ECE 3793
Homework 6 Solution

Spring 2017

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Problem 1)

4.3b) $x(t) = 1 + \cos\left(6\pi t + \frac{\pi}{8}\right) = 1 + \cos\left[6\pi\left(t + \frac{1}{48}\right)\right]$

$1 \leftrightarrow 2\pi\delta(\omega)$ $\qquad \cos 6\pi t \leftrightarrow \pi\left[\delta(\omega - 6\pi) + \delta(\omega + 6\pi)\right]$

$$\cos\left[6\pi\left(t + \tfrac{1}{48}\right)\right] \leftrightarrow e^{j\frac{\omega}{48}}\pi\left[\delta(\omega - 6\pi) + \delta(\omega + 6\pi)\right]$$

$$= \pi e^{j\omega/48}\delta(\omega - 6\pi) + \pi e^{j\omega/48}\delta(\omega + 6\pi)$$

$$= \pi e^{j6\pi/48}\delta(\omega - 6\pi) + \pi e^{-j6\pi/48}\delta(\omega + 6\pi)$$

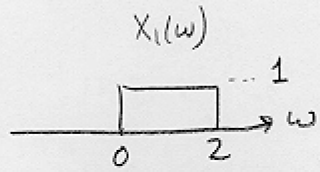
$$= \pi e^{j\pi/8}\delta(\omega - 6\pi) + \pi e^{-j\pi/8}\delta(\omega + 6\pi)$$

$$X(\omega) = 2\pi\delta(\omega) + \pi e^{j\pi/8}\delta(\omega - 6\pi) + \pi e^{-j\pi/8}\delta(\omega + 6\pi)$$

Problem 2)

4.7a)

$X_1(\omega) = u(\omega) - u(\omega - 2)$



$\rightarrow X_1(\omega)$ is not conjugate symmetric $\rightarrow x_1(t)$ not purely real.

$\rightarrow X_1(\omega)$ is not conjugate antisymmetric $\rightarrow x_1(t)$ not purely imaginary.

(i) NEITHER real nor imaginary.

$\rightarrow X_1(\omega)$ is real $\Rightarrow x_1(t)$ is conjugate symmetric

$\rightarrow$ real part even

$\rightarrow$ imaginary part odd.

(ii) NEITHER even nor odd.

Problem 2)…

4.7c) $X_3(\omega) = A(\omega)\, e^{jB(\omega)}$

$$A(\omega) = \frac{\sin 2\omega}{\omega} = \frac{\text{odd}}{\text{odd}} = \text{even}.$$

$$B(\omega) = 2\omega + \pi/2. \rightarrow \text{neither even nor odd.}$$

Let $C(\omega) = 2\omega \rightarrow$ even.

Let $Y(\omega) = A(\omega)\, e^{jC(\omega)}$

$\rightarrow$ spectral magnitude even

$\rightarrow$ spectral phase odd

$\rightarrow$ $Y(\omega)$ is conjugate symmetric

$\Rightarrow$ $y(t)$ is real.

Now, $X_3(\omega) = Y(\omega)\, e^{j\pi/2} = j\,Y(\omega)$

$\rightarrow$ so $x_3(t) = j\,y(t)$, and $y(t)$ is real.

$\Rightarrow$ (i) $x_3(t)$ is imaginary

$\rightarrow$ $X_3(\omega)$ is not pure real $\Rightarrow$ $x_3(t)$ is not odd.

$\rightarrow$ $X_3(\omega)$ is not pure imaginary $\Rightarrow$ $x_3(t)$ is not even

(ii) $x_3(t)$ is NEITHER even nor odd.

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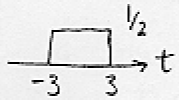
Problem 3)

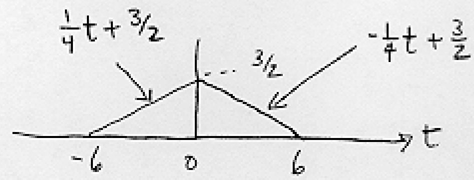
4.18) $H(\omega) = \dfrac{\sin^2 3\omega \cos\omega}{\omega^2}$; $h(t) = \mathcal{F}^{-1}\{H(\omega)\}$.

