ECE
3793

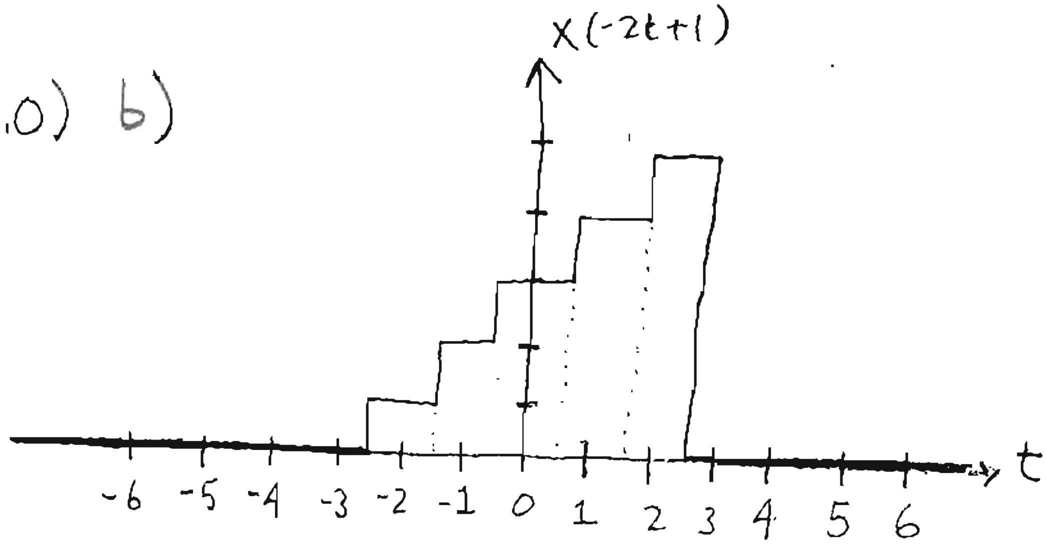
HW 1 SOLUTION

HAVLICEK

1.0) a)

t	$-2t+1$	$x(-2t+1)$
-6 to -5.5	13 to 12	0
-5.5 to -5	12 to 11	0
-5 to -4.5	11 to 10	0
-4.5 to -4	10 to 9	0
-4 to -3.5	9 to 8	0
-3.5 to -3	8 to 7	0
-3 to -2.5	7 to 6	0
-2.5 to -2	6 to 5	1
-2 to -1.5	5 to 4	1
-1.5 to -1	4 to 3	2
-1 to -0.5	3 to 2	2
-0.5 to 0	2 to 1	3
0 to 0.5	1 to 0	3
0.5 to 1	0 to -1	4
1 to 1.5	-1 to -2	4
1.5 to 2	-2 to -3	5
2 to 2.5	-3 to -4	5
2.5 to 3	-4 to -5	0
3 to 3.5	-5 to -6	0
3.5 to 4	-6 to -7	0
4 to 4.5	-7 to -8	0
4.5 to 5	-8 to -9	0
5 to 5.5	-9 to -10	0
5.5 to 6	-10 to -11	0

1.0) b)



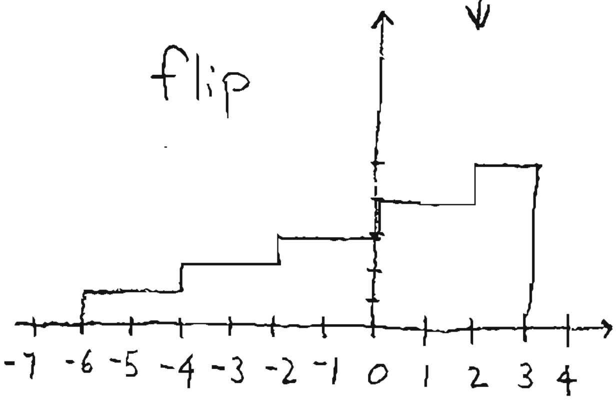
Note: In this problem, there is a scale and a flip. It does not matter if you do "scale then flip", "flip then scale", or "flip and scale all at once".

