

### Formulas from Trigonometry:

$$\sin^2 A + \cos^2 A = 1$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\sin A \sin B = \frac{1}{2} \{ \cos(A - B) - \cos(A + B) \}$$

$$\sin A \cos B = \frac{1}{2} \{ \sin(A - B) + \sin(A + B) \}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

$$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A - \cos B = 2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(B - A)$$

$$\cos A \cos B = \frac{1}{2} \{ \cos(A - B) + \cos(A + B) \}$$

$$\cos(\theta) = \sin(\theta + \pi/2)$$

### Differentiation Formulas:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + \frac{du}{dx} v$$

$$\text{Chain rule: } \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}, \quad \left(-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right)$$

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}, \quad \left(-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right)$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx} \left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

$$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$$

$$\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx} \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}, \quad (0 < \cos^{-1} u < \pi)$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx} \log_a u = \frac{\log_a e}{u} \frac{du}{dx}, \quad a \neq 0, 1$$

### Integration Formulas:

$$\text{Integration by parts: } \int u dv = uv - \int v du$$

$$\int \frac{du}{u} = \ln |u|$$

$$\int a^u du = \frac{a^u}{\ln a}, \quad a > 0, a \neq 1$$

$$\int \cos u du = \sin u$$

$$\int \sin^2 u du = \frac{u}{2} - \frac{\sin 2u}{4} = \frac{1}{2}(u - \sin u \cos u)$$

$$\int \cos^2 u du = \frac{u}{2} + \frac{\sin 2u}{4} = \frac{1}{2}(u + \sin u \cos u)$$

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left( \frac{u-a}{u+a} \right)$$

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2})$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$$

$$\int x \sin ax dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int x^2 \cos ax dx = \frac{2x}{a^2} \cos ax + \left( \frac{x^2}{a} - \frac{2}{a^3} \right) \sin ax$$

$$\int \tan^2 ax dx = \frac{\tan ax}{a} - x$$

$$\int \ln x dx = x \ln x - x$$

$$\int e^u du = e^u$$

$$\int \sin u du = -\cos u$$

$$\int \tan u du = -\ln |\cos u|$$

$$\int \tan^2 u du = \tan u - u$$

$$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a}$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2})$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$$

$$\int x^2 \sin ax dx = \frac{2x}{a^2} \sin ax + \left( \frac{2}{a^3} - \frac{x^2}{a} \right) \cos ax$$

$$\int x \cos ax dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a} \left( x - \frac{1}{a} \right)$$

$$\int x \ln x dx = \frac{x^2}{2} \left( \ln x - \frac{1}{2} \right)$$

**Summation Formulas:**

$$\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1 - \alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1 - a}, \quad |a| < 1$$

$$\sum_{k=0}^{\infty} ka^k = \frac{a}{(1 - a)^2}, \quad |a| < 1$$

$$\sum_{k=0}^n a^k = \frac{1 - a^{n+1}}{1 - a}, \quad a \neq 1$$

$$\sum_{k=0}^n ka^k = \frac{a\{1 - (n + 1)a^n + na^{n+1}\}}{(1 - a)^2}$$

**Signals:**

$$\delta(t) = \frac{d}{dt}u(t) \qquad \delta[n] = u[n] - u[n - 1]$$

$$\langle x_1(t), x_2(t) \rangle = \int_{-\infty}^{\infty} x_1(t)x_2^*(t) dt \qquad \langle x_1[n], x_2[n] \rangle = \sum_{n=-\infty}^{\infty} x_1[n]x_2^*[n]$$

$$\mathcal{E}v\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\} \qquad \mathcal{O}d\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$$

**Complex Exponential Signals:**

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct $\omega_0$	Identical signals for values of $\omega_0$ separated by multiples of $2\pi$
Periodic for any choice of $\omega_0$	Periodic only if $\omega_0/(2\pi) = m/N \in \mathbb{Q}$
Fundamental frequency $\omega_0$	Fundamental frequency $\omega_0/m$
Fundamental period: $\omega_0 = 0$ : undefined $\omega_0 \neq 0$ : $2\pi/\omega_0$	Fundamental period: $\omega_0 = 0$ : one $\omega_0 \neq 0$ : $2\pi m/\omega_0$

**Systems:**

System  $H$  is linear if  $H\{ax_1(t) + bx_2(t)\} = aH\{x_1(t)\} + bH\{x_2(t)\}$ .

System  $H$  is time invariant if  $H\{x(t - t_0)\} = y(t - t_0)$ .

System  $H$  is memoryless if the current output does not depend on future or past inputs.

System  $H$  is invertible if distinct input signals produce distinct output signals.

System  $H$  is invertible if an inverse system  $G$  exists which “undoes” the action of  $H$ .

System  $H$  is causal if the current output does not depend on future inputs.

LTI system  $H$  is causal iff  $h(t) = 0 \forall t < 0$ .

Bounded:  $x(t)$  is bounded if  $\exists B \in \mathbb{R}, B > 0$ , such that  $|x(t)| \leq B \forall t \in \mathbb{R}$ .

System  $H$  is BIBO stable if every bounded input signal produces a bounded output signal.

LTI system  $H$  is BIBO stable iff  $\int_{-\infty}^{\infty} |h(t)| dt < \infty$ .

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau = \int_{-\infty}^{\infty} x(t - \tau)h(\tau) d\tau$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k] = \sum_{k=-\infty}^{\infty} x[n - k]h[k]$$

$$s(t) = \int_{-\infty}^t h(\tau) d\tau \qquad h(t) = \frac{d}{dt}s(t)$$

$$s[n] = \sum_{k=-\infty}^n h[k] \qquad h[n] = s[n] - s[n - 1]$$