Let $x_1(t) =$ 

$\xleftrightarrow{\mathcal{F}}$ $\dfrac{\sin 3\omega}{\omega} = X_1(\omega)$

Convolution Property:

$x_2(t) = x_1(t) * x_1(t) =$



$$X_2(\omega) = X_1^2(\omega) = \frac{\sin^2 3\omega}{\omega^2}$$

$$H(\omega) = \frac{\sin^2 3\omega}{\omega^2}\cos\omega = X_2(\omega)\cos\omega$$

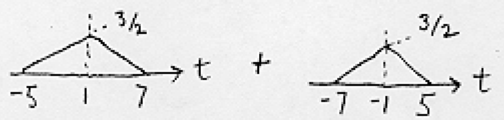
$$= X_2(\omega)\,\frac{e^{-j\omega}+e^{j\omega}}{2}$$

$$= \tfrac{1}{2}e^{-j\omega}X_2(\omega) + \tfrac{1}{2}e^{j\omega}X_2(\omega)$$

Time shift and linearity properties:

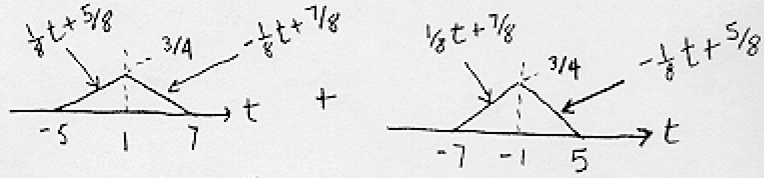
$$h(t) = \tfrac{1}{2}x_2(t-1) + \tfrac{1}{2}x_2(t+1)$$

$= \frac{1}{2}\Big[$



$\Big]$

$=$



For $-7 < t \le -5$, $h(t) = \frac{1}{8}t + 7/8$

$-5 < t \le -1$, $h(t) = \frac{1}{8}t + 5/8 + \frac{1}{8}t + 7/8 = \frac{1}{4}t + \frac{3}{2}$

$-1 < t \le 1$, $h(t) = \frac{1}{8}t + 5/8 - \frac{1}{8}t + \frac{5}{8} = 5/4$

$1 < t \le 5$, $h(t) = -\frac{1}{8}t + 7/8 - \frac{1}{8}t + 5/8 = -\frac{1}{4}t + \frac{3}{2}$

$5 < t \le 7$, $h(t) = -\frac{1}{8}t + 7/8$

$$h(t) = \begin{cases} \frac{1}{8}t + 7/8, & -7 \le t \le -5 \\ \frac{1}{4}t + 3/2, & -5 < t \le -1 \\ 5/4, & -1 < t \le 1 \\ -\frac{1}{4}t + \frac{3}{2}, & 1 < t \le 5 \\ -\frac{1}{8}t + 7/8, & 5 < t \le 7 \\ 0, & \text{otherwise} \end{cases}$$

Problem 4)

4.19)

$$H(\omega) = \frac{1}{3+j\omega}$$

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$

$$Y(\omega) = \mathcal{F}\{e^{-3t}u(t)\} - \mathcal{F}\{e^{-4t}u(t)\}$$

$$= \frac{1}{3+j\omega} - \frac{1}{4+j\omega}$$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{\frac{1}{3+j\omega} - \frac{1}{4+j\omega}}{\frac{1}{3+j\omega}} = 1 - \frac{3+j\omega}{4+j\omega}$$

$$= \frac{4+j\omega-3-j\omega}{4+j\omega} = \frac{1}{4+j\omega}$$

$$x(t) = \mathcal{F}^{-1}\left\{\frac{1}{4+j\omega}\right\} = \underline{\underline{e^{-4t}u(t)}}$$

Problem 5)

4.21a)

$$e^{-\alpha t}u(t) \longleftrightarrow \frac{1}{\alpha+j\omega} \quad \text{(Table)}$$

$$x(t)\cos\omega_0 t \longleftrightarrow \frac{1}{2}X(\omega-\omega_0) + \frac{1}{2}X(\omega+\omega_0) \quad \text{(notes)}$$

$$e^{-\alpha t}\cos\omega_0 t\, u(t) \longleftrightarrow \underline{\underline{\frac{1}{2}\left[\frac{1}{\alpha+j(\omega-\omega_0)} + \frac{1}{\alpha+j(\omega+\omega_0)}\right]}}$$

Problem 5)…

4.21b)

$$x(t) = e^{-3|t|}\sin 2t$$

→ you can write $x(t) = e^{3t}\sin 2t\, u(-t) + e^{-3t}\sin 2t\, u(t)$ and use properties.

