_M = 5000\pi < 10{,}000\pi$ ✓

$x(t)$ can be recovered.

e) $x(t) \in \mathbb{R}$ and $X(\omega) = 0$, $\omega < -15{,}000\pi$.

Again, $X(\omega)$ must be conjugate symmetric, so
this implies that $X(\omega) = 0$, $|\omega| < 15{,}000\pi$.
But $15{,}000\pi > \frac{1}{2}\omega_s$, so $x(t)$ cannot be
recovered.

f) $X(\omega) * X(\omega) = 0$, $|\omega| > 15{,}000\pi$.

This implies that $X(\omega) = 0$ for $|\omega| > \frac{15000\pi}{2} = 7500\pi$.

Since $7500\pi < \frac{1}{2}\omega_s$, $x(t)$ can be recovered.

g) $|X(\omega)| = 0$, $\omega > 5000\pi$.

If $x(t)$ is complex, the spectrum may not be
symmetric. So we don't know anything about what
$X(\omega)$ does when $\omega < 0$.

Cannot guarantee that $x(t)$ is recoverable.

④ 7.30)

a) For the continuous-time system,

$$y_c'(t) + y_c(t) = x_c(t).$$

The input is $x_c(t) = \delta(t)$, so we have

$$y_c'(t) + y_c(t) = \delta(t)$$

Taking Fourier transforms on both sides, we have

$$[1+j\omega]\, Y_c(\omega) = 1,$$

or $Y_c(\omega) = \dfrac{1}{1+j\omega}$

Then $y_c(t) = \mathcal{F}^{-1}\left[\dfrac{1}{1+j\omega}\right] = \underline{\underline{e^{-t}u(t)}}.$

b) We are given in the figure that the sampling interval is T, so that $y[n] = y_c(nT) = e^{-nT}u[n]$

$$= (e^{-T})^n u[n]$$

From Table 5.2, $Y(e^{j\Omega}) = \dfrac{1}{1-e^{-T}e^{-j\Omega}}$.

We want $w[n] = \delta[n] \overset{\mathcal{F}}{\longleftrightarrow} W(e^{j\Omega}) = 1$.

So, for the discrete system,

$$H(e^{j\Omega}) = \frac{W(e^{j\Omega})}{Y(e^{j\Omega})} = \frac{1}{Y(e^{j\Omega})} = 1 - e^{-T}e^{-j\Omega}$$

$h[n] = \mathcal{F}^{-1}[H(e^{j\Omega})] = \underline{\underline{\delta[n] - e^{-T}\delta[n-1]}}$

5 7.31) For the digital filter,

$$y[n] = \tfrac{1}{2}y[n-1] + x[n]$$

$$y[n] - \tfrac{1}{2}y[n-1] = x[n]$$

F.T.: $$\left[1 - \tfrac{1}{2}e^{-j\Omega}\right]Y(e^{j\Omega}) = X(e^{j\Omega})$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

The continuous-time input is $x_c(t) \overset{\mathcal{F}}{\longleftrightarrow} X_c(\omega)$, where $X_c(\omega)=0$ for $|\omega| > \pi/T$. So, by equation (7.23), the fundamental period of $X(e^{j\Omega})$, the F.T. of $x[n]$, is given by

$$X(e^{j\Omega}) = \frac{1}{T}X_c\left(\frac{\Omega}{T}\right), \quad -\pi \le \Omega \le \pi$$

Then $y[n]$, the digital filter output, has F.T. (fundamental period)

$$Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega}) = \frac{\frac{1}{T}X_c\left(\frac{\Omega}{T}\right)}{1-\frac{1}{2}e^{-j\Omega}}, \quad -\pi \le \Omega \le \pi$$

The "impulse train" signal $\tilde{y}(t)$ then has a periodic spectrum $\tilde{Y}(\omega)$ with fundamental period $(\Omega = \omega T)$

$$\tilde{Y}(\omega) = \frac{\frac{1}{T}X_c(\omega)}{1-\frac{1}{2}e^{-j\omega T}}, \quad -\frac{\pi}{T} < \omega < \frac{\pi}{T}.$$