→ In part (c), these all give the same wrong answer.

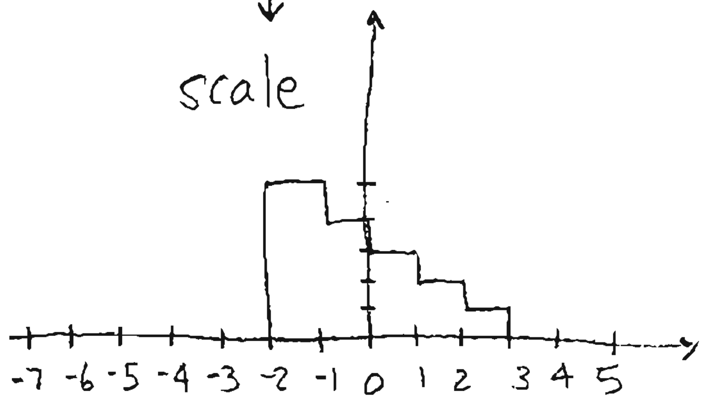
→ In part (d), these all give the same correct answer.

1.0) c) scale/flip first, then shift:

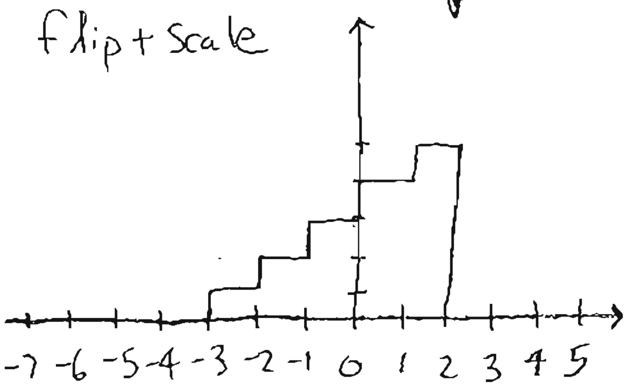
flip



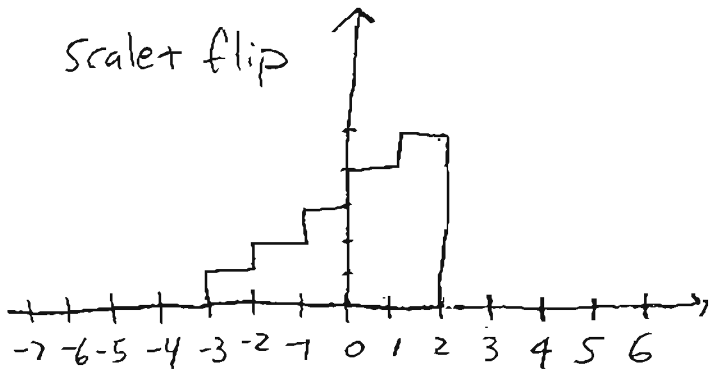
scale



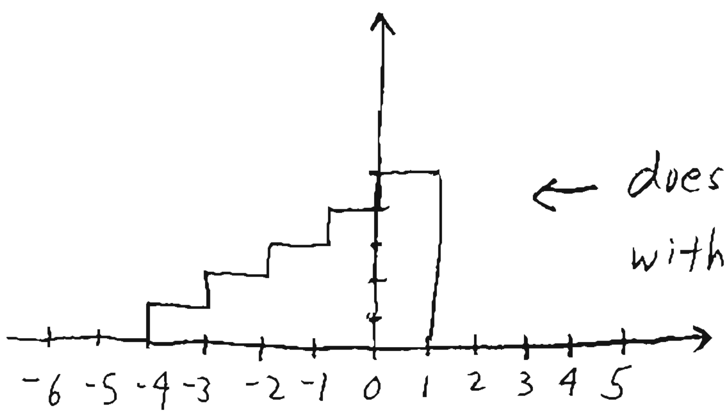
flip + scale



scale + flip



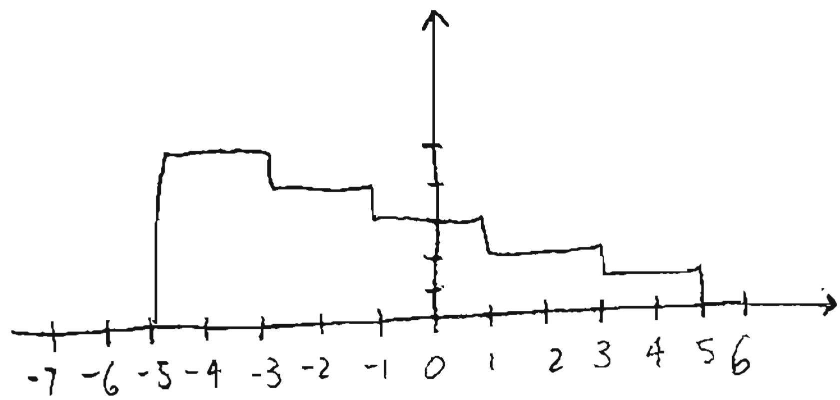
flip + scale + shift = scale + flip + shift = X



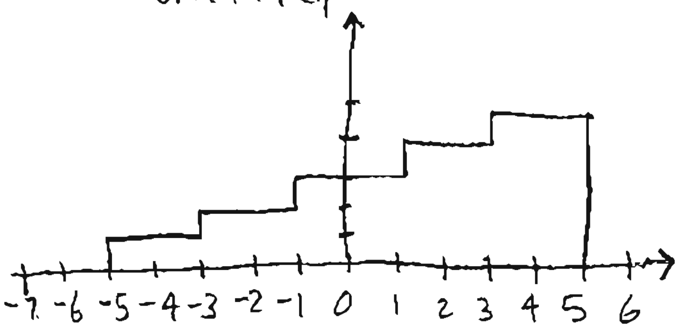
← does <u>not</u> agree with the correct graph in part (b).