→ or: $$X(\omega) = \int_{-\infty}^{\infty} e^{-3|t|}\sin 2t\, e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{0} e^{3t}e^{-j\omega t}\left(\tfrac{1}{2j}\right)\left[e^{j2t}-e^{-j2t}\right]dt + \int_{0}^{\infty} e^{-3t}e^{-j\omega t}\left(\tfrac{1}{2j}\right)\left[e^{j2t}-e^{-j2t}\right]dt$$

$$= \frac{1}{2j}\int_{-\infty}^{0} \exp[3t-j\omega t+j2t] - \exp[3t-j\omega t-j2t]\,dt + \frac{1}{2j}\int_{0}^{\infty} \exp[-3t-j\omega t+j2t] - \exp[-3t-j\omega t-j2t]\,dt$$

$$= \frac{1}{2j}\int_{-\infty}^{0} \exp\{t[3+j(2-\omega)]\} - \exp\{t[3-j(2+\omega)]\}\,dt + \frac{1}{2j}\int_{0}^{\infty} \exp\{t[-3+j(2-\omega)]\} - \exp\{t[-3-j(2+\omega)]\}\,dt$$

$$= \frac{1}{2j}\,\frac{1}{3+j(2-\omega)}[1-0] - \frac{1}{2j}\,\frac{1}{3-j(2+\omega)}[1-0] - \frac{1}{2j}\,\frac{1}{3-j(2-\omega)}[0-1] + \frac{1}{2j}\,\frac{1}{3+j(2+\omega)}[0-1]$$

$$= \frac{1}{2j}\left[\frac{1}{3-j(\omega-2)} + \frac{1}{3+j(\omega-2)}\right] - \frac{1}{2j}\left[\frac{1}{3+j(\omega+2)} + \frac{1}{3-j(\omega+2)}\right]$$

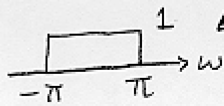
$$= \frac{1}{2j}\left[\frac{3+j(\omega-2)+3-j(\omega-2)}{[3-j(\omega-2)][3+j(\omega-2)]} - \frac{3-j(\omega+2)+3+j(\omega+2)}{[3+j(\omega+2)][3-j(\omega+2)]}\right]$$

$$= \frac{3j}{9+(\omega+2)^2} - \frac{3j}{9+(\omega-2)^2} \quad ///$$

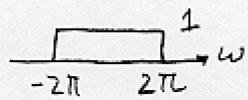
Problem 5)…

4.21f)

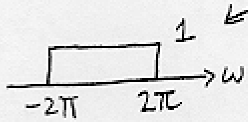
$$x(t) = \left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$$

$$\frac{\sin \pi t}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$$ 

(Table)

$$\frac{\sin 2\pi t}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$$ 

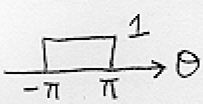
(Table)

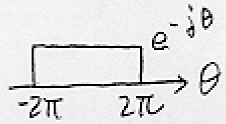
$$\frac{\sin 2\pi(t-1)}{\pi(t-1)} \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega} \times$$ 

(Time Shift property)

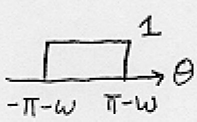
$$X(\omega) = \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega-\theta)\,X_2(\theta)\,d\theta$$

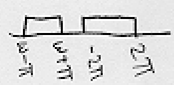
$X_1(\theta)$ $X_2(\theta)$





$X_1(\omega+\theta)$



Case I: $\omega+\pi < -2\pi$: $\omega < -3\pi$ 

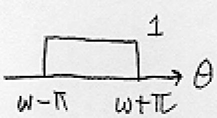
$$X(\omega) = \underline{\underline{0}}.$$

