ECE 4213/5213
Test 1

Monday, October 25, 2010
4:30 PM - 5:45 PM

Fall 2010

Dr. Havlicek

Name: SOLUTION

Student Num:__________

Directions: This test is open book. You may also use a calculator, a clean copy of the course notes, and a clean copy of the formula sheet from the course web site. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

Students enrolled for undergraduate credit: work any four problems. Each problem counts 25 points. Below, Circle the numbers of the four problems you wish to have graded.

Students enrolled for graduate credit: work all five problems. Each problem counts 20 points.

SHOW ALL OF YOUR WORK for maximum partial credit! **GOOD LUCK!**

SCORE:

1. (25/20) ________
2. (25/20) ________
3. (25/20) ________
4. (25/20) ________
5. (25/20) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:__________ Date:__________

1. **25/20 pts.** The input $x[n]$ and output $y[n]$ of a causal discrete-time LTI system H are related by

$$y[n] - \frac{1}{2}y[n-1] = x[n].$$

The system input is given by

$$x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n].$$

(a) **10/8 pts.** Find the system impulse response $h[n]$.

Apply z-transform to both sides of the I/O relation:

$$Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z)$$

$$Y(z)\left[1 - \frac{1}{2}z^{-1}\right] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$$

because H is causal.

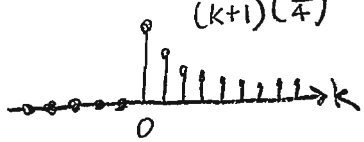
Table:

$$\boxed{h[n] = \left(\frac{1}{2}\right)^n u[n]}$$

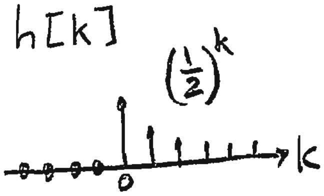
Problem 1, cont...

(b) **15/12 pts:** Use time domain convolution to find the system output $y[n]$.

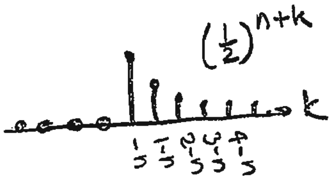
$x[k]$



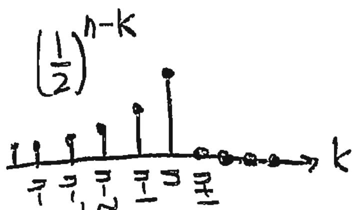
$h[k]$



$h[k - -n] = h[n+k]$

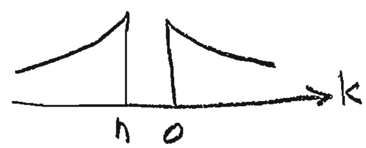


$h[n-k]$



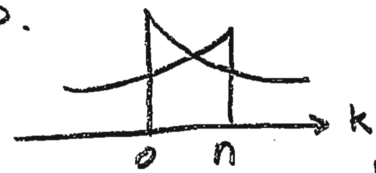
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

Case I) $n<0$:



$$y[n] = \sum_{k=0}^{\infty} 0 = 0.$$

Case II) $n \geq 0$



$$y[n] = \sum_{k=0}^{n} x[k]\,h[n-k] = \sum_{k=0}^{n} (k+1)\left(\tfrac{1}{4}\right)^k \left(\tfrac{1}{2}\right)^{n-k}$$

$$= \sum_{k=0}^{n} (k+1)\left(\tfrac{1}{4}\right)^k \left(\tfrac{1}{2}\right)^{n} \left(\tfrac{1}{2}\right)^{-k} = \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n} (k+1)\left(\tfrac{1}{4}\right)^k 2^k$$

$$= \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n} (k+1)\left(\tfrac{2}{4}\right)^k = \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n} (k+1)\left(\tfrac{1}{2}\right)^k$$

$$= \left(\tfrac{1}{2}\right)^n \left[\sum_{k=0}^{n} k\left(\tfrac{1}{2}\right)^k + \sum_{k=0}^{n}\left(\tfrac{1}{2}\right)^k\right]$$

$$= \left(\tfrac{1}{2}\right)^n \left[\frac{\frac{1}{2}\left\{1-(n+1)\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^{n+1}\right\}}{\left(1-\frac{1}{2}\right)^2} + \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right]$$

