ECE 3793
Matlab Project 2 Solution

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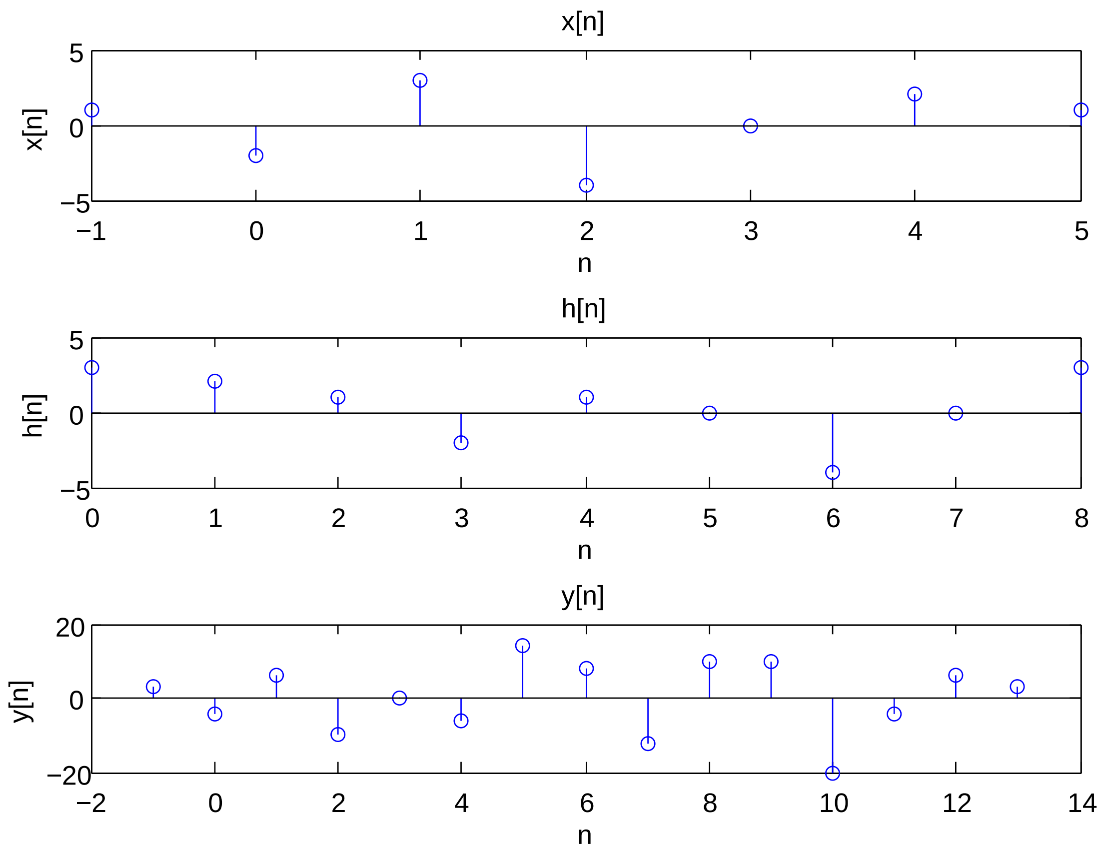
1. (a) Matlab code:

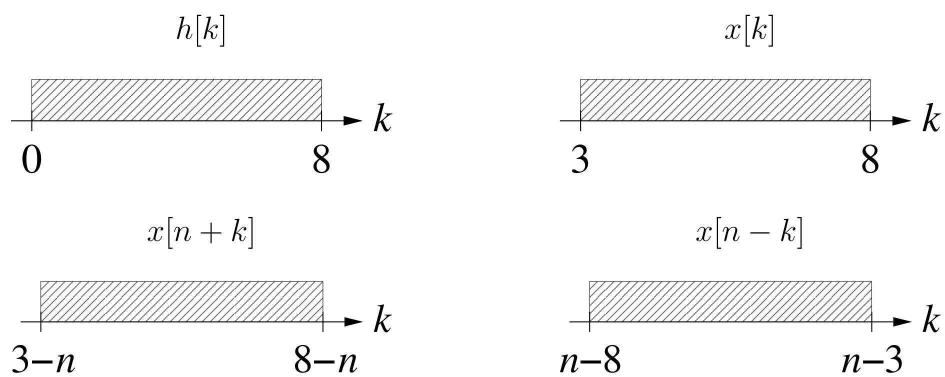
```
%-------------------------------------------------------
% P1a
%
% compute and plot a discrete convolution
%
h = [3 2 1 -2 1 0 -4 0 3];  % Impulse response
x = [1 -2 3 -4 0 2 1];      % Input signal
y = conv(x,h);              % y[n] = x[n] * h[n]
subplot(3,1,1);             % 3x1 array of graphs
stem([-1:5],x);             % plot x[n]
title('x[n]');
xlabel('n');
ylabel('x[n]');
subplot(3,1,2);             % make 2nd graph active
stem([0:8],h);              % plot h[n]
title('h[n]');
xlabel('n');
ylabel('h[n]');
subplot(3,1,3);             % make 3rd graph active
stem([-1:13],y);            % plot y[n]
title('y[n]');
xlabel('n');
ylabel('y[n]');
```



