- The effects of adding feedback to an open loop system can be thought of in terms of how the feedback changes the poles.
- We have already seen that feedback can move the poles of a continuous-time system from the right half of the s-plane to the left half.
- For a discrete system, it can move poles from outside the unit circle to inside.
- The addition of feedback can also make a stable open loop system into an unstable closed loop system

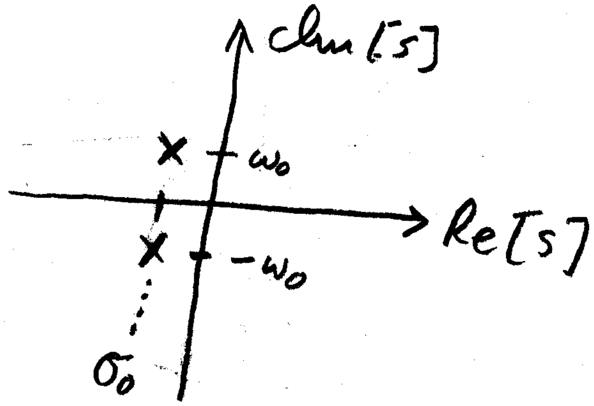
→ EX: Audio feedback of a microphone

→ Sometimes this is done on purpose

- To produce a harmonic oscillator
- controlled feedback of a guitar in rock and roll

- The location of the poles also tells you a lot about the frequency characteristics of a system.

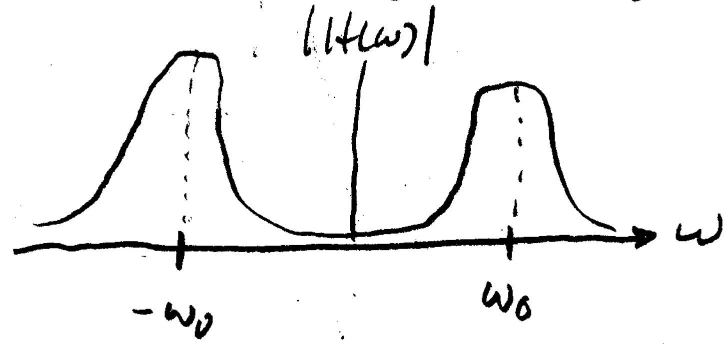
$H(s)$



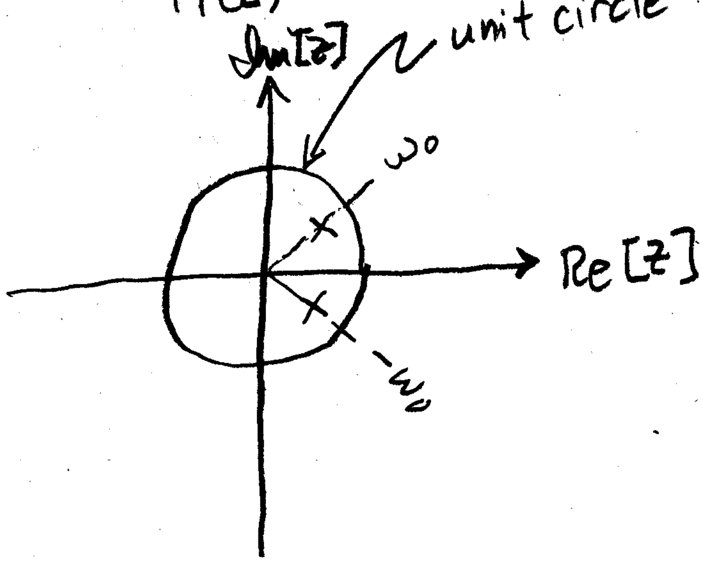
→ The poles are at $s = \sigma_0 \pm \omega_0$

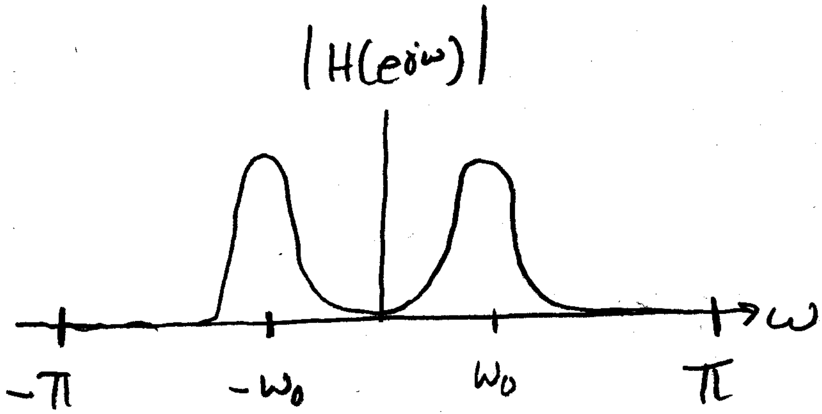
→ At the poles, $H(s) \to \infty$

→ So $H(\omega)$ must be large at $\omega = \pm\omega_0$



$H(z)$





→ So feedback can also be used to alter the frequency characteristic by moving the poles.