After ideal low-pass filtering with cutoff freq. $\omega_c = \frac{\pi}{T}$,

$$Y_c(\omega) = \frac{\frac{1}{T}X_c(\omega)}{1-\frac{1}{2}e^{-j\omega T}}.$$

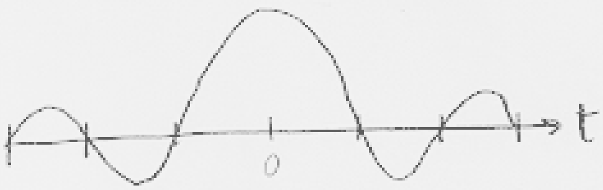
So $$H_c(\omega) = \frac{Y_c(\omega)}{X_c(\omega)} = \frac{1}{X_c(\omega)}Y_c(\omega) = \underline{\underline{\frac{1/T}{1-\frac{1}{2}e^{-j\omega T}}}}$$

6 8.5)

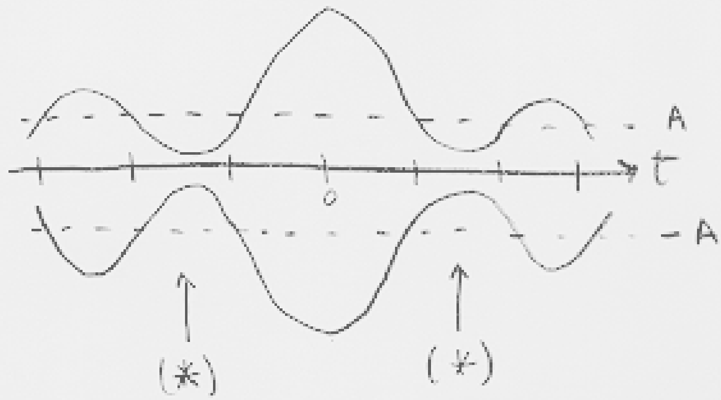
$$x(t) = \frac{\sin 1000\pi t}{\pi t}$$

$$w(t) = [x(t) + A]\cos(10000\pi t)$$

$x(t)$



Envelope of $w(t)$



For synchronous demodulation, A must be larger than the first sidelobe of $x(t)$ so that the height of the envelope remains greater than zero at the times marked "(*)" above.

The first zero-crossing in $x(t)$ is when $1000\pi t = \pi$, or $t = \frac{1}{1000}$.
The 2nd zero-crossing is when $1000\pi t = 2\pi$, or $t = \frac{2}{1000} = \frac{1}{500}$.
The maximum sidelobe excursion is half way between these two, when $t = \frac{1.5}{1000} = \frac{3}{2000}$.
The value of $x(t)$ at this point is $x\left(\frac{3}{2000}\right) = \frac{\sin\frac{3\pi}{2}}{3\pi/2000} = -\frac{2000}{3\pi}$

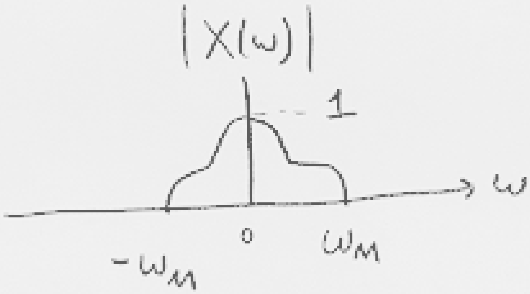
So, for synchronous demodulation, $A > \frac{2000}{3\pi}$, or $A_{min} = \frac{2000}{3\pi}$.

The maximum amplitude of the signal occurs at $t=0$:

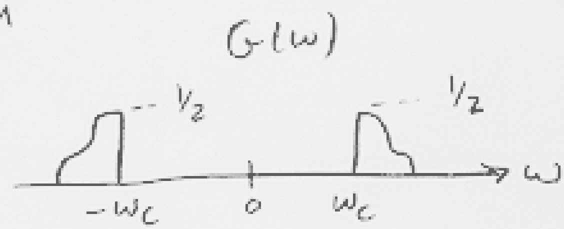
$$x(0) = \frac{d/dt \sin 1000\pi t}{d/dt\ \pi t}\bigg|_{t=0} = \frac{1000\pi \cos 1000\pi t}{\pi}\bigg|_{t=0} = \frac{1000\pi}{\pi} = 1000.$$