1.0) d)

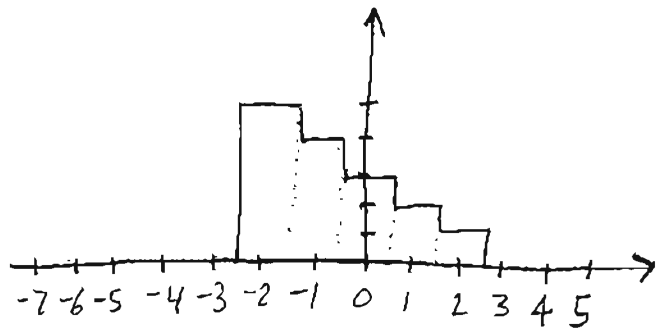
Shift



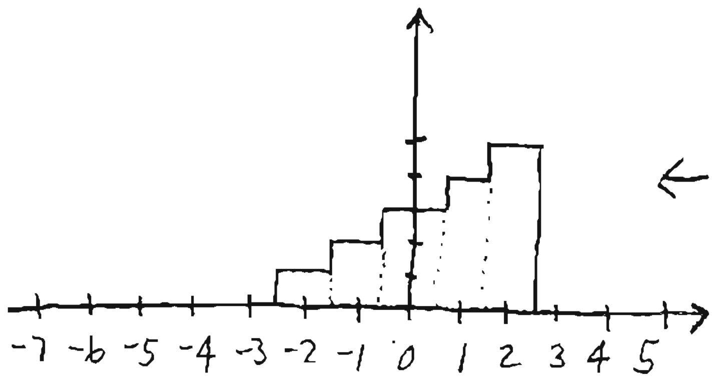
Shift+flip



Shift+Scale



shift+flip+scale = shift+scale+flip = ✓



← agrees with the graph in part (b) ✓

1.0) e)

In the graph of $x(-2t+1)$, the "t-axis" is scaled by a factor of -2 compared to the graph of $x(t)$.

In transforming the graph of $x(t)$ into the graph of $x(-2t+1)$, if we "scale first, then shift", this shift occurs with respect to "t", not the scaled axis variable "$-2t$". So we get the wrong answer this way.