$$= \left(\tfrac{1}{2}\right)^n \left[\frac{\frac{1}{2}\left\{1-(n+1)\left(\frac{1}{2}\right)^n + \frac{1}{2}n\left(\frac{1}{2}\right)^{n}\right\}}{\left(1/2\right)^2} + \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1/2}\right]$$

$$= \left(\tfrac{1}{2}\right)^n \left[\tfrac{4}{2}\left\{1-(n+1)\left(\tfrac{1}{2}\right)^n + \tfrac{n}{2}\left(\tfrac{1}{2}\right)^n\right\} + 2 - 2\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^n\right]$$

$$= \left(\tfrac{1}{2}\right)^n \left[2 - (2n+2)\left(\tfrac{1}{2}\right)^n + n\left(\tfrac{1}{2}\right)^n + 2 - \left(\tfrac{1}{2}\right)^n\right]$$

$$= \left(\tfrac{1}{2}\right)^n \left[4 + (n-1-2n-2)\left(\tfrac{1}{2}\right)^n\right] = \left(\tfrac{1}{2}\right)^n \left[4 + (-n-3)\left(\tfrac{1}{2}\right)^n\right]$$

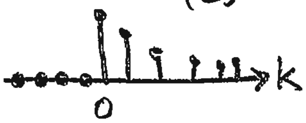
$$= \left(\tfrac{1}{2}\right)^n \left[4 - (n+3)\left(\tfrac{1}{2}\right)^n\right] = 4\left(\tfrac{1}{2}\right)^n - (n+3)\left(\tfrac{1}{2}\right)^n\left(\tfrac{1}{2}\right)^n = 4\left(\tfrac{1}{2}\right)^n - (n+3)\left(\tfrac{1}{2}\cdot\tfrac{1}{2}\right)^n$$

$$= 4\left(\tfrac{1}{2}\right)^n - (n+3)\left(\tfrac{1}{4}\right)^n.$$

All Together: $\boxed{y[n] = \left[4\left(\tfrac{1}{2}\right)^n - (n+3)\left(\tfrac{1}{4}\right)^n\right]u[n]}$

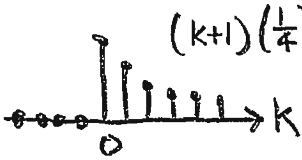
OTHER WAY:

$h[k]$ $\quad \left(\frac{1}{2}\right)^k$



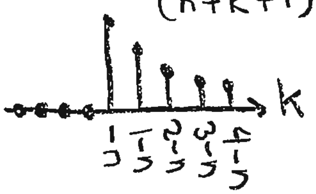
$x[k]$

$(k+1)\left(\frac{1}{4}\right)^k$



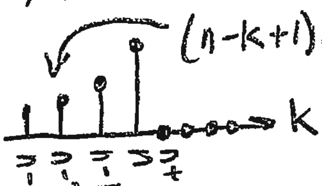
$x[k--n] = x[n+k]$

$(n+k+1)\left(\frac{1}{4}\right)^{n+k}$



$x[n-k]$

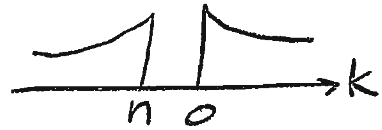
$(n-k+1)\left(\frac{1}{4}\right)^{n-k}$



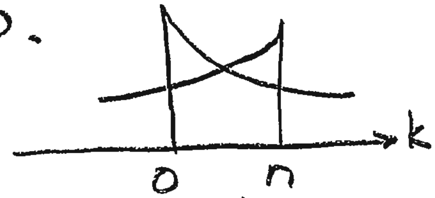
$\boxed{3\text{-}A}$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

case I) $n<0$:



$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0.$$



case II) $n \geq 0$

$$y[n] = \sum_{k=0}^{n} h[k]x[n-k] = \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^k (n-k+1)\left(\tfrac{1}{4}\right)^{n-k}$$

$$= \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^k (n-k+1)\left(\tfrac{1}{4}\right)^{n}\left(\tfrac{1}{4}\right)^{-k}$$

$$= \left(\tfrac{1}{4}\right)^n \sum_{k=0}^{n} (n-k+1)\left(\tfrac{1}{2}\right)^k 4^k$$

$$= \left(\tfrac{1}{4}\right)^n \left[\sum_{k=0}^{n} (n-k+1)\left(\tfrac{4}{2}\right)^k\right] = \left(\tfrac{1}{4}\right)^n \sum_{k=0}^{n} (n-k+1)\, 2^k$$

