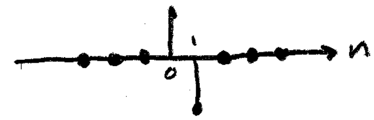
ECE 3793

HOMEWORK 2
SOLUTION

HAVLICEK

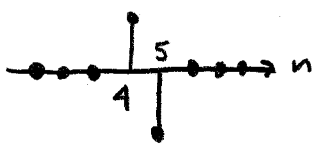
1.6 c) $x_3[n] = \sum_{k=-\infty}^{\infty} \{\delta[n-4k] - \delta[n-1-4k]\}$

- when $k=0$, we get the basic pattern

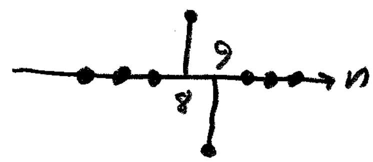
$\delta[n] - \delta[n-1] =$ 

- For each of the remaining terms in the sum, we get another copy of this pattern, but shifted by $4k$:

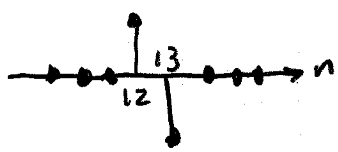
$k=1$: $\delta[n-4] - \delta[n-5]$



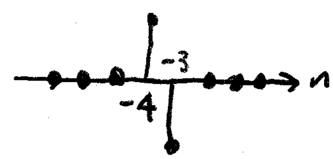
$k=2$: $\delta[n-8] - \delta[n-9]$



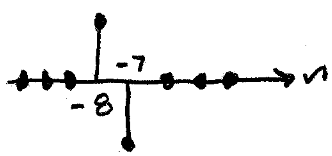
$k=3$: $\delta[n-12] - \delta[n-13]$



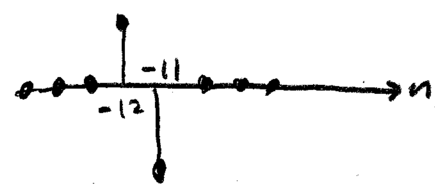
$k=-1$: $\delta[n+4] - \delta[n+3]$



$k=-2$: $\delta[n+8] - \delta[n+7]$

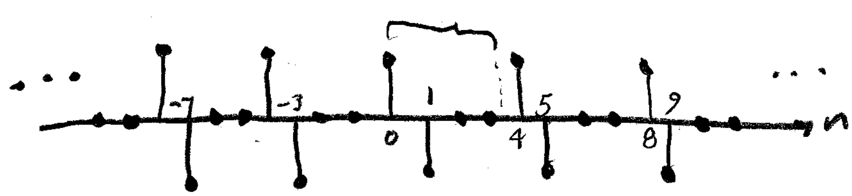


$k=-3$: $\delta[n+12] - \delta[n+11]$



All Together:

one period



So it <u>is periodic</u>. Fundamental period: $N_0 = 4$.

1.10 $x(t) = 2\cos(10t + 1) - \sin(4t - 1)$

For $2\cos(10t+1)$, $\omega_0 = 10$. $T_0 = \left|\frac{2\pi}{\omega_0}\right| = \frac{2\pi}{10} = \frac{\pi}{5}$

For $-\sin(4t-1)$, $\omega_0 = 4$. $T_0 = \left|\frac{2\pi}{\omega_0}\right| = \frac{2\pi}{4} = \frac{\pi}{2}$

For $x(t)$, the fundamental period is the lowest common multiple of $\frac{\pi}{5}$ and $\frac{\pi}{2}$:

$$\underline{\underline{T_0 = \pi}} = 5 \cdot \frac{\pi}{5} = 2 \cdot \frac{\pi}{2}$$

1.17 a) $y(t) = x(\sin(t))$.

The value of the output signal $y(t)$ at $t = -\pi$ is $x(\sin(-\pi)) = x(0)$.

Since the value of the output signal at time $t = -\pi$ depends on the future value of the input signal at $t = 0$, the system is not causal.