If instead we "shift first, then scale", the shift is still done with respect to the "unscaled" t-axis, but the subsequent scale we perform fixes this. So we get the right answer doing it this way.

1.3 a)

$$E_\infty = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_{-\infty}^{\infty} |e^{-2t}u(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} e^{-4t} u^2(t)\, dt = \int_{0}^{\infty} e^{-4t} dt$$

$$= -\frac{1}{4}\left[e^{-4t}\right]_{t=0}^{\infty} = -\frac{1}{4} \lim_{A\to\infty} \left[e^{-4A} - 1\right]$$

$$= -\frac{1}{4}\left[0-1\right] = \underline{\underline{\frac{1}{4}}}$$

$P_\infty = 0$ because $E_\infty < \infty$, but this is not difficult to show directly:

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_1(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} e^{-4t} dt$$

$$= -\frac{1}{4} \lim_{T\to\infty} \frac{1}{2T} \left[e^{-4t}\right]_{t=0}^{T} = -\frac{1}{4} \lim_{T\to\infty} \frac{1}{2T}\left[e^{-4T} - 1\right]$$

$$= \lim_{T\to\infty} \frac{1}{8T} = \underline{\underline{0}}.$$

1.3 b)

$$E_\infty = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \lim_{T\to\infty} \int_{-T}^{T} |e^{j(2t+\pi/4)}|^2 dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} 1^2 dt = \lim_{T\to\infty} \int_{-T}^{T} 1\, dt = \lim_{T\to\infty} \Big[t\Big]_{-T}^{T}$$

$$= \lim_{T\to\infty} 2T \rightarrow \underline{\underline{\infty}}.$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_2(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1\, dt$$

$$= \lim_{T\to\infty} \frac{2T}{2T} = \underline{\underline{1}}.$$

(1.3) c)

$$E_\infty = \int_{-\infty}^{\infty} |x_3(t)|^2 dt = \int_{-\infty}^{\infty} \cos^2(t)\, dt$$

$$= \left[\frac{t}{2} + \frac{\sin 2t}{4}\right]_{-\infty}^{\infty} = \lim_{A\to\infty} \left[\frac{A}{2} + \frac{\sin 2A}{4} - \frac{-A}{2} - \frac{\sin(-2A)}{4}\right]$$

$$= \lim_{A\to\infty} \left[A + \frac{\sin 2A}{2}\right] \longrightarrow \underline{\underline{\infty}}.$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{t}{2} + \frac{\sin 2t}{4}\right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{T}{2} + \frac{\sin 2T}{4} - \frac{-T}{2} - \frac{\sin(-2T)}{4}\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[T + \frac{\sin 2T}{2}\right]$$

$$= \lim_{T\to\infty} \frac{1}{2} + \frac{\sin 2T}{4T} = \frac{1}{2} + 0 = \underline{\underline{\frac{1}{2}}}$$

1.3 d)

$$E_\infty = \sum_{n\in\mathbb{Z}} |x_1[n]|^2 = \sum_{n\in\mathbb{Z}} (2^{-n})^2 u[n] = \sum_{n=0}^{\infty} 2^{-2n}$$

$$= \sum_{n=0}^{\infty} 4^{-n} = \sum_{n=0}^{\infty} \left(\tfrac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}} = \frac{1}{3/4} = \underline{\underline{\frac{4}{3}}}$$

$\underline{P_\infty = 0,}$ since $E_\infty < \infty$.