- often, it is usefull to examine how the poles move when some parameter of the feedback is varied.
- we look at the denominator of the transfer function. The poles are the <u>roots</u> of the denominator.
- The path that a pole (root) follows as the parameter varies is called the <u>locus</u> of the pole (plural = "loci").
- looking at the pole paths in this way is called "Root Locus" analysis.
- It works the same for continuous systems and for discrete systems.
  → For continuous systems, we analyze the root loci with respect to the $j\omega$-axis of the s-plane.

→ For discrete systems we analyze the root loci with respect to the unit circle of the z-plane.

EX : Discrete

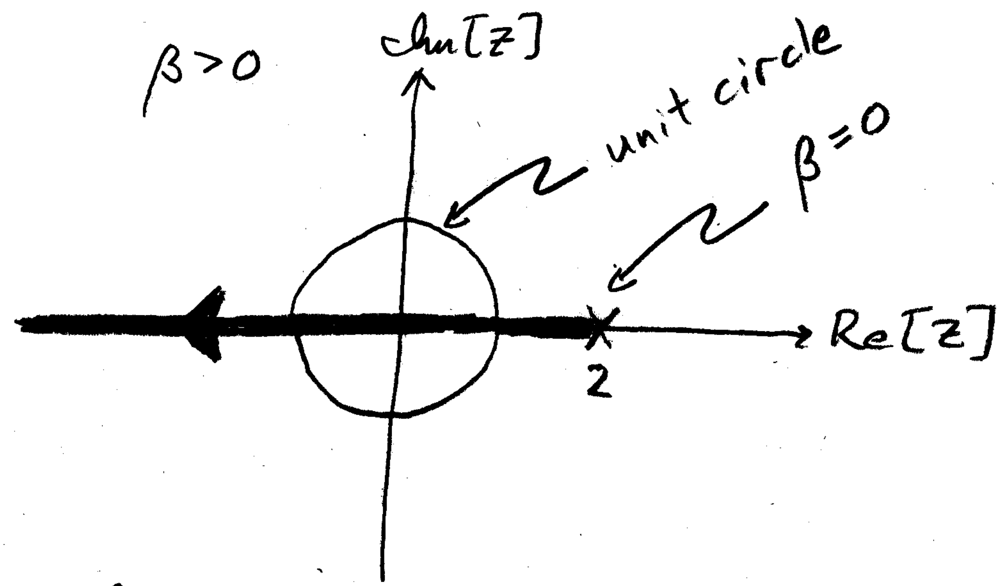
$$H(z) = \frac{1}{1-2z^{-1}} = \frac{z}{z-2}$$

$$G(z) = 2\beta z^{-1} = \frac{2\beta}{z}$$

$$Q(z) = \frac{1}{1-2(1-\beta)z^{-1}} = \frac{z}{z-2(1-\beta)}$$

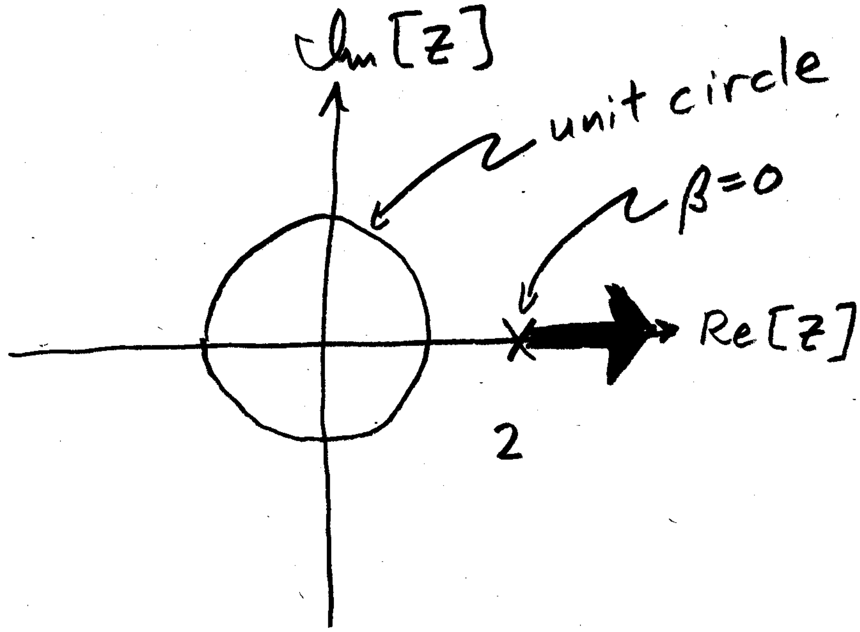
- There is a pole at $z = 2(1-\beta)$.