$$= \left(\tfrac{1}{4}\right)^n n \sum_{k=0}^{n} 2^k - \left(\tfrac{1}{4}\right)^n \sum_{k=0}^{n} k2^k + \left(\tfrac{1}{4}\right)^n \sum_{k=0}^{n} 2^k$$

$$= n\left(\tfrac{1}{4}\right)^n \frac{1-2^{n+1}}{1-2} - \left(\tfrac{1}{4}\right)^n \frac{2\{1-(n+1)2^n + n2^{n+1}\}}{(1-2)^2} + \left(\tfrac{1}{4}\right)^n \frac{1-2^{n+1}}{1-2}$$

$$= (n+1)\left(\tfrac{1}{4}\right)^n \frac{1-2^{n+1}}{-1} - \left(\tfrac{1}{4}\right)^n \frac{2\{1-(n+1)2^n + 2n2^n\}}{(-1)^2}$$

$$= -(n+1)\left(\tfrac{1}{4}\right)^n (1-2\cdot 2^n) - \left(\tfrac{1}{4}\right)^n \left[2-(2n+2)2^n + 4n2^n\right]$$

$$= -(n+1)\left[\left(\tfrac{1}{4}\right)^n - 2\left(\tfrac{2}{4}\right)^n\right] - 2\left(\tfrac{1}{4}\right)^n + (2n+2)\left(\tfrac{2}{4}\right)^n - 4n\left(\tfrac{2}{4}\right)^n$$

$$= (2n+2)\left(\tfrac{1}{2}\right)^n - (n+1)\left(\tfrac{1}{4}\right)^n - 2\left(\tfrac{1}{4}\right)^n + (2n+2)\left(\tfrac{1}{2}\right)^n - 4n\left(\tfrac{1}{2}\right)^n$$

$$= (2n+2+2n+2-4n)\left(\tfrac{1}{2}\right)^n - (n+1+2)\left(\tfrac{1}{4}\right)^n$$

$$= 4\left(\tfrac{1}{2}\right)^n - (n+3)\left(\tfrac{1}{4}\right)^n$$

All Together:

$$\boxed{y[n] = \left[4\left(\tfrac{1}{2}\right)^n - (n+3)\left(\tfrac{1}{4}\right)^n\right] u[n]}$$

2. **25/20 pts.** A causal discrete-time LTI system H has impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

The system input is given by

$$x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n].$$

Use the DTFT convolution property to find the system output $y[n]$.

$$H(e^{j\omega}) = \mathcal{F}\left\{\left(\tfrac{1}{2}\right)^n u[n]\right\} \stackrel{\text{Table}}{=} \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$$X(e^{j\omega}) = \mathcal{F}\left\{(n+1)\left(\tfrac{1}{4}\right)^n u[n]\right\} \stackrel{\text{Table}}{=} \frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}\,\frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

$$= \frac{A}{1-\frac{1}{2}e^{-j\omega}} + \frac{B}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2} + \frac{C}{\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

PFE:

$$\frac{1}{\left(1-\frac{1}{2}\theta\right)\left(1-\frac{1}{4}\theta\right)^2} = \frac{A}{1-\frac{1}{2}\theta} + \frac{B}{\left(1-\frac{1}{4}\theta\right)^2} + \frac{C}{1-\frac{1}{4}\theta}$$

$$A = \left.\frac{1}{\left(1-\frac{1}{4}\theta\right)^2}\right|_{\theta=2} = \frac{1}{\left(1-\frac{1}{2}\right)^2} = \frac{1}{(1/2)^2} = \frac{1}{1/4} = 4$$

$$B = \left.\frac{1}{1-\frac{1}{2}\theta}\right|_{\theta=4} = \frac{1}{1-\frac{4}{2}} = \frac{1}{1-2} = \frac{1}{-1} = -1$$

$$\left.\frac{d}{d\theta}\left(1-\tfrac{1}{2}\theta\right)^{-1}\right|_{\theta=4} = \left.\frac{d}{d\theta}C\left(1-\tfrac{1}{4}\theta\right)\right|_{\theta=4}$$

$$-\left.\left(1-\tfrac{1}{2}\theta\right)^{-2}\left(-\tfrac{1}{2}\right)\right|_{\theta=4} = -\tfrac{1}{4}C$$

$$-(1-2)^{-2}\left(-\tfrac{1}{2}\right) = -\tfrac{1}{4}C$$

$$\tfrac{1}{2} = -\tfrac{1}{4}C$$

$$C = -2$$

$$Y(e^{j\omega}) = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2} - \frac{2}{1-\frac{1}{4}e^{-j\omega}}$$

More Workspace for Problem 2...