1.17 b) Let $x_1(t)$ and $x_2(t)$ be two arbitrary input signals. Let c_1 and c_2 be two arbitrary constants.

When the input is $x_1(t)$, the output is

$$y_1(t) = H\{x_1(t)\} = x_1(\sin(t)).$$

When the input is $x_2(t)$, the output is

$$y_2(t) = H\{x_2(t)\} = x_2(\sin(t)).$$

Now let $x_3(t) = c_1 x_1(t) + c_2 x_2(t)$.

When the input is $x_3(t)$, the output is

$$y_3(t) = H\{x_3(t)\} = x_3(\sin(t))$$

$$= c_1 x_1(\sin(t)) + c_2 x_2(\sin(t)) \checkmark$$

The system is linear.

(1.17) c) Let $x(t)$ be a bounded input signal. Then $\exists B \in \mathbb{R}$, $B > 0$, s.t. $|x(t)| \leq B \;\; \forall t \in \mathbb{R}$.

Now, $\sin(t) \in \mathbb{R} \;\; \forall t \in \mathbb{R}$.

So $|y(t)| = |x(\sin(t))| \leq B$.

So $y(t)$ is bounded by B.

→ Every bounded input signal produces a bounded output signal.

→ The system is stable.

(1.17) d) Use the "rule of thumb" on page 1.86 of the notes.

If we put the signal through the system and then shift, we have

$$x_1(t) \longrightarrow \boxed{H} \longrightarrow y_1(t) \overset{\text{shift}}{\longmapsto} y_1(t-t_0)$$

The first transformation is $t \longmapsto \sin(t)$

The second transformation is $t \longmapsto t - t_0$

$$y_1(t-t_0) = y_1(t)\Big|_{t=t-t_0}$$

$$= x_1(\sin(t))\Big|_{t=t-t_0}$$

$$= x_1(\sin(t-t_0))$$

1.17 d... If we shift first to let $x_2(t) = x_1(t-t_0)$, and then put $x_2(t)$ through the system, we have

$$x_1(t) \xmapsto{\text{shift}} x_2(t) = x_1(t-t_0) \longrightarrow \boxed{H} \longrightarrow y_2(t)$$

The first transformation is $t \mapsto t - t_0$

The second transformation is $t \mapsto \sin(t)$

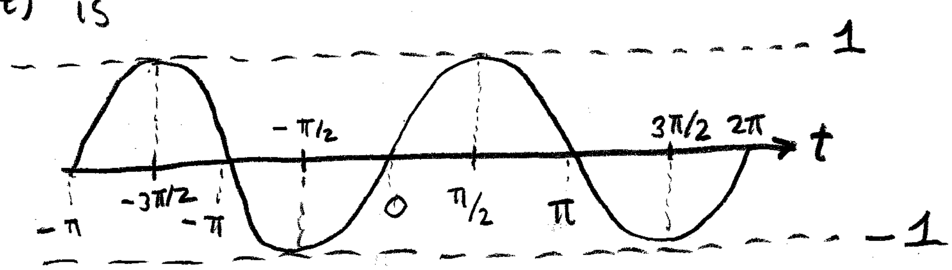
By the rule of thumb,

$$y_2(t) = x_1(t-t_0)\Big|_{t=\sin(t)} = x_1(\sin(t) - t_0)$$

Since $y_2(t) \neq y_1(t-t_0)$, the system is <u>not</u> time invariant. //

Here's how you would alternatively solve this one from first principles without using the "rule of thumb":

The graph of $\sin(t)$ is



Since $y(t) = x(\sin(t))$, and $-1 \leq \sin(t) \leq 1 \quad \forall\, t \in \mathbb{R}$,