(b) In this case, the input signal $x[n]$ starts at $n = 3$, ends at $n = 8$, and has length $8 - 3 + 1 = 6$. The impulse reponse $h[n]$ is unchanged from part (a): it starts at $n = 0$, ends at $n = 8$, and has length $8 - 0 + 1 = 9$ (as before). Therefore, the output signal $y[n]$ returned by `conv` will have length $6 + 9 - 1 = 14$.

But we still need to figure out the values of n when $y[n]$ starts and ends.

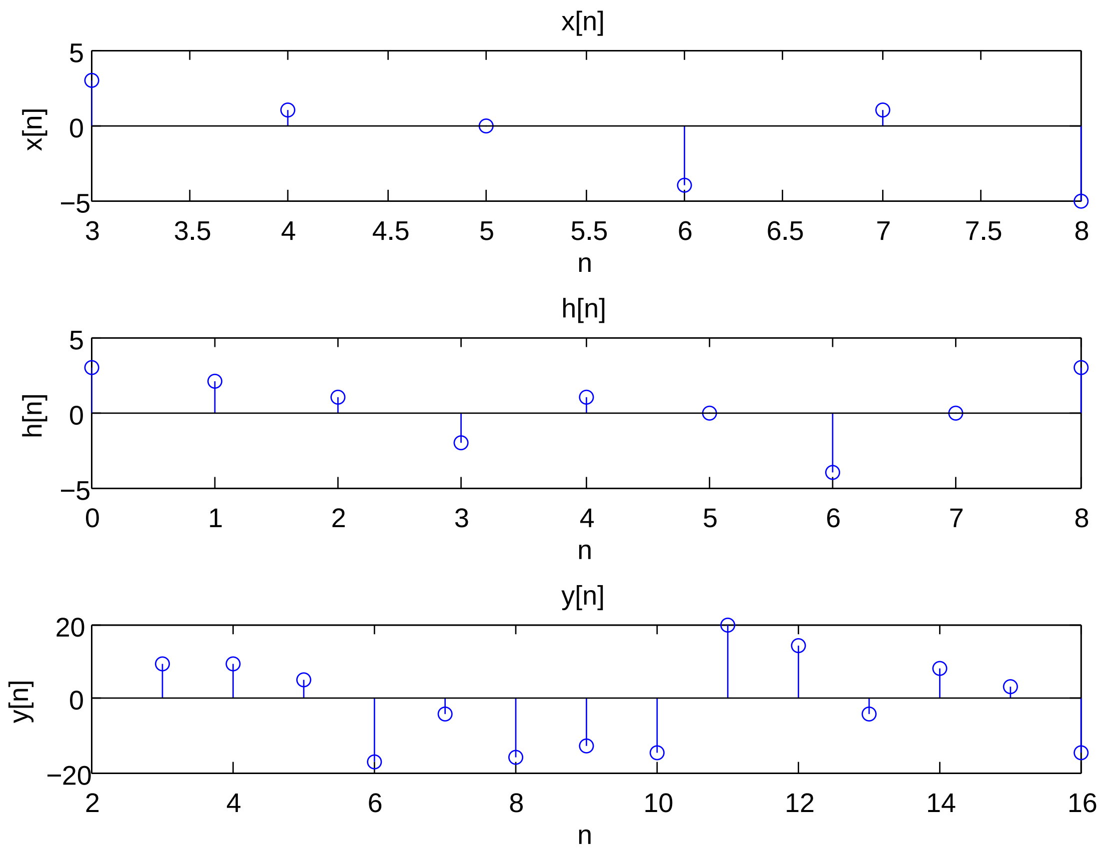
The figures below show the starting and stopping values of k for the signals $h[k]$, $x[k]$, $x[n + k]$, and $x[n - k]$:



From these figures, we see that the first time $y[n]$ can be nonzero is when $n - 3 = 0$, or $n = 3$. The last time that $y[n]$ can be nonzero is when $n - 8 = 8$, or $n = 16$. Note that this again gives us that the length of $y[n]$ is $16 - 3 + 1 = 14$.

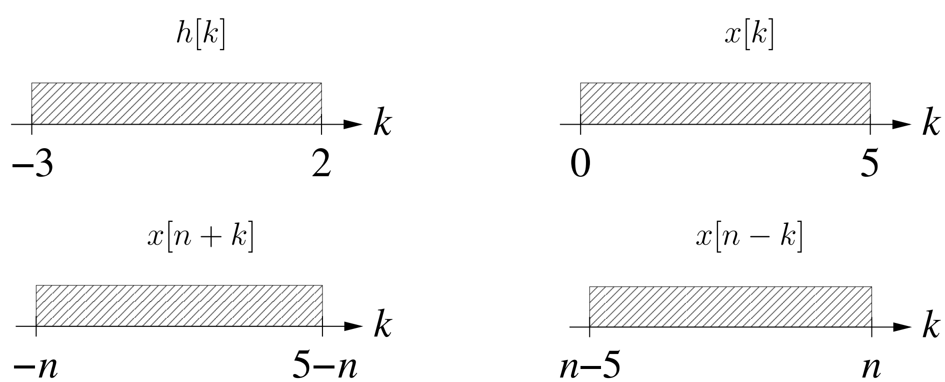
Matlab code:

```
%-------------------------------------------------------
% P1b
%
% compute and plot another discrete convolution
%
h = [3 2 1 -2 1 0 -4 0 3];  % Same as before
x = [3 1 0 -4 1 -5];        % Input signal
y = conv(x,h);              % y[n] = x[n] * h[n]
subplot(3,1,1);             % 3x1 array of graphs
stem([3:8],x);              % plot x[n]
title('x[n]');
xlabel('n');
ylabel('x[n]');
subplot(3,1,2);             % make 2nd graph active
stem([0:8],h);              % plot h[n]
title('h[n]');
xlabel('n');
ylabel('h[n]');
subplot(3,1,3);             % make 3rd graph active
stem([3:16],y);             % plot y[n]
title('y[n]');
xlabel('n');
ylabel('y[n]');
```



(c) For this part, the input signal $x[n]$ starts at $n = 0$, ends at $n = 5$, and has length $5 - 0 + 1 = 6$. The impulse reponse $h[n]$ starts at $n = -3$, ends at $n = 2$, and has length $2 - (-3) + 1 = 6$. So the output signal $y[n]$ returned by `conv` will have length $6 + 6 - 1 = 11$.