Previous Page:

$$Y(e^{j\omega}) = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{(1-\frac{1}{4}e^{-j\omega})^2} - \frac{2}{1-\frac{1}{4}e^{-j\omega}}$$

Table:

$$y[n] = 4\left(\tfrac{1}{2}\right)^n u[n] - (n+1)\left(\tfrac{1}{4}\right)^n u[n] - 2\left(\tfrac{1}{4}\right)^n u[n]$$

$$= 4\left(\tfrac{1}{2}\right)^n u[n] - (n+1+2)\left(\tfrac{1}{4}\right)^n u[n]$$

$$= 4\left(\tfrac{1}{2}\right)^n u[n] - (n+3)\left(\tfrac{1}{4}\right)^n u[n]$$

$$= \left[4\left(\tfrac{1}{2}\right)^n - (n+3)\left(\tfrac{1}{4}\right)^n\right] u[n]$$

$$\boxed{y[n] = \left[4\left(\tfrac{1}{2}\right)^n - (n+3)\left(\tfrac{1}{4}\right)^n\right] u[n]}$$

3. **25/20 pts.** The input $x[n]$ and output $y[n]$ of a causal discrete-time LTI system H are related by

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n] + \frac{11}{3}x[n-1] - \frac{1}{2}x[n-2] - \frac{1}{6}x[n-3].$$

Find the system impulse response $h[n]$.

Z on both sides:

$$Y(z)\left[1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right] = X(z)\left[1 + \frac{11}{3}z^{-1} - \frac{1}{2}z^{-2} - \frac{1}{6}z^{-3}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{11}{3}z^{-1} - \frac{1}{2}z^{-2} - \frac{1}{6}z^{-3}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$

To find h[n], I must do a PFE. But this is an Improper Fraction. So I will use long division to make it proper, then use a PFE on the remainder.

$$H(z) = \frac{\text{numer}}{\text{denom}} = \text{denom}\,\overline{)\,\text{numer}}$$

$$\begin{array}{r} z^{-1} + 2 \\ -\frac{1}{6}z^{-2} - \frac{1}{6}z^{-1} + 1 \;\overline{)\, -\frac{1}{6}z^{-3} - \frac{1}{2}z^{-2} + \frac{11}{3}z^{-1} + 1} \\ -\frac{1}{6}z^{-3} - \frac{1}{6}z^{-2} + z^{-1} \\ \hline -\frac{1}{3}z^{-2} + \frac{8}{3}z^{-1} + 1 \\ -\frac{1}{3}z^{-2} - \frac{1}{3}z^{-1} + 2 \\ \hline 3z^{-1} - 1 \end{array}$$

remainder is now order in z^{-1}. 1st

$$H(z) = 2 + z^{-1} + \frac{-1 + 3z^{-1}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$

$\longrightarrow$

PFE on the fraction:

$$\frac{-1+3z^{-1}}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}} = \frac{-1+3z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}}$$

$$A = \left.\frac{-1+3\theta}{1+\frac{1}{3}\theta}\right|_{\theta=2} = \frac{-1+6}{1+2/3} = \frac{5}{5/3} = 3$$

$$B = \left.\frac{-1+3\theta}{1-\frac{1}{2}\theta}\right|_{\theta=-3} = \frac{-1-9}{1+3/2} = \frac{-10}{5/2} = -10\cdot\frac{2}{5} = -4$$

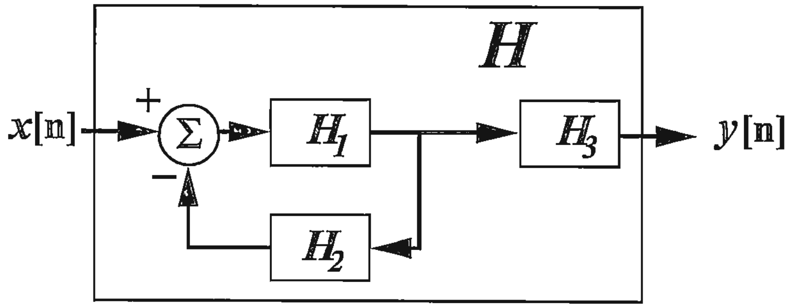
$$H(z) = \underbrace{2}_{\text{all } z} + \underbrace{z^{-1}}_{|z|>0} + \underbrace{\frac{3}{1-\frac{1}{2}z^{-1}}}_{|z|>\frac{1}{2}} - \underbrace{\frac{4}{1+\frac{1}{3}z^{-1}}}_{|z|>\frac{1}{3}}$$