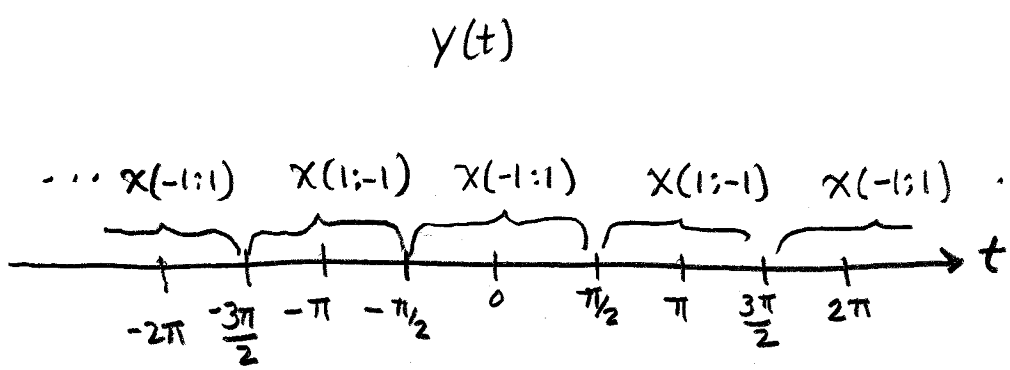
the output signal $y(t)$ will always be equal to some value of $x(t)$ from $t \in [-1, 1]$. The system ignores (or throws away) the rest of the input signal.

1.17 d... when t goes from $-\pi/2$ to $\pi/2$, $\sin(t)$ goes from -1 to $+1$. So the output from $t=-\pi/2$ to $\pi/2$ is equal to the input signal from $t=-1$ to $+1$, with some nonlinear stretching and squishing due to the sine. Call this $x(-1:1)$.

When t goes from $\pi/2$ to $3\pi/2$, $\sin(t)$ goes from $+1$ to -1. So the output signal from $t=\pi/2$ to $3\pi/2$ is the "flip" of what we just had. Call this $x(1:-1)$.

For the rest of the t's, this behavior just repeats. For example, when t goes from $3\pi/2$ to $5\pi/2$, $\sin(t)$ goes from -1 to $+1$ again. So we get $x(-1:1)$ again at the output.

So for input $x(t)$, the graph of the output signal looks like:

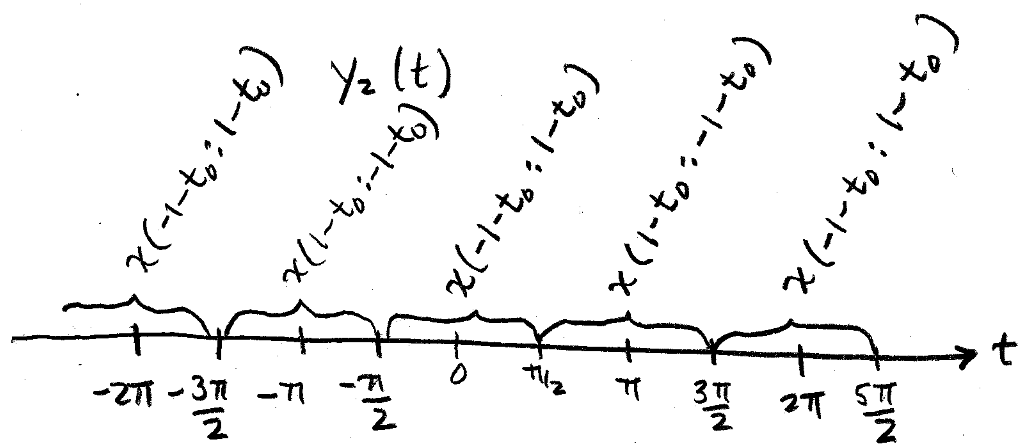


→ Then $y(t-t_0)$ is just this graph shifted right by t_0.

1.17 d... Now let's see what happens if we instead shift first, then put the shifted signal through the system.

Let $x_2(t) = x(t - t_0)$.

When t goes from $-\pi/2$ to $\pi/2$, $\sin(t)$ goes from -1 to $+1$ as before. So this time, the output $y_2(t)$ will be $x_2(-1:1)$. That's $x(-1-t_0 : 1-t_0)$. This is a different part of the input signal that was thrown away before. Doing things this way, the graph of the output signal is



→ This graph is using different parts of the input signal than before, so it's <u>not</u> the same as $y(t-t_0)$ in general.