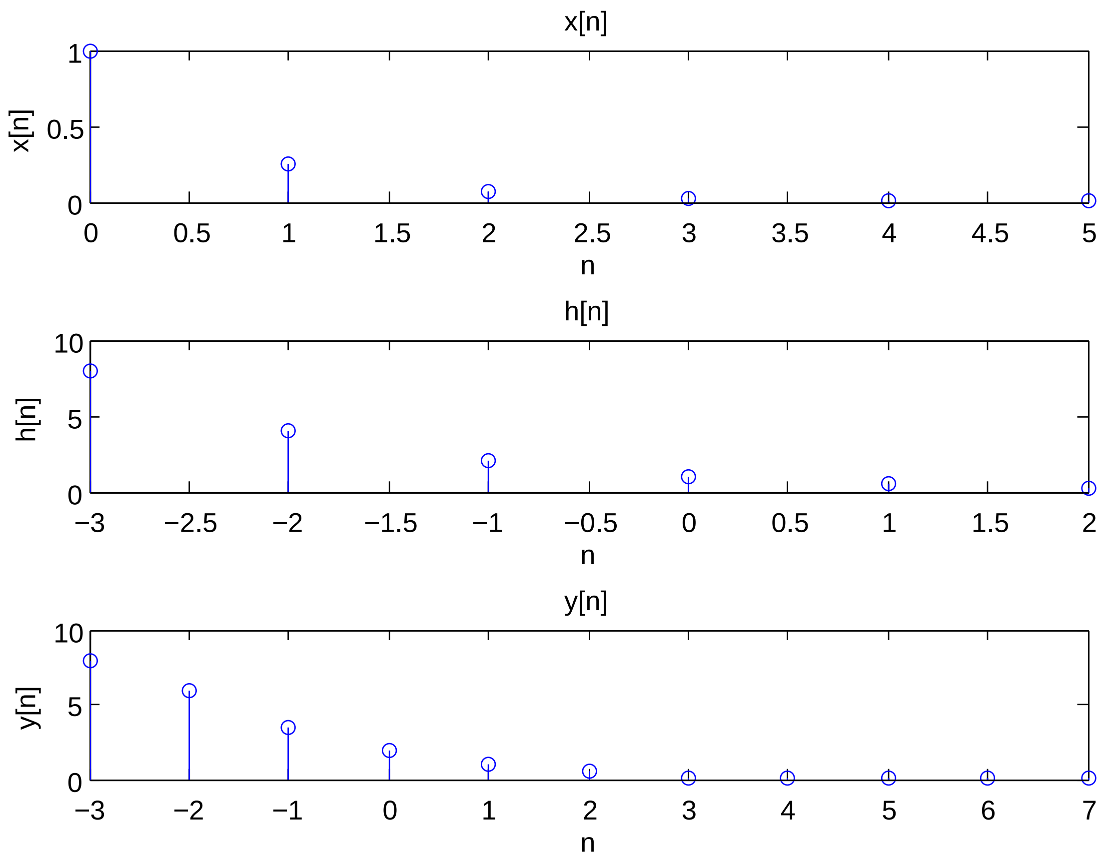
The figures below show the starting and stopping values of k for the signals $h[k]$, $x[k]$, $x[n+k]$, and $x[n-k]$:



From these figures, we see that the first time $y[n]$ can be nonzero is when $n = -3$. The last time that $y[n]$ can be nonzero is when $n - 5 = 2$, or $n = 7$. Note that this again gives us that the length of $y[n]$ is $7 - (-3) + 1 = 11$.

Matlab code:

```
%-------------------------------------------------------
% P1c
%
% compute and plot yet another discrete convolution
%
n = 0:5;                       % values of "n" for x[n]
x = 0.25.^n;                   % input signal
subplot(3,1,1);                % 3x1 array of graphs
stem(n,x);                     % plot x[n]
title('x[n]');
xlabel('n');
ylabel('x[n]');
n = -3:2;                      % values of "n" for h[n]
h = 0.5.^n;                    % impulse response
subplot(3,1,2);                % make 2nd graph active
stem(n,h);                     % plot h[n]
title('h[n]');
xlabel('n');
ylabel('h[n]');
y = conv(x,h);                 % y[n] = x[n] * h[n]
n = -3:7;                      % values of "n" for y[n]
subplot(3,1,3);                % make 3rd graph active
stem(n,y);                     % plot y[n]
title('y[n]');
xlabel('n');
ylabel('y[n]');
```



2. (a) (Text problem 4.33). The given input/output relation is

$$y''(t) + 6y'(t) + 8y(t) = 2x(t).$$

Taking the Fourier transform of both sides of the input/output relation, we obtain

$$(j\omega)^2 Y(\omega) + 6j\omega Y(\omega) + 8Y(\omega) = 2X(\omega)$$

$$\left[(j\omega)^2 + 6j\omega + 8\right] Y(\omega) = 2X(\omega),$$

so

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{(j\omega)^2 + 6j\omega + 8}.$$

4.33a) We are asked to find the system impulse reponse $h(t)$. The following Matlab session log computes the partial fraction expansion for $H(\omega)$:

```
numer = [2];
denom = [1 6 8];
[r p k] = residue(numer,denom)

r =
```

```
   -1
    1


p =

   -4
   -2


k =

    []
```