The maximum modulation index is $m_{max} = \frac{\max|x(t)|}{A_{min}} = \frac{1000}{2000/3\pi} = \frac{3\pi}{2}$

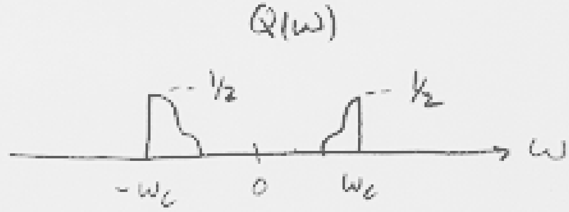
7 8.7) Suppose the spectrum of $x(t)$ is



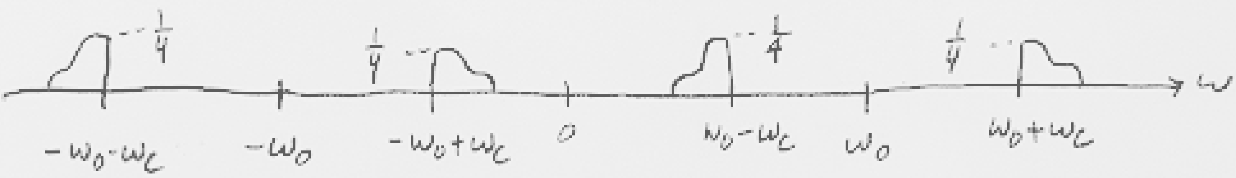
Then the spectrum of $g(t)$ is



The spectrum of $q(t)$ is



The spectrum of $g(t)\cos\omega_0 t$ is



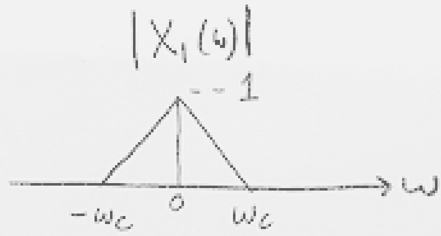
← Lowpass Filter.

- The filter gain must be $\boxed{A = 2,}$ since $2\left(\frac{1}{4}\right) = \frac{1}{2}$ will give us the spectrum of $q(t)$ from $g(t)\cos\omega_0 t$.

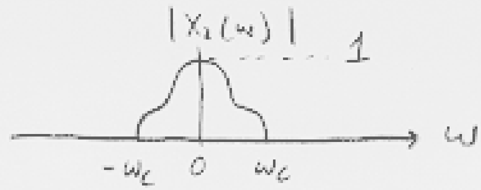
- We need $\boxed{\omega_0 \geq 2\omega_c}$ so that the two lower side bands don't cross $\omega = 0$ and "bump into each other" in $g(t)\cos\omega_0 t$.

8) 8.9)

Suppose $x_1(t)$ has spectrum

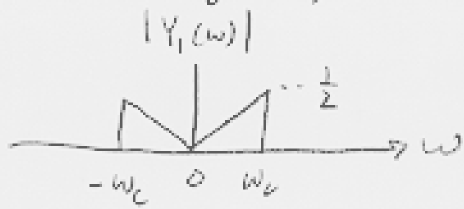


and $x_2(t)$ has spectrum



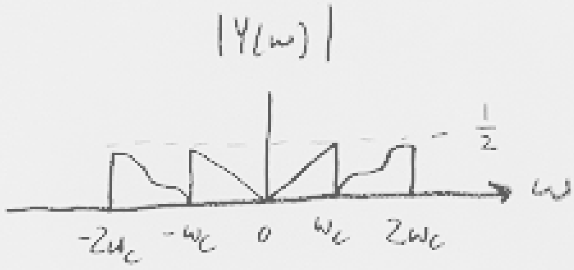
Let $y_1(t)$ be the AM-SSB/SC signal generated from $x_1(t)$ with carrier frequency ω_c and $y_2(t)$ be the AM-SSB/SC signal generated from $x_2(t)$ with carrier frequency $2\omega_c$.

Then $y_1(t)$ has spectrum



while $y_2(t)$ has spectrum

For the FDM signal $y(t) = y_1(t) + y_2(t)$, the spectrum is



a) Clearly, $Y(\omega) = 0$ for $|\omega| > 2\omega_c$.