Therefore, the system is <u>not</u> time invariant

(1.18) $$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k], \quad n_0 \in \mathbb{N}.$$

a) Linear?

Let $x_1[n]$ and $x_2[n]$ be two input signals and let c_1 and c_2 be two constants.

Then $y_1[n] = H\{x_1[n]\} = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

and $y_2[n] = H\{x_2[n]\} = \sum_{k=n-n_0}^{n+n_0} x_2[k]$.

Let $x_3[n] = c_1 x_1[n] + c_2 x_2[n]$.

Then $$y_3[n] = \sum_{k=n-n_0}^{n+n_0} x_3[k]$$

$$= \sum_{k=n-n_0}^{n+n_0} c_1 x_1[k] + c_2 x_2[k]$$

$$= c_1 \sum_{k=n-n_0}^{n+n_0} x_1[k] + c_2 \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$= c_1 y_1[n] + c_2 y_2[n] \checkmark$$

The system is linear.

1.18 b) Time invariant ?

Let $y_1[n] = H\{x_1[n]\} = \sum_{k=n-n_0}^{n+n_0} x_1[k]$.

Let $p \in \mathbb{Z}$ be the time shift.

Then $y_1[n-p] = \sum_{k=n-p-n_0}^{n-p+n_0} x_1[k]$

let $m = k+p$
$k = m-p$
when $k = n-p-n_0$, $m = n-n_0$
when $k = n-p+n_0$, $m = n+n_0$

$$= \sum_{m=n-n_0}^{n+n_0} x_1[m-p]$$

Now let $k=m$ and rewrite again

$$= \sum_{k=n-n_0}^{n+n_0} x_1[k-p].$$

Now let $x_2[n] = x_1[n-p]$. Then

$$y_2[n] = H\{x_2[n]\} = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$= \sum_{k=n-n_0}^{n+n_0} x_1[k-p] = y_1[n-p] \checkmark$$

The system is time invariant.

(1.18) c) Stable ?

Let $x[n]$ be a bounded input signal.
Then $\exists B \in \mathbb{R}$, $B > 0$, s.t. $|x[n]| \leq B \;\; \forall n \in \mathbb{Z}$.

Now,

$$|y[n]| = \left| \sum_{k=n-n_0}^{n+n_0} x[n] \right| \leq \sum_{k=n-n_0}^{n+n_0} |x[k]|$$

$$\leq \sum_{k=n-n_0}^{n+n_0} B$$

$$= (2n_0 + 1) B.$$

So the output signal is bounded by $C = (2n_0+1)B$.

→ Every bounded input signal produces a bounded output signal.

The system <u>is</u> stable.

(1.18) d) Causal ? For $n=0$, the value of the output signal is $y[0] = \sum_{k=n-n_0}^{n+n_0} x[k]$.

Since this depends on the <u>future</u> value of the input signal $x[n+n_0]$, the system is <u>not</u> causal.

(1.19) a) $y(t) = t^2 x(t-1)$.

Linear? Let $x_1(t)$ and $x_2(t)$ be input signals and let c_1 and c_2 be constants.

Then $y_1(t) = H\{x_1(t)\} = t^2 x_1(t-1)$

and $y_2(t) = H\{x_2(t)\} = t^2 x_2(t-1)$.

Now let $x_3(t) = c_1 x_1(t) + c_2 x_2(t)$.

$$\begin{aligned} y_3(t) = H\{x_3(t)\} &= t^2 x_3(t-1) \\ &= t^2 [c_1 x_1(t-1) + c_2 x_2(t-1)] \\ &= c_1 t^2 x_1(t-1) + c_2 t^2 x_2(t-1) \\ &= c_1 y_1(t) + c_2 y_2(t) \quad ✓ \end{aligned}$$

The system is linear. //

Time invariant? This one is simple enough that we don't need to use the rule of thumb on page 1.86 of the notes (but you could).

