Formulas from Trigonometry:

$\sin^2 A + \cos^2 A = 1$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$\sin \frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$

$\cos \frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$

$\tan \frac{A}{2} = \frac{\sin A}{1+\cos A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$

$\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$

$\sin A - \sin B = 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$

$\cos A + \cos B = 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$

$\cos A - \cos B = 2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(B-A)$

$\sin A \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$

$\cos A \cos B = \frac{1}{2}\{\cos(A-B) + \cos(A+B)\}$

$\sin A \cos B = \frac{1}{2}\{\sin(A-B) + \sin(A+B)\}$

$\cos(\theta) = \sin(\theta + \pi/2)$

Differentiation Formulas:

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$

$\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

$\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$

$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \ \left(-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right)$

$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}, \ (0 < \cos^{-1} u < \pi)$

$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}, \ \left(-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right)$

$\frac{d}{dx}e^u = e^u \frac{du}{dx}$

$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$

$\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx}, \ a \neq 0, 1$

Integration Formulas:

Integration by parts: $\int u\,dv = uv - \int v\,du$

$\int \frac{du}{u} = \ln|u|$

$\int e^u\,du = e^u$

$\int a^u\,du = \frac{a^u}{\ln a}, \quad a > 0,\ a \neq 1$

$\int \sin u\,du = -\cos u$

$\int \cos u\,du = \sin u$

$\int \tan u\,du = -\ln \cos u$

$\int \sin^2 u\,du = \frac{u}{2} - \frac{\sin 2u}{4} = \frac{1}{2}(u - \sin u \cos u)$

$\int \tan^2 u\,du = \tan u - u$

$\int \cos^2 u\,du = \frac{u}{2} + \frac{\sin 2u}{4} = \frac{1}{2}(u + \sin u \cos u)$

$\int \frac{du}{u^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a}$

$\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left(\frac{u-a}{u+a}\right)$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a}$

$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2})$

$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2})$

$\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$

$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$

$\int x \sin ax\,dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$

$\int x^2 \sin ax\,dx = \frac{2x}{a^2}\sin ax + \left(\frac{2}{a^3} - \frac{x^2}{a}\right)\cos ax$

$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$

$\int x \cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$

$\int x^2 \cos ax\,dx = \frac{2x}{a^2}\cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\sin ax$

$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$

$\int \tan^2 ax\,dx = \frac{\tan ax}{a} - x$

$\int x e^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$

$\int \ln x\,dx = x \ln x - x$

$\int x \ln x\,dx = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right)$

Summation Formulas:

$$\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1-\alpha}, \quad \alpha \neq 1 \qquad \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}, \quad |a| < 1$$

$$\sum_{k=0}^{\infty} k a^k = \frac{a}{(1-a)^2}, \quad |a| < 1 \qquad \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}, \quad a \neq 1$$

$$\sum_{k=0}^{n} k a^k = \frac{a\{1-(n+1)a^n + n a^{n+1}\}}{(1-a)^2}$$

Signals:

$$\delta(t) = \tfrac{d}{dt}u(t) \qquad \delta[n] = u[n] - u[n-1]$$

$$\langle x_1(t), x_2(t)\rangle = \int_{-\infty}^{\infty} x_1(t) x_2^*(t)\, dt \qquad \langle x_1[n], x_2[n]\rangle = \sum_{n=-\infty}^{\infty} x_1[n] x_2^*[n]$$

$$\mathcal{E}v\{x(t)\} = \tfrac{1}{2}\{x(t) + x(-t)\} \qquad \mathcal{O}d\{x(t)\} = \tfrac{1}{2}\{x(t) - x(-t)\}$$

Complex Exponential Signals:

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct w_0	Identical signals for values of w_0 separated by multiples of 2π
Periodic for any choice of w_0	Periodic only if $w_0/(2\pi) = m/N \in \mathbb{Q}$
Fundamental frequency w_0	Fundamental frequency w_0/m
Fundamental period: $w_0 = 0$: undefined $w_0 \neq 0$: $2\pi/w_0$	Fundamental period: $w_0 = 0$: one $w_0 \neq 0$: $2\pi m/w_0$