1.3 e)

$$E_\infty = \sum_{n=-\infty}^{\infty} \left| e^{j(\pi/2 n + \pi/8)} \right|^2 = \sum_{n=-\infty}^{\infty} 1 \longrightarrow \underline{\underline{\infty}}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j(\pi/2 n + \pi/8)} \right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1 = \lim_{N\to\infty} \frac{2N+1}{2N+1}$$

$$= \lim_{N\to\infty} 1 = \underline{\underline{1}} .$$

1.3 f)

$$E_\infty = \sum_{n\in\mathbb{Z}} |x_3[n]|^2 = \sum_{n\in\mathbb{Z}} \left|\cos\frac{\pi n}{4}\right|^2 = \sum_{n\in\mathbb{Z}} \cos^2\frac{\pi n}{4}$$

$$= \sum_{n\in\mathbb{Z}} \left[\frac{1}{2} + \frac{1}{2}\cos\frac{\pi n}{2}\right] = \sum_{n\in\mathbb{Z}} \frac{1}{2} + \sum_{n\in\mathbb{Z}} \frac{1}{2}\cos\frac{\pi n}{2}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2} + \sum_{n=-\infty}^{-1} \frac{1}{2}\cos\frac{\pi n}{2} + \underbrace{\frac{1}{2}}_{n=0 \text{ term}} + \sum_{n=1}^{\infty} \frac{1}{2}\cos\frac{\pi n}{2}$$

these sums are the same b/c cosine is even

$$= \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} + 2\sum_{n=1}^{\infty} \frac{1}{2}\cos\frac{\pi n}{2} = \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} + \sum_{n=1}^{\infty} \cos\frac{\pi n}{2}$$

→ now, in the last sum, let $m = 4n$ and sum on m instead. Then <u>each</u> term of the new "m" sum will include <u>four terms</u> of the old "n" sum:

$$= \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} + \sum_{m=1}^{\infty} \left[\cos\frac{\pi(4m-3)}{2} + \cos\frac{\pi(4m-2)}{2} + \cos\frac{\pi(4m-1)}{2} + \cos\frac{\pi 4m}{2}\right]$$

$$= \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} + \sum_{m=1}^{\infty} \left[\cos\left(-\frac{3\pi}{2} + 2\pi m\right) + \cos(-\pi + 2\pi m) + \cos\left(-\frac{\pi}{2} + 2\pi m\right) + \cos(2\pi m)\right]$$

$$\rightarrow \cos(\theta + 2\pi) = \cos\theta$$

$$= \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} + \sum_{m=1}^{\infty} \left[\cos\left(\frac{-3\pi}{2}\right) + \cos(-\pi) + \cos\left(-\frac{\pi}{2}\right) + \cos(0)\right]$$

$$\longrightarrow$$

(1.3) f...

$$\ldots E_\infty = \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} + \sum_{m=1}^{\infty} \left[0 + (-1) + 0 + 1 \right]$$

$$= \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} + \sum_{m=1}^{\infty} 0$$

$$= \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} + 0 = \frac{1}{2} + \sum_{n=-\infty}^{\infty} \frac{1}{2} \longrightarrow \underline{\underline{\infty}}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_3[n]|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\cos\frac{\pi n}{4}\right|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\frac{\pi n}{4}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[\frac{1}{2} + \frac{1}{2}\cos\frac{\pi n}{2} \right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left\{ \sum_{n=-N}^{N} \frac{1}{2} + \sum_{n=-N}^{N} \frac{1}{2}\cos\frac{\pi n}{2} \right\}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left\{ (2N+1)\frac{1}{2} + \sum_{n=-N}^{-1} \frac{1}{2}\cos\frac{\pi n}{2} + \frac{1}{2} + \sum_{n=1}^{N} \frac{1}{2}\cos\frac{\pi n}{2} \right\}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left\{ (2N+2)\frac{1}{2} + 2\sum_{n=1}^{N} \frac{1}{2}\cos\frac{\pi n}{2} \right\}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left\{ (2N+2)\frac{1}{2} + \sum_{n=1}^{N} \cos\frac{\pi n}{2} \right\}$$

$$= \lim_{N\to\infty} \frac{\frac{1}{2}(2N+2)}{2N+1} + \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=1}^{N} \cos\frac{\pi n}{2}$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1} + \lim_{N\to\infty} \frac{1}{2N+1} \underbrace{\sum_{n=1}^{\infty} \cos\frac{\pi n}{2}}$$

This is zero, as shown above in the solution for E_∞

$\longrightarrow$

1.3 f, ...