- When the input is $x_1(t)$, the output is

$$y_1(t) = t^2 x_1(t-1).$$

So $y_1(t-t_0) = (t-t_0)^2 x_1(t-t_0-1)$.

- Now let $x_2(t) = x_1(t-t_0)$. The output is

$$y_2(t) = t^2 x_2(t-1) = t^2 x_1(t-t_0-1) \neq y_1(t-t_0).$$

So the system is not time invariant. //

(1.19) c) $y[n] = x[n+1] - x[n-1]$.

Linear? Let $x_1[n]$ and $x_2[n]$ be two input signals and let c_1 and c_2 be constants.

When $x_1[n]$ is the input, the output is given by

$$y_1[n] = x_1[n+1] - x_1[n-1].$$

When $x_2[n]$ is the input, the output is

$$y_2[n] = x_2[n+1] - x_2[n-1].$$

Now let the input be $x_3[n] = c_1 x_1[n] + c_2 x_2[n]$.
The output is given by

$$\begin{aligned} y_3[n] &= x_3[n+1] - x_3[n-1] \\ &= \left(c_1 x_1[n+1] + c_2 x_2[n+1]\right) - \left(c_1 x_1[n-1] + c_2 x_2[n-1]\right) \\ &= \left(c_1 x_1[n+1] - c_1 x_1[n-1]\right) + \left(c_2 x_2[n+1] - c_2 x_2[n-1]\right) \\ &= c_1\left(x_1[n+1] - x_1[n-1]\right) + c_2\left(x_2[n+1] - x_2[n-1]\right) \\ &= c_1 y_1[n] + c_2 y_2[n] \ \checkmark \end{aligned}$$

The system <u>is</u> linear. //

(1.19) c)... Time invariant ?

Let $x_1[n]$ be the input. The output is

$$y_1[n] = x_1[n+1] - x_1[n-1].$$

Let $n_0 \in \mathbb{N}$. Then

$$y_1[n-n_0] = x_1[n-n_0+1] - x_1[n-n_0-1].$$

Now let $x_2[n] = x_1[n-n_0]$. The output is

$$\begin{aligned} y_2[n] &= x_2[n+1] - x_2[n-1] \\ &= x_1[n-n_0+1] - x_1[n-n_0-1] \\ &= y_1[n-n_0] \checkmark \end{aligned}$$

The system <u>is</u> time invariant. //

1.25 b) $x(t) = e^{j(\pi t - 1)} = \cos(\pi t - 1) + j\sin(\pi t - 1)$

→ It is a complex sinusoid with a phase offset, so it is periodic.

The frequency is $\omega_0 = \pi$ rad/sec.

so the fundamental period is

$$T_0 = \left|\frac{2\pi}{\omega_0}\right| = \frac{2\pi}{\pi} = \underline{\underline{2}}$$

1.25 d) $x(t) = \mathcal{E}v\{\cos(4\pi t)\,u(t)\}$

$$= \tfrac{1}{2}\cos(4\pi t)u(t) + \tfrac{1}{2}\cos(-4\pi t)u(-t)$$

$$= \tfrac{1}{2}\cos(4\pi t)u(t) + \tfrac{1}{2}\cos(4\pi t)u(-t)$$

$$= \tfrac{1}{2}\cos(4\pi t)\left[u(t) + u(-t)\right].$$

⇒ The answer depends on how you define $u(t)$ at $t=0$.

- if you say $u(0) = 1$, then the signal is not periodic.
- if you say $u(0) = 0$, then it also is not periodic.
- but if you say $u(0) = \frac{1}{2}$, then $x(t) = \cos 4\pi t$ and it is periodic. In this case, the frequency is $\omega_0 = 4\pi$, so the fundamental period is

$$T_0 = \left|\frac{2\pi}{\omega_0}\right| = \frac{2\pi}{4\pi} = \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$$

(1.26) c) $x[n] = \cos\left(\frac{\pi}{8} n^2\right)$.