Systems:

System H is linear if $H\{a x_1(t) + b x_2(t)\} = aH\{x_1(t)\} + bH\{x_2(t)\}$.
System H is time invariant if $H\{x(t-t_0)\} = y(t-t_0)$.
System H is memoryless if the current output does not depend on future or past inputs.
System H is invertible if distinct input signals produce distinct output signals.
System H is invertible if an inverse system G exists which "undoes" the action of H.
System H is causal if the current output does not depend on future inputs.
LTI system H is causal iff $h(t) = 0 \ \forall\ t < 0$.
Bounded: $x(t)$ is bounded if $\exists\ B \in \mathbb{R}$, $B > 0$, such that $|x(t)| \leq B \ \forall\ t \in \mathbb{R}$.
System H is BIBO stable if every bounded input signal produces a bounded output signal.
LTI system H is BIBO stable iff $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t-\tau) h(\tau)\, d\tau$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k] h[k]$$

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau \qquad h(t) = \tfrac{d}{dt}s(t)$$

$$s[n] = \sum_{k=-\infty}^{n} h[k] \qquad h[n] = s[n] - s[n-1]$$

Fourier Series Representation for a Periodic $x(t)$ with Period T:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t} \qquad\qquad X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_s)$$

$$a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-jk\omega_s t}\, dt \qquad\qquad \omega_s = \frac{2\pi}{T}$$

Sampling:

$$\text{sampling interval: } T \qquad\qquad \text{sampling frequency: } \omega_s = \frac{2\pi}{T}$$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t + nT) \qquad\qquad P(j\omega) = \frac{2\pi}{T}\sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT) \qquad\qquad X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$

$$\omega_s > 2\omega_M$$

Discrete-Time Processing of Continuous-Time Signals:

$$\text{continuous-time frequency: } \omega \qquad\qquad \omega = \frac{\Omega}{T}$$

$$\text{discrete-time frequency: } \Omega \qquad\qquad \Omega = \omega T$$

$$H_c(j\omega) = \begin{cases} H_d\left(e^{j\omega T}\right), & |\omega| < \frac{\omega_s}{2} \\ 0, & |\omega| > \frac{\omega_s}{2} \end{cases}$$

$$H_d\left(e^{j\Omega}\right) = H_c\left(\frac{\Omega}{T}\right) \qquad \text{(fundamental period)}$$

$$H_d\left(e^{j\Omega}\right) = \sum_{k=-\infty}^{\infty} H_c\left(\frac{\Omega - 2\pi k}{T}\right)$$

Properties of the Fourier Transform:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt \qquad\qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

Property	**Aperiodic signal**	**Fourier transform**
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time Reversal	$x(-t)$	$X(-j\omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) * Y(j\omega)$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \mathrm{Re}\{X(j\omega)\} = \mathrm{Re}\{X(-j\omega)\} \\ \mathrm{Im}\{X(j\omega)\} = -\mathrm{Im}\{X(-j\omega)\} \\ \|X(j\omega)\| = \|X(-j\omega)\| \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\mathrm{Re}\{X(j\omega)\}$ $j\mathrm{Im}\{X(j\omega)\}$

Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2\,d\omega$$

Basic Fourier Transform Pairs:

Signal	Fourier transform
$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \omega_0)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$x(t) = 1$	$2\pi\delta(\omega)$
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \leq \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{\infty} \frac{2\sin k\omega_0 T_1}{k}\delta(\omega - k\omega_0)$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$
$x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2\sin\omega T_1}{\omega}$
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
$\delta(t - t_0)$	$e^{-j\omega t_0}$
$e^{-at}u(t), \quad \mathrm{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
$te^{-at}u(t), \quad \mathrm{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t), \quad \mathrm{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$

Properties of the Discrete-Time Fourier Transform:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad\qquad x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