$$\ldots P_\infty = \lim_{N\to\infty} \frac{N+1}{2N+1} + 0$$

$$= \lim_{N\to\infty} \frac{N}{2N}$$

$$= \lim_{N\to\infty} \frac{1}{2} = \frac{1}{2}$$

1.3f) A slightly easier way to work out E_∞:

$$E_\infty = \sum_{n=-\infty}^{\infty} |x_3[n]|^2 = \sum_{n=-\infty}^{\infty} \left| \cos \frac{\pi n}{4} \right|^2 = \sum_{n=-\infty}^{\infty} \cos^2 \frac{\pi n}{4}$$

$$\geq \sum_{n=0}^{\infty} \cos^2 \frac{\pi n}{4}$$

Since all terms in the sum are ≥ 0, it can't get bigger if we throw away half of the terms.

Let $m = 4n$. Each term of the new sum on "m" contains 4 terms from the old sum on "n":

$$= \sum_{m=0}^{\infty} \left[\cos^2 \frac{4m\pi}{4} + \cos^2 \frac{(4m+1)\pi}{4} + \cos^2 \frac{(4m+2)\pi}{4} + \cos^2 \frac{(4m+3)\pi}{4} \right]$$

$$\geq \sum_{m=0}^{\infty} \cos^2 \frac{4m\pi}{4}$$

again, all terms are ≥ 0, so it can't get bigger if we throw away some of the terms.

$$= \sum_{m=0}^{\infty} \cos^2(m\pi)$$

$$= \sum_{m=0}^{\infty} 1 \longrightarrow \infty$$

$\Longrightarrow$ So $E_\infty \geq \infty$

That means $E_\infty \to \infty$.

Although this method is easier, it doesn't help very much with finding P_∞.

1.4 a)

$x[n]$ is nonzero only for $n \in [-2, 4]$.

The graph of $x[n-3]$ is the same as the graph of $x[n]$ shifted right by three.

So $x[n-3]$ is nonzero only for $n \in [1, 7]$.

$\rightarrow$ $x[n-3] = 0 \quad \forall n < 1$ and $\forall n > 7$.

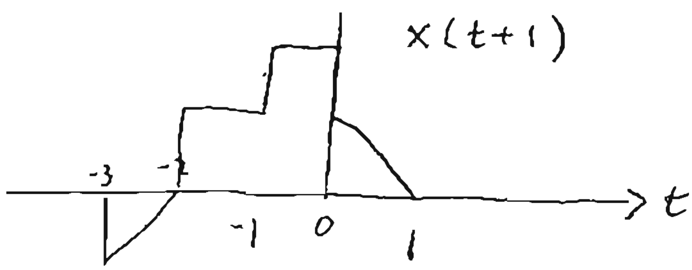
1.4 d)

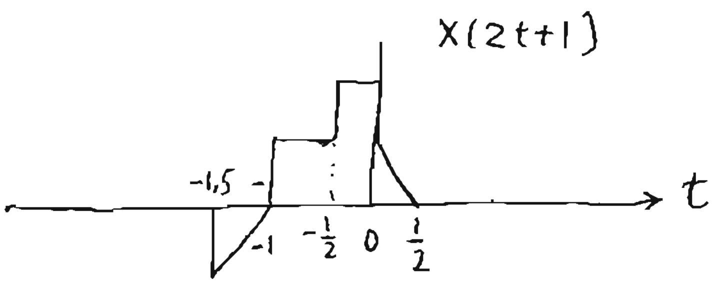
- The graph of $x[n]$ is nonzero only for $n \in [-2, 4]$
- The graph of $x[n+2]$ is shifted left by 2, so it is nonzero only for $n \in [-4, 2]$.
- The graph of $x[-n+2]$ is the <u>flip</u> of $x[n+2]$, so it is nonzero only for $n \in [-2, 4]$.

→ $x[-n+2] = 0 \quad \forall\, n < -2$ and $\forall\, n > 4$.

1.21 c)

Remember, do the shift <u>first</u> and the scale <u>second</u>.