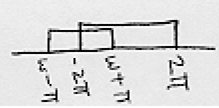
Case II: $\omega+\pi \geq -2\pi$ and $\omega-\pi < -2\pi$

$\omega \geq -3\pi$ and $\omega < -\pi$

$$-3\pi \leq \omega < -\pi$$

$X_1(\omega-\theta)$



$\longrightarrow$ 

Problem 5)…

$$X(\omega) = \frac{1}{2\pi}\int_{-2\pi}^{\omega+\pi} e^{-j\theta}\,d\theta = \frac{1}{2\pi}\left(\frac{-1}{j}\right)\left[e^{-j\theta}\right]_{\theta=-2\pi}^{\omega+\pi}$$

$$= \frac{j}{2\pi}\left[e^{-j(\omega+\pi)} - e^{j2\pi}\right]$$

$$= \frac{j}{2\pi}\left[e^{-j\omega}e^{-j\pi} - (\cos 2\pi + j\sin 2\pi)\right]$$

$$= \frac{j}{2\pi}\left[e^{-j\omega}(\cos\pi - j\sin\pi) - 1\right]$$

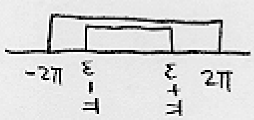
$$= \frac{j}{2\pi}\left[e^{-j\omega}(-1-0) - 1\right]$$

$$= \frac{j}{2\pi}\left[-e^{-j\omega} - 1\right] = \underline{\underline{\frac{1}{j2\pi}\left[e^{-j\omega} + 1\right]}}$$

case III: $\omega+\pi < 2\pi$ and $\omega-\pi \geq -2\pi$

$\omega < \pi$ and $\omega \geq -\pi$

$-\pi \leq \omega < \pi$



$$X(\omega) = \int_{\omega-\pi}^{\omega+\pi} e^{-j\theta}\,d\theta = \frac{j}{2\pi}\left[e^{-j\theta}\right]_{\theta=\omega-\pi}^{\omega+\pi}$$

$$= \frac{j}{2\pi}\left[e^{-j(\omega+\pi)} - e^{-j(\omega-\pi)}\right]$$

$$= \frac{j}{2\pi}e^{-j\omega}\left[e^{-j\pi} - e^{j\pi}\right] = \frac{e^{-j\omega}}{\pi}\left[\frac{e^{j\pi} - e^{-j\pi}}{2j}\right]$$

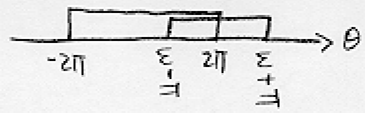
$$= \frac{e^{-j\omega}}{\pi}\sin\pi = \underline{\underline{0}}.$$

Problem 5)…

case IV : $\omega + \pi \geq 2\pi$ and $\omega - \pi < 2\pi$

$\omega \geq \pi$ and $\omega < 3\pi$

$\pi \leq \omega < 3\pi$



$$X(\omega) = \frac{1}{2\pi}\int_{\omega-\pi}^{2\pi} e^{-j\theta}\,d\theta$$

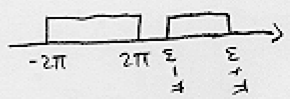
$$= \frac{j}{2\pi}\left[e^{-j\theta}\right]_{\theta=\omega-\pi}^{2\pi} = \frac{j}{2\pi}\left[e^{-j2\pi} - e^{-j(\omega-\pi)}\right]$$

$$= \frac{j}{2\pi}\left[\cos 2\pi - j\sin 2\pi - e^{-j\omega}e^{j\pi}\right]$$

$$= \frac{j}{2\pi}\left[1 - 0 - e^{-j\omega}(\cos\pi - j\sin\pi)\right]$$

$$= \underline{\underline{\frac{j}{2\pi}\left[1 + e^{-j\omega}\right]}}$$

case V : $\omega \geq 3\pi$:



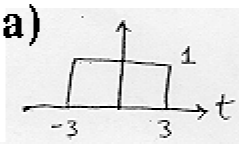
; $X(\omega) = 0$

all together:

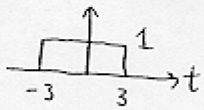
$$X(\omega) = \begin{cases} \frac{1}{j2\pi}\left[e^{-j\omega} + 1\right], & -3\pi \leq \omega < -\pi \\ \frac{j}{2\pi}\left[e^{-j\omega} + 1\right], & \pi \leq \omega < 3\pi \\ 0, & \text{otherwise} \end{cases}$$

Problem 6)

4.22a)



$$\xleftrightarrow{\mathcal{F}} \quad \frac{2\sin 3\omega}{\omega}$$

$e^{j2\pi t} \times$ 

$$\xleftrightarrow{\mathcal{F}} \quad \frac{2\sin[3(\omega-2\pi)]}{\omega-2\pi}$$

$$x(t) = \begin{cases} e^{j2\pi t}, & |t| \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

4.22d)

$$x(t) = \mathcal{F}^{-1}[X(\omega)]$$

$$= \mathcal{F}^{-1}\left\{2[\delta(\omega-1)-\delta(\omega+1)] + 3[\delta(\omega-2\pi)+\delta(\omega+2\pi)]\right\}$$

$$= \mathcal{F}^{-1}\left\{\frac{2j}{\pi}\cdot\frac{\pi}{j}[\delta(\omega-1)-\delta(\omega+1)]\right\} + \mathcal{F}^{-1}\left\{\frac{3}{\pi}\,\pi[\delta(\omega-2\pi)+\delta(\omega+2\pi)]\right\}$$

$$= \frac{2j}{\pi}\sin t + \frac{3}{\pi}\cos 2\pi t$$

Problem 7)

4.26a) (i)

$$x(t) = te^{-2t}u(t) \xleftrightarrow{\mathcal{F}} \frac{1}{(2+j\omega)^2} \quad \text{(table)}$$

$$h(t) = e^{-4t}u(t) \xleftrightarrow{\mathcal{F}} \frac{1}{4+j\omega} \quad \text{(table)}$$

$$Y(\omega) = X(\omega)H(\omega) = \frac{1}{(4+j\omega)(2+j\omega)^2}$$

$$= \frac{A}{4+j\omega} + \frac{B}{(2+j\omega)^2} + \frac{C}{(2+j\omega)}$$

$$A = \left.\frac{1}{(2+\theta)^2}\right|_{\theta=-4} = \frac{1}{4}$$

$$B = \left.\frac{1}{4+\theta}\right|_{\theta=-2} = \frac{1}{2}$$

$$\frac{d}{d\theta}(4+\theta)^{-1} = \frac{d}{d\theta}(2+\theta)C$$

$$-(4+\theta)^{-2}\Big|_{\theta=-2} = -\frac{1}{4} = C$$

$$Y(\omega) = \frac{1/4}{4+j\omega} + \frac{1/2}{(2+j\omega)^2} - \frac{1/4}{2+j\omega}$$

$$y(t) = \left[\frac{1}{4}e^{-4t} + \frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t}\right]u(t)$$

Problem 7)…

4.26a) (iii)

$$x(t) = e^{-t}u(t)$$

$$X(\omega) = \frac{1}{1+j\omega}$$

$$h(t) = x(-t) \Rightarrow H(\omega) = X(-\omega) = \frac{1}{1-j\omega}$$

$$Y(\omega) = X(\omega)H(\omega)$$

$$= \frac{1}{1+j\omega}\,\frac{1}{1-j\omega}$$

$$= \frac{A}{1+j\omega} + \frac{B}{1-j\omega}$$

$$A = \left.\frac{1}{1-\theta}\right|_{\theta=-1} = \frac{1}{2}$$

$$B = \left.\frac{1}{1+\theta}\right|_{\theta=1} = \frac{1}{2}$$

$$Y(\omega) = \frac{1/2}{1+j\omega} + \frac{1/2}{1-j\omega}$$

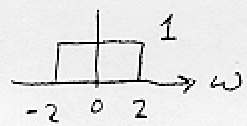
$$y(t) = \tfrac{1}{2}e^{-t}u(t) + \tfrac{1}{2}e^{t}u(-t)$$

$$= \tfrac{1}{2}e^{-|t|}$$

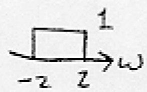
Problem 8)