ROCs follow from the fact that H is causal

Table:

$$\boxed{h[n] = 2\delta[n] + \delta[n-1] + 3\left(\tfrac{1}{2}\right)^n u[n] - 4\left(-\tfrac{1}{3}\right)^n u[n]}$$

4. **25/20 pts.** The **BIBO stable** discrete-time LTI system H is formed by interconnecting three discrete-time LTI systems H_1, H_2, and H_3 as shown in the figure below.



The transfer functions $H_1(z)$, $H_2(z)$, and $H_3(z)$ are given by

$$H(z) = \frac{H_1(z)H_3(z)}{1+H_1(z)H_2(z)} \quad (*)$$

$$\begin{aligned} H_1(z) &= \frac{1}{1-\frac{1}{\sqrt{2}}z^{-1}}, \\ H_2(z) &= \frac{1}{1+\frac{1}{\sqrt{2}}z^{-1}}, \\ H_3(z) &= \frac{1}{1-3z^{-1}}. \end{aligned}$$

Find the system impulse response $h[n]$. **Hint:** use the fact that the system is BIBO stable to determine the ROC of $H(z)$.

$$\frac{H_1(z)}{1+H_1(z)H_2(z)} = \frac{\frac{1}{1-\frac{1}{\sqrt{2}}z^{-1}}}{1+\frac{1}{(1-\frac{1}{\sqrt{2}}z^{-1})(1+\frac{1}{\sqrt{2}}z^{-1})}} \cdot \left\{\frac{(1-\frac{1}{\sqrt{2}}z^{-1})(1+\frac{1}{\sqrt{2}}z^{-1})}{(1-\frac{1}{\sqrt{2}}z^{-1})(1+\frac{1}{\sqrt{2}}z^{-1})}\right\} = 1$$

$$= \frac{1+\frac{1}{\sqrt{2}}z^{-1}}{(1-\frac{1}{\sqrt{2}}z^{-1})(1+\frac{1}{\sqrt{2}}z^{-1})+1} = \frac{1+\frac{1}{\sqrt{2}}z^{-1}}{1-\frac{1}{2}z^{-2}+1} = \frac{1+\frac{1}{\sqrt{2}}z^{-1}}{2-\frac{1}{2}z^{-2}} \cdot \frac{1/2}{1/2}$$

$$= \frac{\frac{1}{2}(1+\frac{1}{\sqrt{2}}z^{-1})}{1-\frac{1}{4}z^{-2}} = \frac{1/2\,(1+\frac{1}{\sqrt{2}}z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})}$$

plug this into $(*)$:

$$H(z) = \frac{1/2\,(1+\frac{1}{\sqrt{2}}z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})(1-3z^{-1})} = \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}} + \frac{C}{1-3z^{-1}}$$

→

More Workspace for Problem 4...

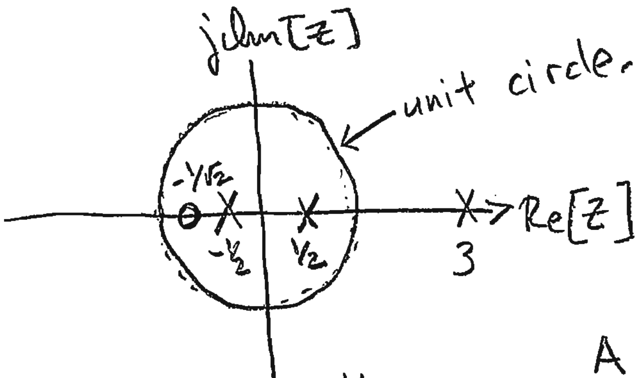
$$A = \left.\frac{\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\theta\right)}{\left(1-\frac{1}{2}\theta\right)\left(1-3\theta\right)}\right|_{\theta=-2} = \frac{\frac{1}{2}\left(1-\frac{2}{\sqrt{2}}\right)}{(1+1)(1+6)} = \frac{\frac{1}{2}\left(1-\sqrt{2}\right)}{2\cdot 7} = \frac{1}{28}\left(1-\sqrt{2}\right)$$