Property	**Aperiodic signal**	**Fourier transform**
Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
Time Shifting	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Time Reversal	$x[-n]$	$X(e^{-j\omega})$
Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})\,d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1-e^{-j\omega})X(e^{j\omega})$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{-j\omega}}X(e^{j\omega})$
Differentiation in Frequency	$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \text{Re}\{X(e^{j\omega})\} = \text{Re}\{X(e^{-j\omega})\} \\ \text{Im}\{X(e^{j\omega})\} = -\text{Im}\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
Symmetry for Real and Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
Symmetry for Real and Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ [$x[n]$ real] $x_o[n] = \mathcal{O}d\{x[n]\}$ [$x[n]$ real]	$\text{Re}\{X(e^{j\omega})\}$ $j\text{Im}\{X(e^{j\omega})\}$

Parseval's Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2\,d\omega$$

Basic Discrete-Time Fourier Transform Pairs:

Signal	Fourier transform
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l)$
Periodic square wave $x[n] = \begin{cases} 1, & \lvert n\rvert < N_1 \\ 0, & N_1 < \lvert n\rvert \leq N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$\sum_{k=-\infty}^{\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$
$a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{1 - ae^{-j\omega}}$
$x[n] = \begin{cases} 1, & \lvert n\rvert \leq N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$
$\frac{\sin Wn}{\pi n}, \quad 0 < W < \pi$	$X(e^{j\omega}) = \begin{cases} 1, & 0 \leq \lvert\omega\rvert \leq W \\ 0, & W < \lvert\omega\rvert \leq \pi \end{cases}$ $X(e^{j\omega})$ is 2π-periodic
$\delta[n]$	1
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2\pi k)$
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$(n+1)a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$

Properties of the Laplace Transform:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt \qquad\qquad x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$$

Property	**Signal**	**Laplace Transform**	**ROC**
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time Shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
s-Domain Shifting	$e^{s_0 t}x(t)$	$X(s - s_0)$	$R + s_0$
Time Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	aR
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Time Differentiation	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
s-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Time Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathrm{Re}\{s\} > 0\}$

Laplace Transforms of Elementary Functions:

Signal	Transform	ROC
$\delta(t)$	1	All s
$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} > 0$
$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} < 0$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\mathrm{Re}\{s\} > 0$
$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\mathrm{Re}\{s\} < 0$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\mathrm{Re}\{s\} > -\alpha$
$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\mathrm{Re}\{s\} < -\alpha$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\mathrm{Re}\{s\} > -\alpha$
$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\mathrm{Re}\{s\} < -\alpha$
$\delta(t-T)$	e^{-sT}	All s
$[\cos\omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\} > 0$
$[\sin\omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\} > 0$
$[e^{-\alpha t}\cos\omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\mathrm{Re}\{s\} > -\alpha$
$[e^{-\alpha t}\sin\omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\mathrm{Re}\{s\} > -\alpha$
$u_n(t) = \frac{d^n}{dt^n}\delta(t)$	s^n	All s
$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\mathrm{Re}\{s\} > 0$

Properties of the Unilateral Laplace Transform:

$$\mathcal{X}(s) = \int_{0^-}^{\infty} x(t)e^{-st}\,dt \qquad\qquad x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \mathcal{X}(s)e^{st}\,ds$$

Property	**Signal**	**Unilateral Laplace Transform**
	$x(t)$ $x_1(t)$ $x_2(t)$	$\mathcal{X}(s)$ $\mathcal{X}_1(s)$ $\mathcal{X}_2(s)$
Linearity	$ax_1(t) + bx_2(t)$	$a\mathcal{X}_1(s) + b\mathcal{X}_2(s)$
s-Domain Shifting	$e^{s_0 t}x(t)$	$\mathcal{X}(s - s_0)$
Time Scaling	$x(at), \quad a > 0$	$\frac{1}{a}\mathcal{X}\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$\mathcal{X}^*(s^*)$
Convolution (assumes $x_1(t)$ and $x_2(t)$ are zero $\forall\, t < 0$).	$x_1(t) * x_2(t)$	$\mathcal{X}_1(s)\mathcal{X}_2(s)$
Time Differentiation	$\frac{d}{dt}x(t)$	$s\mathcal{X}(s) - x(0^-)$
	$\frac{d^n}{dt^n}x(t)$	$s^n\mathcal{X}(s) - s^{n-1}x(0^-) - \cdots - x^{(n-1)}(0^-)$
s-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}\mathcal{X}(s)$
Time Integration	$\int_{0^-}^{t} x(\tau)\,d\tau$	$\frac{1}{s}\mathcal{X}(s)$