The values of `[r p k]` returned by the `residue` function give the partial fraction expansion

$$H(\omega) = -\frac{1}{j\omega + 4} + \frac{1}{j\omega + 2}.$$

The impulse response can then be written down directly from Table 4.2:

$$h(t) = -e^{-4t}u(t) + e^{-2t}u(t).$$

4.33b) In part (b), we are asked to find the system output $y(t)$ when the input is

$$x(t) = te^{-2t}u(t).$$

From Table 4.2, the Fourier transform of the input is

$$X(\omega) = \frac{1}{(j\omega + 2)^2}.$$

So the Fourier transform of the output is given by

$$\begin{aligned} Y(\omega) = X(\omega)H(\omega) &= \frac{1}{(j\omega+2)^2} \quad \frac{2}{(j\omega)^2 + 6j\omega + 8} \\ &= \frac{1}{(j\omega)^2 + 4j\omega + 4} \quad \frac{2}{(j\omega)^2 + 6j\omega + 8} \\ &= \frac{2}{(j\omega)^4 + 10(j\omega)^3 + 36(j\omega)^2 + 56j\omega + 32}. \end{aligned}$$

The following Matlab session log determines the partial fraction expansion for $Y(\omega)$:

```
numer = [2];
denom = [1 10 36 56 32];
[r p k] = residue(numer,denom)
```

```
r =

  -0.2500
   0.2500
  -0.5000
   1.0000


p =

  -4.0000
  -2.0000
  -2.0000
  -2.0000


k =

     []
```

From the values of `[r p k]` returned by `residue`, the partial fraction expansion for $Y(\omega)$ is given by

$$Y(\omega) = \frac{-\frac{1}{4}}{j\omega + 4} + \frac{\frac{1}{4}}{j\omega + 2} + \frac{-\frac{1}{2}}{(j\omega + 2)^2} + \frac{1}{(j\omega + 2)^3}.$$

From the partial fraction expansion, we can then use Table 4.2 to write down the solution for $y(t)$ directly as

$$y(t) = \frac{1}{4}\left[-e^{-4t} + e^{-2t} - 2te^{-2t} + 2t^2e^{-2t}\right] u(t).$$

4.33c) Part (c) asks us to find the impulse response $h(t)$ for the causal, stable LTI system H with input/output relation

$$y''(t) + \sqrt{2}y'(t) + y(t) = 2x''(t) - 2x(t).$$

Taking Fourier transforms on both sides, we obtain

$$\left[(j\omega)^2 + \sqrt{2}j\omega + 1\right] Y(\omega) = \left[2(j\omega)^2 - 2\right] X(\omega),$$

from which we obtain the frequency response as

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2(j\omega)^2 - 2}{(j\omega)^2 + \sqrt{2}j\omega + 1}.$$

The following Matlab session log finds the partial fraction expansion for $H(\omega)$:

```
numer = [2 0 -2];
denom = [1 sqrt(2) 1];

r =

  -1.4142 + 1.4142i
  -1.4142 - 1.4142i


p =

  -0.7071 + 0.7071i
  -0.7071 - 0.7071i


k =

     2
```

According to the `[r p k]` values returned by `residue`, the partial fraction expansion is

$$H(\omega) = 2 + \frac{-\sqrt{2} + j\sqrt{2}}{j\omega + \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)} + \frac{-\sqrt{2} - j\sqrt{2}}{j\omega + \left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)}.$$

The impulse response $h(t)$ is then obtained using Table 4.2:

$$\begin{aligned}
h(t) &= 2\delta(t) + \left(-\sqrt{2} + j\sqrt{2}\right) e^{-\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)t} u(t) + \left(-\sqrt{2} - j\sqrt{2}\right) e^{-\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)t} u(t) \\
&= 2\delta(t) + \left\{\left(-\sqrt{2} + j\sqrt{2}\right) e^{-\frac{\sqrt{2}}{2}t} e^{j\frac{\sqrt{2}}{2}t} + \left(-\sqrt{2} - j\sqrt{2}\right) e^{-\frac{\sqrt{2}}{2}t} e^{-j\frac{\sqrt{2}}{2}t}\right\} u(t) \\
&= 2\delta(t) + \left\{-\sqrt{2}e^{-\frac{\sqrt{2}}{2}t} e^{j\frac{\sqrt{2}}{2}t} + j\sqrt{2}e^{-\frac{\sqrt{2}}{2}t} e^{j\frac{\sqrt{2}}{2}t}\right. \\
&\qquad \left. - \sqrt{2}e^{-\frac{\sqrt{2}}{2}t} e^{-j\frac{\sqrt{2}}{2}t} - j\sqrt{2}e^{-\frac{\sqrt{2}}{2}t} e^{-j\frac{\sqrt{2}}{2}t}\right\} u(t) \\
&= 2\delta(t) + \left\{-\sqrt{2}e^{-\frac{\sqrt{2}}{2}t}\left[e^{j\frac{\sqrt{2}}{2}t} + e^{-j\frac{\sqrt{2}}{2}t}\right] + j\sqrt{2}e^{-\frac{\sqrt{2}}{2}t}\left[e^{j\frac{\sqrt{2}}{2}t} - e^{-j\frac{\sqrt{2}}{2}t}\right]\right\} u(t) \\
&= 2\delta(t) + \left\{-2\sqrt{2}e^{-\frac{\sqrt{2}}{2}t}\cos\left(\frac{\sqrt{2}}{2}t\right) - 2\sqrt{2}e^{-\frac{\sqrt{2}}{2}t}\sin\left(\frac{\sqrt{2}}{2}t\right)\right\} u(t) \\
&= 2\delta(t) - 2\sqrt{2}e^{-\frac{\sqrt{2}}{2}t}\left[\cos\left(\frac{\sqrt{2}}{2}t\right) + \sin\left(\frac{\sqrt{2}}{2}t\right)\right] u(t) \\
&= 2\delta(t) - 2\sqrt{2}e^{-t/\sqrt{2}}\left[\cos\left(\frac{t}{\sqrt{2}}\right) + \sin\left(\frac{t}{\sqrt{2}}\right)\right] u(t).
\end{aligned}$$

(b) (Text problem 4.34(b)). We are given a causal and stable LTI system with frequency response

$$H(\omega) = \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6}$$

and asked to find the impulse response $h(t)$. The following Matlab session log finds the partial fraction expansion for $H(\omega)$:

```
numer = [1 4];
denom = [1 5 6];
[r p k] = residue(numer,denom)

r =

   -1.0000
    2.0000


p =

   -3.0000
   -2.0000


k =

     []
```

According to the values of `[r p k]` returned by `residue`, the partial fraction expansion is

$$H(\omega) = -\frac{1}{j\omega + 3} + \frac{2}{j\omega + 2}.$$

Using Table 4.2, the impulse response is given by

$$h(t) = -e^{-3t}u(t) + 2e^{-2t}u(t).$$

3. (Text Problem 4.3(a)). Matlab session log:

```
syms t w
xt = sin(2*pi*t + pi/4);
Xw = fourier(xt,t,w)

Xw =

-pi*(2^(1/2)*dirac(w - 2*pi)*(1/2 + i/2)
                            + 2^(1/2)*dirac(2*pi + w)*(- 1/2 + i/2))*i
```

4. (Text Problem 4.1(a)). Matlab session log:

```
syms t w
xt = exp(-2*(t-1)) * heaviside(t-1)

xt =

heaviside(t - 1)*exp(2 - 2*t)

Xw = fourier(xt,t,w)

Xw =

exp(-w*i)/(w*i + 2)

pretty(Xw)

  exp(-w i)
  ---------
   2 + w i
```

5. (Text Problem 4.26(a)(iii)). Matlab session log:

```
syms t w
xt = exp(-t)*heaviside(t);
ht = exp(t)*heaviside(-t);
Xw = fourier(xt,t,w)

Xw =

1/(w*i + 1)

Hw = fourier(ht,t,w)

Hw =

-1/(w*i - 1)

Yw = Xw * Hw

Yw =

-1/((w*i - 1)*(w*i + 1))
```

```
yt = ifourier(Yw,w,t)

yt =

(pi*exp(-t)*heaviside(t) + pi*heaviside(-t)*exp(t))/(2*pi)

simplify(yt)

ans =

(exp(-t)*heaviside(t))/2 + (heaviside(-t)*exp(t))/2
```