This signal is <u>not</u> a pure sinusoid, so we have to go back to first principles. A discrete-time signal is periodic if $\exists N \in \mathbb{N}$ s.t. $x[n] = x[n+N] \; \forall n \in \mathbb{Z}$.

If more than one such N exists, then the smallest one is the fundamental period.

- So we need to solve $\cos\left(\frac{\pi}{8} n^2\right) = \cos\left(\frac{\pi}{8}(n+N)^2\right)$ for N.
- The equality holds $\forall n$ if $\frac{\pi}{8} n^2$ and $\frac{\pi}{8}(n+N)^2$ always differ by an integer multiple of 2π. In other words,

$$\frac{\pi}{8}(n+N)^2 - \frac{\pi}{8} n^2 = 2\pi k, \quad k \in \mathbb{Z}.$$

$$\frac{\pi}{8}(n^2 + 2nN + N^2) - \frac{\pi}{8} n^2 = 2\pi k, \quad k \in \mathbb{Z}$$

$$n^2 \frac{\pi}{8} + 2nN\frac{\pi}{8} + N^2 \frac{\pi}{8} - n^2 \frac{\pi}{8} = 2\pi k, \quad k \in \mathbb{Z}$$

$$2nN\frac{\pi}{8} + N^2\frac{\pi}{8} = 2\pi k, \quad k \in \mathbb{Z}$$

$$\frac{2nN}{8} + \frac{N^2}{8} = 2k, \quad k \in \mathbb{Z}$$

$$\frac{nN}{8} + \frac{N^2}{16} = k = \text{any integer} \quad (*)$$

(*) is satisfied if N is any integer multiple of 8. To see this, let $m \in \mathbb{Z}$ and let $N = 8m$. Then (*) becomes $\frac{nm8}{8} + \frac{64m^2}{8} = nm + 8m^2 = \text{integer}$ ✓

- <u>So the signal is periodic.</u> The fundamental period is the smallest non-negative N such that $N = 8m$, which is obtained with $m = 1$. So $\underline{N_0 = 8}$

1.26 d) $x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

$\rightarrow \cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

with $A = \frac{\pi}{2}n$ and $B = \frac{\pi}{4}n$, we have

$$x[n] = \frac{1}{2}\left[\cos\left(\frac{\pi}{2}n - \frac{\pi}{4}n\right) + \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}n\right)\right]$$

$$= \frac{1}{2}\cos\left(\frac{\pi}{4}n\right) + \frac{1}{2}\cos\left(\frac{3\pi}{4}n\right)$$

The sum of two periodic discrete-time signals <u>is</u> periodic.

Notes page 1.55:

- For $\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)$, $\omega_0 = \frac{\pi}{4} \rightarrow \frac{\omega_0}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8} = \frac{m}{N_0}$

  Fundamental period = 8.

- For $\frac{1}{2}\cos\left(\frac{3\pi}{4}n\right)$, $\omega_0 = \frac{3\pi}{4} \rightarrow \frac{\omega_0}{2\pi} = \frac{3\pi/4}{2\pi} = \frac{3}{8} = \frac{m}{N_0}$

  Fundamental period = 8.

For $x[n]$, the fundamental period is

$$\operatorname{lcm}(8,8) = \underline{\underline{8}}$$

(1.26) e) $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$.

$\rightarrow$ $2\cos\left(\frac{\pi}{4}n\right)$, $\sin\left(\frac{\pi}{8}n\right)$, and $-2\cos\left(\frac{\pi}{2}n+\frac{\pi}{6}\right)$ are all discrete time sinusoids.

For $2\cos\left(\frac{\pi}{4}n\right)$, $\omega_0 = \frac{\pi}{4}$. So $\frac{\omega_0}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8} = \frac{m}{N} \in \mathbb{Q}$

$\rightarrow$ periodic with fundamental period $N = 8$

For $\sin\left(\frac{\pi}{8}n\right)$, $\omega_0 = \frac{\pi}{8}$. So $\frac{\omega_0}{2\pi} = \frac{\pi/8}{2\pi} = \frac{1}{16} = \frac{m}{N} \in \mathbb{Q}$

$\rightarrow$ periodic with fundamental period $N=16$.