- For $0 \le \beta < \infty$, the root locus is



This locus (path) starts at $z=2$ when $\beta=0$ and goes toward $z=-\infty$ as $\beta \to \infty$.

- For $-\infty < \beta \leq 0$, the locus of $2(1-\beta)$ again starts at $z=2$ when $\beta=0$, but goes toward $z=+\infty$ as $\beta \to -\infty$:



Overall, we see that the system is stable when $-1 < 2(1-\beta) < 1$

$$-\tfrac{1}{2} < 1-\beta < \tfrac{1}{2}$$

$$-\tfrac{3}{2} < -\beta < -\tfrac{1}{2}$$

$$\tfrac{3}{2} > \beta > \tfrac{1}{2}$$

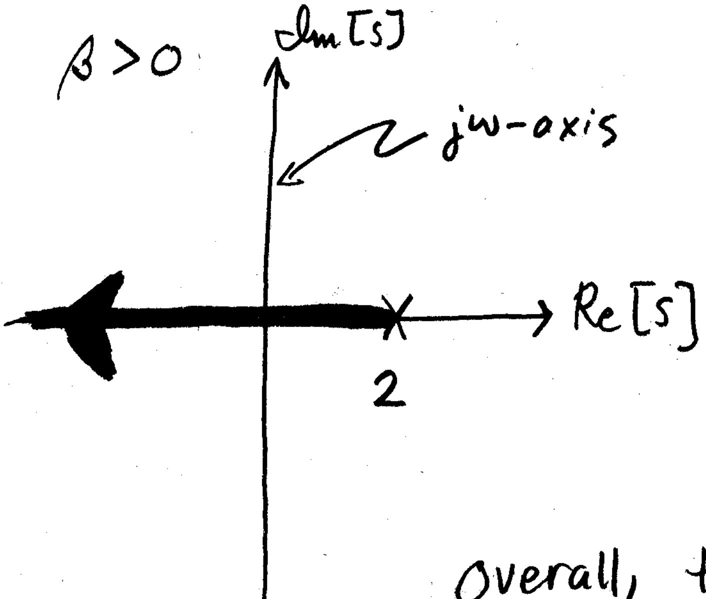
- The same analysis can be applied to a continuous-time system with

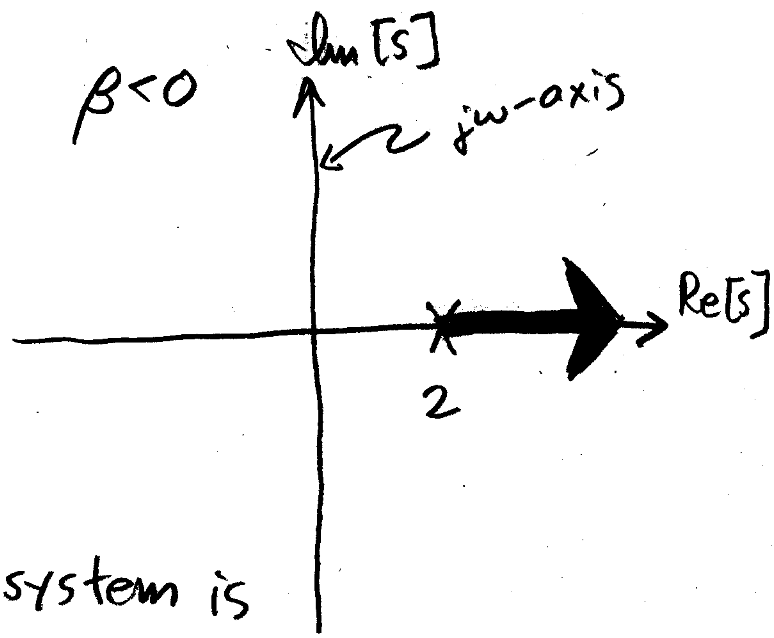
$$H(s) = \frac{s}{s-2}$$

$$G(s) = \frac{2\beta}{s}$$

$$Q(s) = \frac{s}{s-2(1-\beta)}$$

- The pole is at $s = 2(1-\beta)$
- For $\beta > 0$, the locus starts at $s=2$ when $\beta = 0$ and goes toward $s = -\infty$ as $\beta \to \infty$.
- For $\beta < 0$, the locus starts at $s=2$ when $\beta=0$ and goes toward $s = +\infty$ as $\beta \to -\infty$.