$$B = \left.\frac{\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\theta\right)}{\left(1+\frac{1}{2}\theta\right)\left(1-3\theta\right)}\right|_{\theta=2} = \frac{\frac{1}{2}\left(1+\frac{2}{\sqrt{2}}\right)}{(1+1)(1-6)} = \frac{\frac{1}{2}\left(1+\sqrt{2}\right)}{2(-5)} = -\frac{1}{20}\left(1+\sqrt{2}\right)$$

$$C = \left.\frac{\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\theta\right)}{\left(1+\frac{1}{2}\theta\right)\left(1-\frac{1}{2}\theta\right)}\right|_{\theta=\frac{1}{3}} = \frac{\frac{1}{2}\left(1+\frac{1}{3\sqrt{2}}\right)}{\left(1+\frac{1}{6}\right)\left(1-\frac{1}{6}\right)} = \frac{\frac{1}{2}\left(1+\frac{1}{3\sqrt{2}}\right)}{\frac{7}{6}\cdot\frac{5}{6}} = \frac{1}{2}\,\frac{36}{35}\left(1+\frac{1}{3\sqrt{2}}\right)$$

$$= \frac{18}{35}\left(1+\frac{1}{3\sqrt{2}}\right) = \frac{18}{35}\left(1+\frac{\sqrt{2}}{6}\right)$$

Now, the poles of $H(z)$ are at the roots of the denominator, or $z = \pm\frac{1}{2}, 3$. The zero is at $z = -\frac{1}{\sqrt{2}}$



$\Rightarrow$ Because H is BIBO stable, the ROC must include the unit circle.

ROC: $\frac{1}{2} < |z| < 3$

$$H(z) = \underbrace{\frac{A}{1+\frac{1}{2}z^{-1}}}_{|z|>1/2} + \underbrace{\frac{B}{1-\frac{1}{2}z^{-1}}}_{|z|>1/2} + \underbrace{\frac{C}{1-3z^{-1}}}_{|z|<3}$$

Table: $h[n] = A\left(-\frac{1}{2}\right)^n u[n] + B\left(\frac{1}{2}\right)^n u[n] - C\,3^n u[-n-1]$

$$\boxed{h[n] = \frac{1}{28}\left(1-\sqrt{2}\right)\left(-\frac{1}{2}\right)^n u[n] - \frac{1}{20}\left(1+\sqrt{2}\right)\left(\frac{1}{2}\right)^n u[n] - \frac{18}{35}\left(1+\frac{\sqrt{2}}{6}\right)3^n u[-n-1]}$$

5. **25/20 pts.** Consider a 4-point discrete-time signal $x_4[n]$ given by

$$x_4[n] = [4\ \ 3\ \ 2\ \ 1] = 4\delta[n] + 3\delta[n-1] + 2\delta[n-2] + \delta[n-3],\ 0 \le n \le 3.$$

The signal $x_4[n]$ is zero padded on the right to a length of $N = 6$ to obtain a new 6-point signal $x_6[n]$ given by

$$x_6[n] = [x_4[n]\ \ 0\ \ 0] = [4\ \ 3\ \ 2\ \ 1\ \ 0\ \ 0],\ 0 \le n \le 5.$$

Let $X_6[k] = \mathrm{DFT}_6\{x_6[n]\}$. Find the 6-point time sequence

$$y[n] = \mathrm{DFT}_6^{-1}\{W_6^{4k} X_6[k]\}.$$

The DFT time shift property on p. 5.43 of the notes says that $x[\langle n-n_0\rangle_N] \overset{DFT}{\longleftrightarrow} W_N^{kn_0} X[k]$. Here, we have $N=6$ and $n_0=4$.

So $y[n] = x_6[\langle n-4\rangle_6]$.

$y[0] = x_6[\langle -4\rangle_6] = x_6[2] = 2$

$y[1] = x_6[\langle -3\rangle_6] = x_6[3] = 1$

$y[2] = x_6[\langle -2\rangle_6] = x_6[4] = 0$

$y[3] = x_6[\langle -1\rangle_6] = x_6[5] = 0$

$y[4] = x_6[\langle 0\rangle_6] = x_6[0] = 4$

$y[5] = x_6[\langle 1\rangle_6] = x_6[1] = 3$

$$\boxed{\begin{aligned} y[n] &= [2\ \ 1\ \ 0\ \ 0\ \ 4\ \ 3] \\ &= 2\delta[n] + \delta[n-1] + 4\delta[n-4] + 3\delta[n-5], \\ &\qquad 0 \le n \le 5 \end{aligned}}$$