For $-2\cos\left(\frac{\pi}{2}n+\frac{\pi}{6}\right)$, $\omega_0 = \frac{\pi}{2}$. So $\frac{\omega_0}{2\pi}$ $\frac{\pi/2}{2\pi} = \frac{1}{4} = \frac{m}{N} \in \mathbb{Q}$

$\rightarrow$ periodic with fundamental period $N=4$.

Since $x[n]$ is a sum of discrete-time periodic signals, it <u>is</u> periodic. //

The fundamental period of $x[n]$ is the lowest common multiple

$$N_0 = \text{lcm}(8, 16, 4) = \underline{\underline{16}}$$

(1,27) b) $y(t) = \cos(3t)\, x(t)$.

(1) Memoryless: at any time t, the value of the output signal $y(t)$ depends only on the current value $x(t)$ of the input signal, and not on any future values $x(\tau)$ of the input signal for $\tau > t$ and not on any past values $x(\tau)$ for $\tau < 0$. So the system is memoryless. //

(2) Time invariant: let the input signal be $x_1(t)$. Then the output signal is $y_1(t) = \cos(3t)\, x_1(t)$. Then $y_1(t-t_0) = \cos[3(t-t_0)]\, x_1(t-t_0)$.

Now let the input be $x_2(t) = x_1(t-t_0)$. Then the output is

$$y_2(t) = \cos(3t)\, x_2(t) = \cos(3t)\, x_1(t-t_0)$$
$$\neq y_1(t-t_0).$$

So the system is not time invariant. //

(1.27) b... (3) Linear:

Let $y_1(t) = H\{x_1(t)\} = \cos(3t)x_1(t)$.

Let $y_2(t) = H\{x_2(t)\} = \cos(3t)\,x_2(t)$.

Let c_1 and c_2 be constants and let

$x_3(t) = c_1x_1(t) + c_2x_2(t)$.

Then
$$
\begin{aligned}
y_3(t) = H\{x_3(t)\} &= \cos(3t)\,x_3(t) \\
&= \cos(3t)\left[c_1x_1(t) + c_2x_2(t)\right] \\
&= c_1\cos(3t)x_1(t) + c_2\cos(3t)x_2(t) \\
&= c_1y_1(t) + c_2y_2(t) \checkmark
\end{aligned}
$$

So the system <u>is</u> linear. //

(4) Causal: as shown in (1) for memoryless, the current value of the output signal does not depend on any future values of the input signal. So the system <u>is</u> causal. //

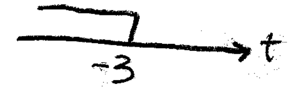
(5) Stable: let $x(t)$ be a bounded input. Then $\exists B \in \mathbb{R}$, $B > 0$, s.t. $|x(t)| \leq B \ \forall t \in \mathbb{R}$.

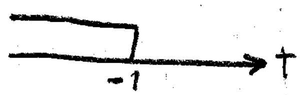
Then
$$
\begin{aligned}
|y(t)| &= |\cos(3t)\,x(t)| \\
&= |\cos(3t)| \cdot |x(t)| \\
&\leq 1 \cdot |x(t)| \\
&\leq |x(t)| \leq B.
\end{aligned}
$$

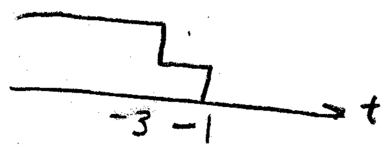
Therefore, $y(t)$ is bounded. The system <u>is</u> stable because every bounded input signal produces a bounded output signal. //

(1.27) d) $y(t) = \begin{cases} 0 & , t < 0 \\ x(t) + x(t-2) & , t \geq 0 \end{cases}$

(1) The system is <u>not</u> memoryless, when $t = 5$, the value of the output signal $y(5)$ depends on the past value $x(3)$ of the input signal.