Properties of the z-Transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad\qquad x[n] = \frac{1}{2\pi j} \oint_C X(z) z^{n-1}\, dz$$

Property	Signal	z-Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least $R_1 \cap R_2$
Time Shifting	$x[n - n_0]$	$z^{-n_0} X(z)$	R, except possibly $z = 0$
z-Domain Scaling	$e^{j\omega_0 n} x[n]$	$X(e^{-j\omega_0} z)$	R
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
	$a^n x[n]$	$X\left(\frac{z}{a}\right)$	$\|a\|R$
Time Reversal	$x[-n]$	$X(z^{-1})$	R^{-1}
Time Expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases} \quad r \in \mathbb{Z}$	$X(z^k)$	$R^{\frac{1}{k}}$
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Convolution	$x_1[n] * x_2[n]$	$X_1(z) X_2(z)$	At least $R_1 \cap R_2$
First Difference	$x[n] - x[n-1]$	$(1 - z^{-1}) X(z)$	At least $R \cap \{\|z\| > 0\}$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - z^{-1}} X(z)$	At least $R \cap \{\|z\| > 1\}$
z-Domain Differentiation	$nx[n]$	$-z \frac{d}{dz} X(z)$	R

Common z-Transform Pairs:

Signal	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z, except 0 (if $m>0$) or ∞ (if $m<0$)
$\alpha^n u[n]$	$\dfrac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-\alpha^n u[-n-1]$	$\dfrac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$n\alpha^n u[n]$	$\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-n\alpha^n u[-n-1]$	$\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$
$[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
$[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$

Properties of the Unilateral z-Transform:

$$\mathcal{X}(z) = \sum_{n=0}^{\infty} x[n] z^{-n} \qquad\qquad x[n] = \frac{1}{2\pi j} \oint_C \mathcal{X}(z) z^{n-1}\, dz$$

Property	**Signal**	**Unilateral z-Transform**
	$x[n]$ $x_1[n]$ $x_2[n]$	$\mathcal{X}(z)$ $\mathcal{X}_1(z)$ $\mathcal{X}_2(z)$
Linearity	$ax_1[n] + bx_2[n]$	$a\mathcal{X}_1(z) + b\mathcal{X}_2(z)$
Time Delay	$x[n-1]$	$z^{-1}\mathcal{X}(z) + x[-1]$
Time Advance	$x[n+1]$	$z\mathcal{X}(z) - zx[0]$
z-Domain Scaling	$e^{j\omega_0 n} x[n]$	$\mathcal{X}(e^{-j\omega_0} z)$
	$z_0^n x[n]$	$\mathcal{X}(z/z_0)$
	$a^n x[n]$	$\mathcal{X}(z/a)$
Time Expansion	$x_{(k)}[n] = \begin{cases} x[m], & n = mk \\ 0, & n \neq mk \end{cases} \quad m \in \mathbb{Z}$	$\mathcal{X}(z^k)$
Conjugation	$x^*[n]$	$\mathcal{X}^*(z^*)$
Convolution (assumes $x_1[n]$ and $x_2[n]$ are zero $\forall\ n < 0$).	$x_1[n] * x_2[n]$	$\mathcal{X}_1(z)\mathcal{X}_2(z)$
First Difference	$x[n] - x[n-1]$	$(1 - z^{-1})\mathcal{X}(z) - x[-1]$
Accumulation	$\sum_{k=0}^{n} x[k]$	$\frac{1}{1 - z^{-1}}\mathcal{X}(z)$
z-Domain Differentiation	$nx[n]$	$-z\frac{d}{dz}\mathcal{X}(z)$