4.32d)

$$\frac{\sin 2t}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$$



$$x_4(t) = \left[\frac{\sin 2t}{\pi t}\right]^2 \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi}\Big\{$$



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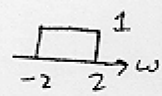
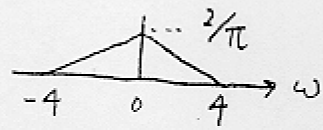


$$\Big\}$$

$$=$$



$$Y_4(\omega) = X_4(\omega)\,H(\omega)$$

$$= X_4(\omega)\,e^{-j\omega}$$

$$y_4(t) = \mathcal{F}^{-1}\{Y_4(\omega)\} = x_4(t-1)$$

$$= \left[\frac{\sin 2(t-1)}{\pi(t-1)}\right]^2$$

Problem 9)

4.33a) $y''(t) + 6y'(t) + 8y(t) = 2x(t)$

$$-\omega^2 Y(\omega) + 6j\omega Y(\omega) + 8Y(\omega) = 2X(\omega)$$

$$Y(\omega)\left[-\omega^2 + 6j\omega + 8\right] = 2X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{-\omega^2 + 6j\omega + 8}$$

$$H(\omega) = \frac{2}{(j\omega+4)(j\omega+2)} = \frac{A}{j\omega+4} + \frac{B}{j\omega+2}$$

$$A = \left.\frac{2}{\theta+2}\right|_{\theta=-4} = \frac{2}{-2} = -1$$

$$B = \left.\frac{2}{\theta+4}\right|_{\theta=-2} = \frac{2}{2} = 1$$

$$H(\omega) = \frac{1}{j\omega+2} - \frac{1}{j\omega+4}$$

$$h(t) = \mathcal{F}^{-1}\{H(\omega)\} = \underline{\underline{e^{-2t}u(t) - e^{-4t}u(t)}}$$

4.33b) $x(t) = te^{-2t}u(t)$

$$X(\omega) = \frac{1}{(j\omega+2)^2}$$

Problem 9)…

$$Y(\omega) = X(\omega)H(\omega) = \frac{2}{(j\omega+4)(j\omega+2)^3}$$

$$= \frac{A}{j\omega+2} + \frac{B}{(j\omega+2)^2} + \frac{C}{(j\omega+2)^3} + \frac{D}{j\omega+4}$$

Write in terms of $\theta = j\omega$:

$$\frac{2}{(\theta+4)(\theta+2)^3} = \frac{A}{\theta+2} + \frac{B}{(\theta+2)^2} + \frac{C}{(\theta+2)^3} + \frac{D}{\theta+4}$$

$$D = \left.\frac{2}{(\theta+2)^3}\right|_{\theta=-4} = \frac{2}{(-2)^3} = \frac{2}{-8} = -\frac{1}{4}$$

$$C = \left.\frac{2}{\theta+4}\right|_{\theta=-2} = \frac{2}{2} = 1$$

For B, multiply both sides by $(\theta+2)^3$, differentiate, and evaluate at $\theta=-2$:

$$\frac{d}{d\theta}\left[2(\theta+4)^{-1}\right]_{\theta=-2} = \frac{d}{d\theta}\left[A(\theta+2)^2\right]_{\theta=-2} + \frac{d}{d\theta}\left[B(\theta+2)\right]_{\theta=-2}$$

$$+ \left.\frac{d}{d\theta}C\right|_{\theta=-2} + \frac{d}{d\theta}\left[D(\theta+4)^{-1}(\theta+2)^3\right]_{\theta=-2}$$

$$\left[(2)(-1)(\theta+4)^{-2}\right]_{\theta=-2} = \left[A(2)(\theta+2)\right]_{\theta=-2} + B + 0$$

$$+ D\left[(-1)(\theta+4)^{-2}(\theta+2)^3 + (\theta+4)^{-1}(3)(\theta+2)^2\right]_{\theta=-2}$$