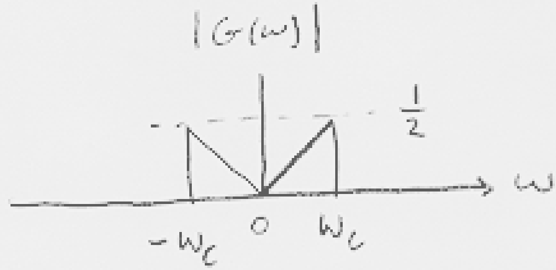
8 8.9) We want to recover $x_1(t)$ from the FDM signal $y(t)$ by

$$x_1(t) = \left[\left\{ y(t) * \frac{\sin \omega_0 t}{\pi t} \right\} \cos \omega_0 t \right] * \frac{A \sin \omega_c t}{\pi t}$$

We are asked to design ω_0 and A.

$\Rightarrow$ We need to eliminate the contribution of $x_2(t)$ from $y(t)$. To do this, we need $\omega_0 = \omega_c$, so that

$$g(t) = y(t) * \frac{\sin \omega_c t}{\pi t} \text{ has spectrum}$$

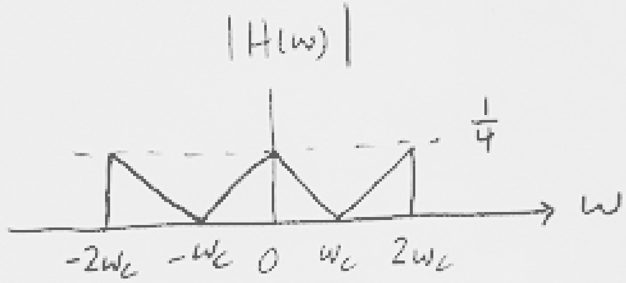


Then let $h(t) = g(t) \cos \omega_0 t$

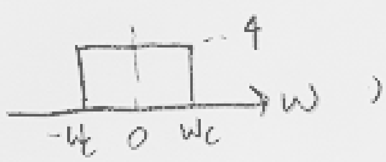
$$= g(t) \cos \omega_c t$$

$$= \left\{ y(t) * \frac{\sin \omega_c t}{\pi t} \right\} \cos \omega_c t$$

will have spectrum



To get back $x_1(t)$, we need to put $h(t)$ through a filter

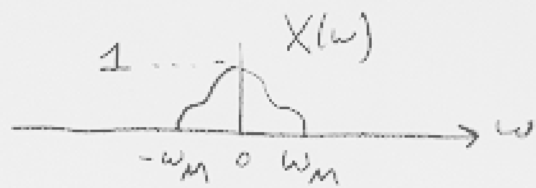
with freq. response  , so $\underline{\underline{A = 4}}$

9) 8.21)

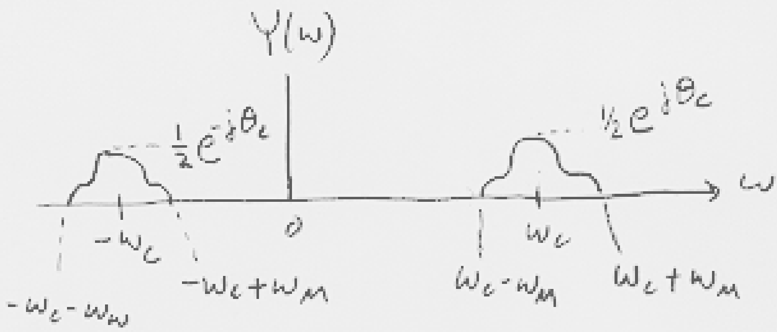
a) $y(t) = x(t)\cos(\omega_c t + \theta_c)$

$$\begin{aligned} w(t) &= y(t)\cos(\omega_c t + \theta_c) \\ &= x(t)\cos^2(\omega_c t + \theta_c) \\ &= x(t)\left[\tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega_c t + 2\theta_c)\right] \\ &= \tfrac{1}{2}x(t) + \tfrac{1}{2}x(t)\cos(2\omega_c t + 2\theta_c). \end{aligned}$$