Overall, the system is stable when

$$\left.\begin{array}{c} 2(1-\beta) < 0 \\ 2 < 2\beta \end{array}\right\} \underline{\underline{\beta > 1}}$$

- For practical systems, it is generally not easy to solve for the closed loop poles.

- The root locus technique is a set of rules that can be used to sketch the root loci without solving for the poles explicitly.

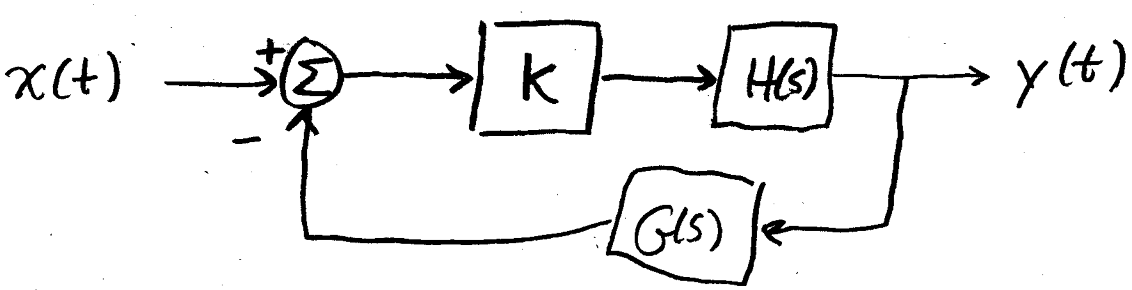
  → These "rules" are somewhat complicated, but not inherently difficult.

- We will talk about this with respect to $Q(s)$ and the loci of its poles in the s-plane.

- The analysis is the same for $H(z)$, but you compare to the unit circle of the z-plane instead of the $j\omega$-axis of the s-plane.

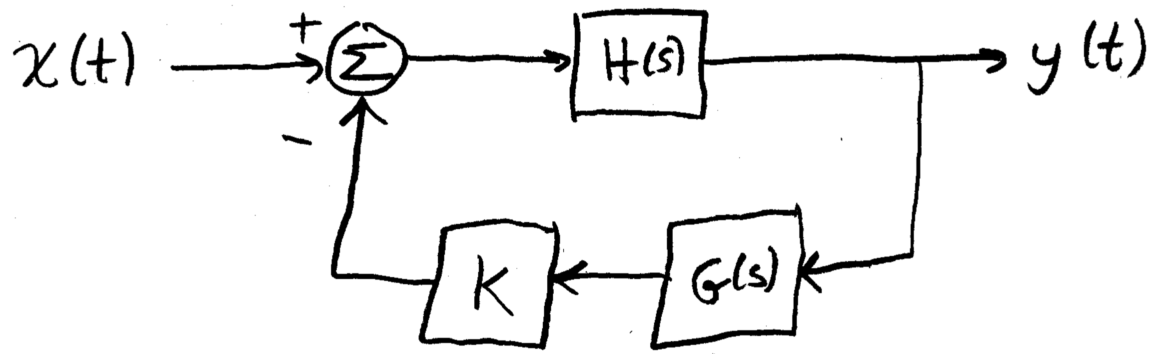
- The root locus analysis is done as a function of a variable gain parameter K.

→ K can be in the forward path:



$$Q(s) = \frac{KH(s)}{1 + KH(s)G(s)}$$

→ or in the reverse path:



$$Q(s) = \frac{H(s)}{1+KH(s)G(s)}$$

- The denominator of Q(s) is the same in both cases.
- The poles are the roots of $1+KH(s)G(s)$;

→ The poles are the solutions of

$$1 + KH(s)G(s) = 0 \qquad (11.44)$$

$$G(s)H(s) = -\frac{1}{K} \qquad (11.45)$$

⇒ Before we can look into plotting the solutions of (11.44), (11.45), we need to discuss a graphical technique for analyzing the pole-zero plot.