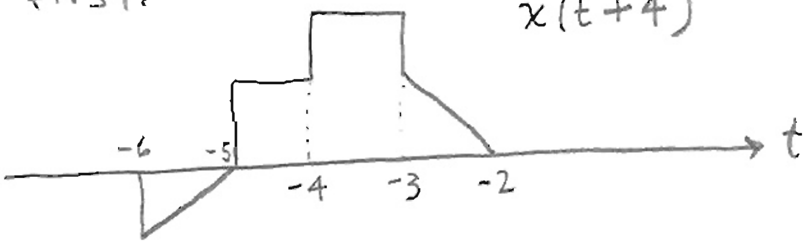




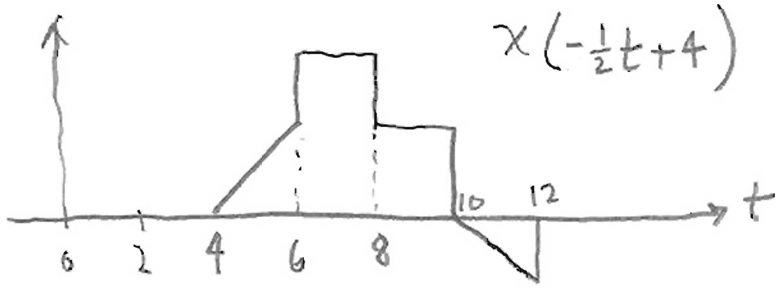
1.21d)

$$x(4-\tfrac{1}{2}t) = x\left(-\tfrac{1}{2}t-(-4)\right) = x\left(-\tfrac{1}{2}t+4\right)$$

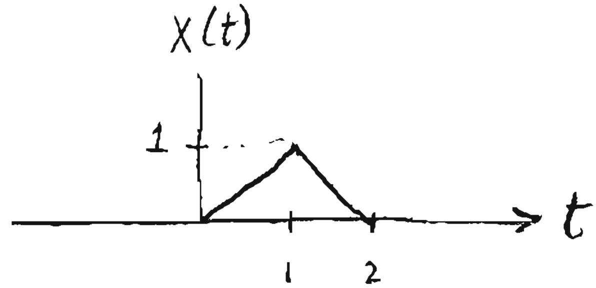
Shift first:

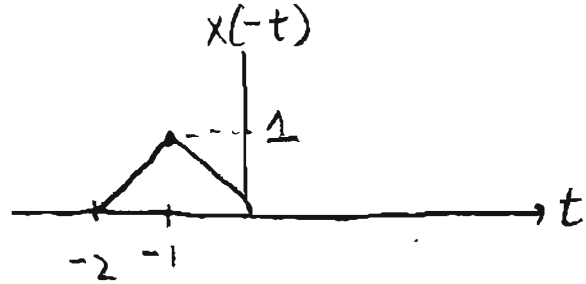


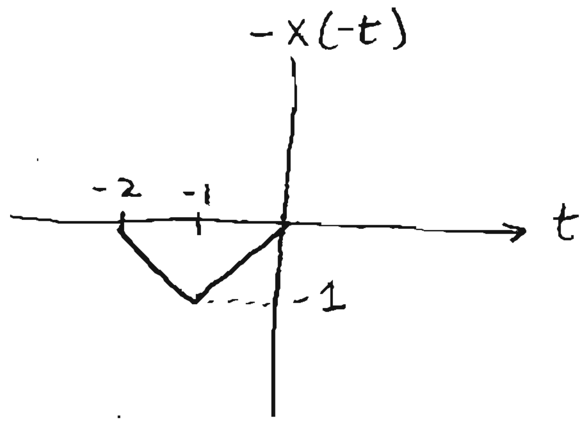
flip/scale second:



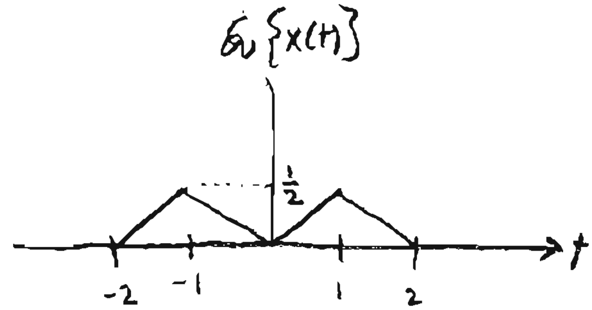
1.23 a)