(2) Time invariant: Let $x_1(t) = u(-t-3) =$ 

Then $x_1(t-2) =$ 

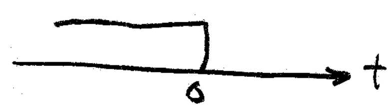
$x_1(t) + x_1(t-2) =$ 

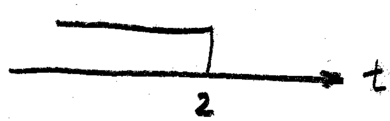
$= \begin{cases} 2 & , t < -3 \\ 1 & , -3 < t < -1 \\ 0 & , t > -1 \end{cases}$

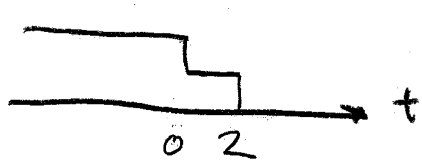
So $x_1(t) + x_1(t-2) = 0 \ \forall\ t \geq 0$.

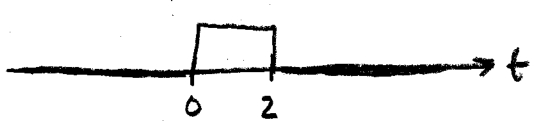
So $y_1(t) = 0 \ \forall\ t$.

Now let $t_0 = 3$. Then $y_1(t - t_0) = 0 \ \forall t$.

Let $x_2(t) = x_1(t - t_0) = x_1(t-3) =$ 

Then $x_2(t-2) =$ 

$x_2(t) + x_2(t-2) =$ 

Then $y_2(t) =$ 

The system is <u>not</u> time invariant because $x_1(t - t_0) \neq x_2(t)$.

1.27 d... (3) Linear:

Let $y_1(t) = H\{x_1(t)\} = \begin{cases} 0, & t<0 \\ x_1(t)+x_1(t-2), & t\geq 0 \end{cases}$.

Let $y_2(t) = H\{x_2(t)\} = \begin{cases} 0, & t<0 \\ x_2(t)+x_2(t-2), & t\geq 0 \end{cases}$.

Then for constants c_1 and c_2,

$$c_1 y_1(t) + c_2 y_2(t) = \begin{cases} 0, & t<0 \\ c_1 x_1(t) + c_1 x_1(t-2) \\ \quad + c_2 x_2(t) + c_2 x_2(t-2), & t\geq 0. \end{cases}$$

Now let $x_3(t) = c_1 x_1(t) + c_2 x_2(t)$.

Then $y_3(t) = H\{x_3(t)\} = \begin{cases} 0, & t<0 \\ x_3(t)+x_3(t-2), & t\geq 0 \end{cases}$

$$= \begin{cases} 0, & t<0 \\ c_1 x_1(t) + c_2 x_2(t) + c_1 x_1(t-2) \\ \quad + c_2 x_2(t-2), & t\geq 0 \end{cases}$$

$$= c_1 y_1(t) + c_2 y_2(t). \checkmark$$

The system is linear //

1.27 d...

(4) Causal: when $t<0$, $y(t)$ does not depend on the input at all. When $t \geq 0$, the value of the output signal $y(t)$ depends on the current value of the input signal $x(t)$ and on a past value of the input signal $x(t-2)$, but not on any future value of the input signal. Therefore, the system <u>is</u> causal.

(5) Stable: let $x(t)$ be a bounded input signal. Then $\exists B \in \mathbb{R}$, $B>0$, s.t. $|x(t)| \leq B \ \forall t \in \mathbb{R}$.

Then
$$|y(t)| \leq |x(t)+x(t+2)| \leq |x(t)| + |x(t+2)| \leq B+B \leq 2B.$$

Therefore, the output signal $y(t)$ is bounded. Since every bounded input signal produces a bounded output signal, the system <u>is</u> stable.//