Problem 9)…

$$\frac{-2}{(2)^2} = A\cdot 0 + B + D\left[0+0\right]$$

$$B = \frac{-2}{4} = -\frac{1}{2}$$

For A, multiply both sides by $(\theta+2)^3$, differentiate twice, and evaluate at $\theta=-2$:

$$\frac{d^2}{d\theta^2}\left[2(\theta+4)^{-1}\right]_{\theta=-2} = \frac{d^2}{d\theta^2}\left[A(\theta+2)^2\right]_{\theta=-2} + \frac{d^2}{d\theta^2}\left[B(\theta+2)\right]_{\theta=-2}$$
$$+ \frac{d^2}{d\theta^2}C\Big|_{\theta=-2} + \frac{d^2}{d\theta^2}\left[D(\theta+4)^{-1}(\theta+2)^3\right]_{\theta=-2}$$

$$\frac{d}{d\theta}\left[(-2)(\theta+4)^{-2}\right]_{\theta=-2} = 2A\frac{d}{d\theta}\left[(\theta+2)\right]_{\theta=-2} + \frac{d}{d\theta}B\Big|_{\theta=-2}$$
$$+ D\frac{d}{d\theta}\left[(-1)(\theta+4)^{-2}(\theta+2)^3 + (\theta+4)^{-1}(3)(\theta+2)^2\right]_{\theta=-2}$$

$$(4)(\theta+4)^{-3}\Big|_{\theta=-2} = 2A\Big|_{\theta=-2} + 0 + D\Big[(-1)(-2)(\theta+4)^{-3}(\theta+2) + (-1)(\theta+4)^{-2}(3)(\theta+2)^2$$
$$+ (3)(-1)(\theta+4)^{-2}(\theta+2)^2 + (3)(\theta+4)^{-1}(2)(\theta+2)\Big]_{\theta=-2}$$

$$\frac{4}{(2)^3} = 2A + D\left[0+0+0+0\right]$$

$$2A = \frac{1}{2}$$

$$A = \frac{1}{4}$$

$$Y(\omega) = \frac{1/4}{j\omega+2} - \frac{1/2}{(j\omega+2)^2} + \frac{1}{(j\omega+2)^3} - \frac{1/4}{j\omega+4}$$

$$\underline{\underline{y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + t^2e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t)}}$$

Problem 10)

4.34a)

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6} = \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6}$$

$$\left[(j\omega)^2 + 5j\omega + 6\right] Y(\omega) = \left[j\omega + 4\right] X(\omega)$$

$$(j\omega)^2 Y(\omega) + 5j\omega Y(\omega) + 6Y(\omega) = j\omega X(\omega) + 4X(\omega)$$

$$\underline{\underline{y''(t) + 5y'(t) + 6y(t) = x'(t) + 4x(t)}}$$

4.34b)

$$H(\omega) = \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6} = \frac{j\omega + 4}{(3+j\omega)(2+j\omega)} = \frac{A}{3+j\omega} + \frac{B}{2+j\omega}$$

$$A = \left.\frac{\theta + 4}{2+\theta}\right|_{\theta=-3} = \frac{1}{-1} = -1$$

$$B = \left.\frac{\theta + 4}{3+\theta}\right|_{\theta=-2} = \frac{2}{1} = 2$$

$$H(\omega) = \frac{2}{2+j\omega} - \frac{1}{3+j\omega} \Longrightarrow \underline{\underline{h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)}}$$

Problem 10)…

4.34c) $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$

$$X(\omega) = \frac{1}{j\omega+4} - \frac{1}{(j\omega+4)^2} = \frac{j\omega+4-1}{(j\omega+4)^2} = \frac{3+j\omega}{(4+j\omega)^2}$$

$$Y(\omega) = X(\omega)H(\omega) = \frac{3+j\omega}{(4+j\omega)^2}\,\frac{j\omega+4}{(3+j\omega)(2+j\omega)} = \frac{1}{(4+j\omega)(2+j\omega)}$$

$$= \frac{A}{2+j\omega} + \frac{B}{4+j\omega}$$

$$A = \left.\frac{1}{4+\theta}\right|_{\theta=-2} = \frac{1}{2}$$

$$B = \left.\frac{1}{2+\theta}\right|_{\theta=-4} = -\frac{1}{2}$$

$$Y(\omega) = \frac{1/2}{2+j\omega} - \frac{1/2}{4+j\omega}$$

$$y(t) = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-4t}u(t)$$