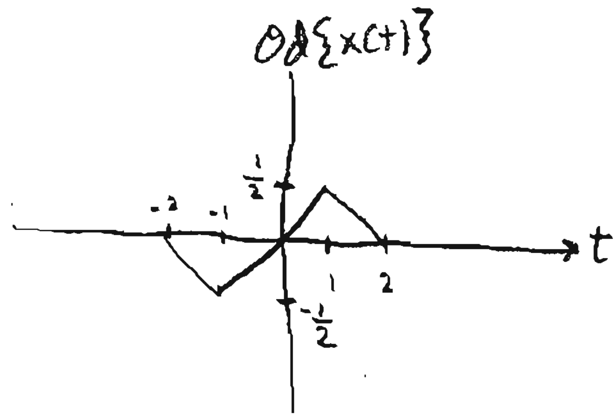




$\mathcal{E}v\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right] =$

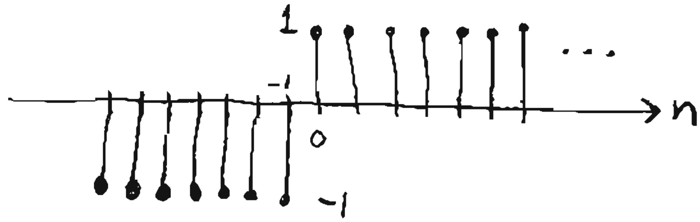


$\mathcal{O}d\{x(t)\} = \frac{1}{2}\left[x(t) - x(-t)\right] =$



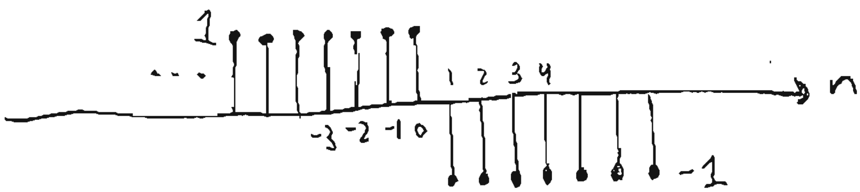
(1.24) a)

$x[n]$



$$x[n] = u[n] - u[-n-1]$$

$$x[-n] = u[-n] - u[n-1]$$



$$\mathcal{E}v\{x[n]\} = \frac{1}{2}\{x[n] + x[-n]\}$$

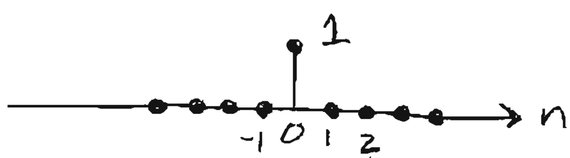
$$= \frac{1}{2}\{u[n] - u[-n-1] + u[-n] - u[n-1]\}$$

$$= \frac{1}{2}\{u[n] - u[n-1] + u[-n] - u[-n-1]\}$$

$$= \frac{1}{2}\{\delta[n] + \delta[n]\}$$

$$= \delta[n]$$

$\mathcal{E}v\{x[n]\}$



⟶ continues

1.24 a) ...

$$Od\{x[n]\} = \frac{1}{2}\{x[n] - x[-n]\}$$

$$= \frac{1}{2}\{u[n] - u[-n-1] - u[-n] + u[n-1]\}$$

$$= \frac{1}{2}\{u[n] + u[n-1] - (u[-n] + u[-n-1])\}$$

$$= \frac{1}{2}\{2u[n] - \delta[n] - (2u[-n] - \delta[n])\}$$

$$= \frac{1}{2}\{2u[n] - 2u[-n] - \delta[n] + \delta[n]\}$$

$$= u[n] - u[-n]$$

$Od\{x[n]\}$