b) Now $x(t)$ is bandlimited to ω_M. Suppose the spectrum of $x(t)$ is

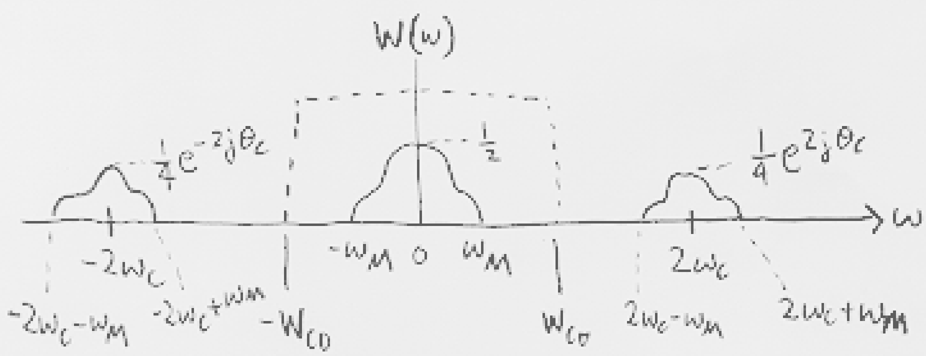


Then the spectrum of $y(t) = x(t)\cos(\omega_c t + \theta_c)$ is



$\Rightarrow$ we must have $\omega_c > \omega_M$ to avoid overlapping the spectra.

The spectrum of $w(t) = y(t)\cos(\omega_c t + \theta_c)$ is shown below. Note that the phase offsets $e^{-j\theta_c}$ and $e^{j\theta_c}$ are <u>cancelled</u>.



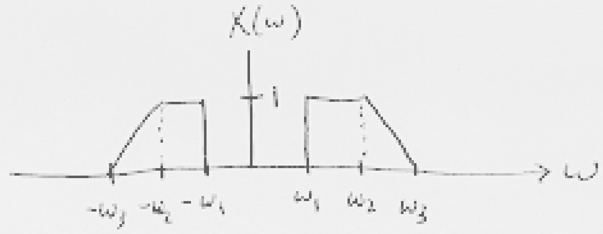
We need

$$\omega_M \le W_{co} \le 2\omega_c - \omega_M$$

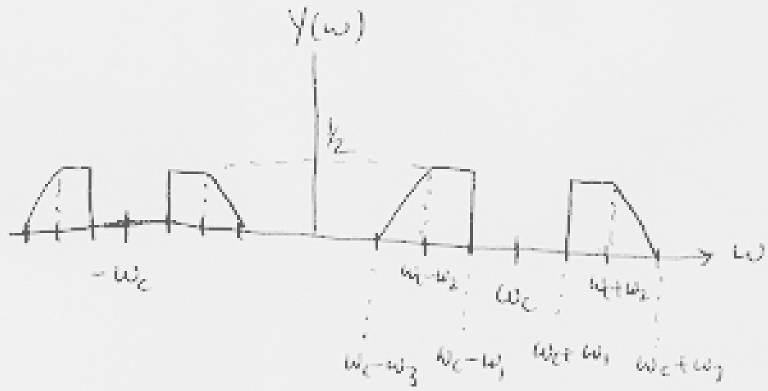
Answer is independent of θ_c.

8 8.29) a)

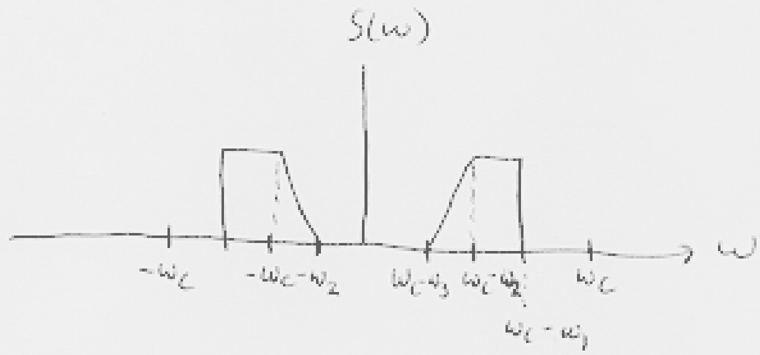
The spectrum of the baseband signal is



After multiplication by the carrier, $y(t) = x(t)c(t)$ and $Y(\omega)$ is



- For the top system, we low-pass filter to retain the lower sidebands:



- For the bottom system, we high-pass filter to retain the upper sidebands: