ECE 4213/5213
Test 1

Monday, November 5, 2012
4:30 PM - 5:45 PM

Fall 2012 Name: SOLUTION

Dr. Havlicek Student Num:__________

Directions: This test is open book. You may also use a calculator, a clean copy of the course notes, and clean copies of the formula sheet and z-transform quick start guide from the course web site. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

Students enrolled for undergraduate credit: work any four problems. Each problem counts 25 points. Below, Circle the numbers of the four problems you wish to have graded.

Students enrolled for graduate credit: work all five problems. Each problem counts 20 points.

SHOW ALL OF YOUR WORK for maximum partial credit! **GOOD LUCK!**

SCORE:

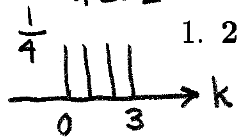
1. (25/20) ________
2. (25/20) ________
3. (25/20) ________
4. (25/20) ________
5. (25/20) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:__________ Date:__________

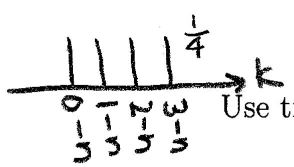
$h[k]$



1. **25/20 pts**. A discrete-time LTI system H has impulse response $h[n]$ given by

$$h[n] = \frac{1}{4}\left(u[n] - u[n-4]\right) = \begin{cases} \frac{1}{4} & 0 \le n \le 3, \\ 0 & \text{otherwise.} \end{cases}$$

$h[n--k] = h[n+k]$



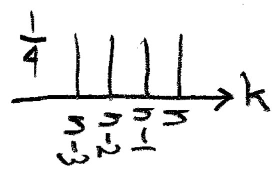
The system input is given by

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

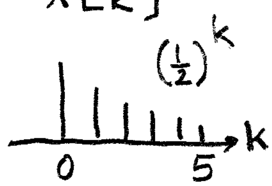
$$x[n] = \left(\frac{1}{2}\right)^n \left(u[n] - u[n-6]\right) = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \le n \le 5, \\ 0 & \text{otherwise.} \end{cases}$$

Use time domain convolution to find the system output $y[n]$.

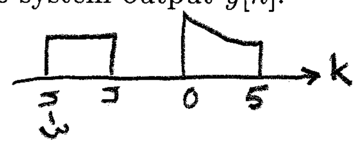
$h[n-k]$



$x[k]$



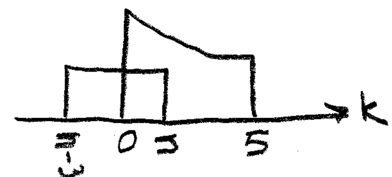
case I: $n<0$:



$x[k]h[n-k]=0$

$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$$

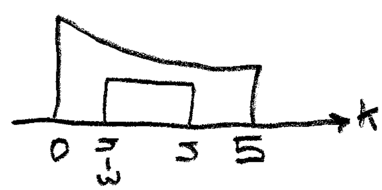
case II : $n \geq 0$ and $n-3<0$: $0 \leq n < 3$



$$y[n] = \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^k \tfrac{1}{4} = \tfrac{1}{4}\left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right] = 2\left(\tfrac{1}{4}\right)\left[1 - \tfrac{1}{2}\left(\tfrac{1}{2}\right)^n\right]$$

$$= \tfrac{1}{2} - \tfrac{1}{4}\left(\tfrac{1}{2}\right)^n = \tfrac{1}{2} - \left(\tfrac{1}{2}\right)^{n+2}$$

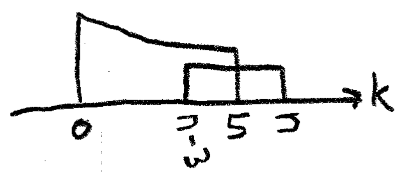
case III : $n-3 \geq 0$ and $n<5$: $3 \leq n < 5$



$$y[n] = \sum_{k=n-3}^{n} \left(\tfrac{1}{2}\right)^k \tfrac{1}{4} = \tfrac{1}{4}\left[\frac{\left(\frac{1}{2}\right)^{n-3} - \left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right]$$

$$= 2\left(\tfrac{1}{4}\right)\left[2^3\left(\tfrac{1}{2}\right)^n - \tfrac{1}{2}\left(\tfrac{1}{2}\right)^n\right] = \tfrac{16}{4}\left(\tfrac{1}{2}\right)^n - \tfrac{1}{4}\left(\tfrac{1}{2}\right)^n = \tfrac{15}{4}\left(\tfrac{1}{2}\right)^n = 15\left(\tfrac{1}{2}\right)^{n+2}$$

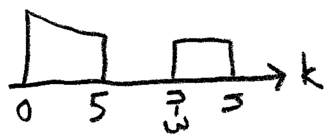
case IV : $n \geq 5$ and $n-3<6$: $5 \leq n < 9$



$$y[n] = \sum_{k=n-3}^{5} \left(\tfrac{1}{2}\right)^k \tfrac{1}{4} = \tfrac{1}{4}\left[\frac{\left(\frac{1}{2}\right)^{n-3} - \left(\frac{1}{2}\right)^{6}}{1-\frac{1}{2}}\right]$$

$$= 2\left(\tfrac{1}{4}\right)\left[2^3\left(\tfrac{1}{2}\right)^n - \left(\tfrac{1}{2}\right)^6\right] = \tfrac{16}{4}\left(\tfrac{1}{2}\right)^n - 2\left(\tfrac{1}{2}\right)^8 = 4\left(\tfrac{1}{2}\right)^n - \tfrac{1}{128}$$

case V : $n \geq 9$



$x[k]h[n-k]=0$

$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$$

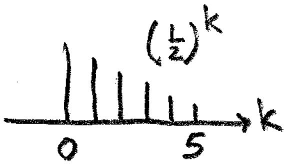
ALL TOGETHER:
$$y[n] = \begin{cases} 0, & n<0 \\ \frac{1}{2} - \left(\frac{1}{2}\right)^{n+2}, & 0 \leq n < 3 \\ 15\left(\frac{1}{2}\right)^{n+2}, & 3 \leq n < 5 \\ 4\left(\frac{1}{2}\right)^n - \frac{1}{128}, & 5 \leq n < 9 \\ 0, & n \geq 9 \end{cases}$$

"OTHER WAY"

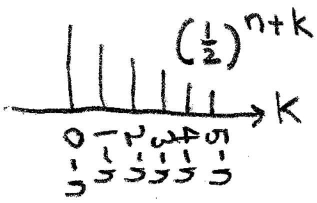
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

More Workspace for Problem 1...

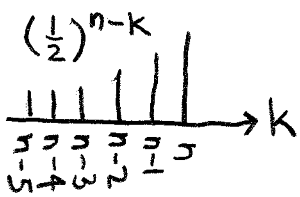
$x[k]$



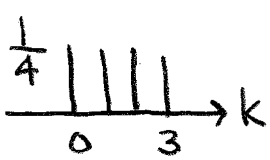
$x[k--n] = x[n+k]$



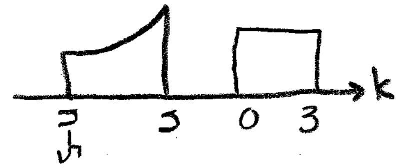
$x[n-k]$



$h[k]$



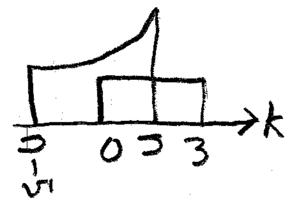
case I : $n<0$:



$h[k]x[n-k]=0$

$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$$

Case II : $n \geq 0$ and $n < 3$: $0 \leq n < 3$

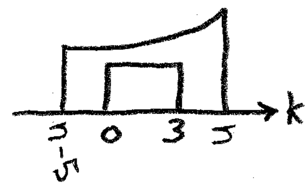


$$y[n] = \sum_{k=0}^{n} \frac{1}{4}\left(\frac{1}{2}\right)^{n-k} = \frac{1}{4}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k$$

$$= \frac{1}{4}\left(\frac{1}{2}\right)^n \left[\frac{1-2^{n+1}}{1-2}\right] = \frac{1}{4}\left(\frac{1}{2}\right)^n 2^{n+1} - \frac{1}{4}\left(\frac{1}{2}\right)^n$$

$$= \frac{1}{2} - \left(\frac{1}{2}\right)^{n+2}$$

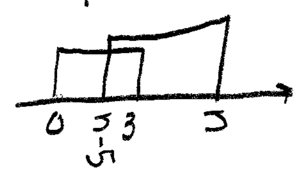
case III : $n \geq 3$ and $n-5<0$: $3 \leq n < 5$



$$y[n] = \sum_{k=0}^{3} \frac{1}{4}\left(\frac{1}{2}\right)^{n-k} = \frac{1}{4}\left(\frac{1}{2}\right)^n \sum_{k=0}^{3} 2^k$$

$$= \frac{1}{4}\left(\frac{1}{2}\right)^n \left[\frac{1-2^4}{1-2}\right] = \frac{1}{4}\left(\frac{1}{2}\right)^n [16-1] = \frac{15}{4}\left(\frac{1}{2}\right)^n = 15\left(\frac{1}{2}\right)^{n+2}$$

case IV : $n-5 \geq 0$ and $n-5<4$: $5 \leq n < 9$

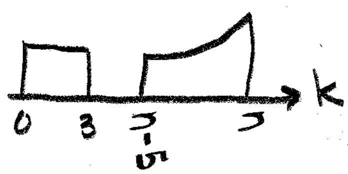


$$y[n] = \sum_{k=n-5}^{3} \frac{1}{4}\left(\frac{1}{2}\right)^{n-k} = \frac{1}{4}\left(\frac{1}{2}\right)^n \sum_{k=n-5}^{3} 2^k$$

$$= \frac{1}{4}\left(\frac{1}{2}\right)^n \left[\frac{2^{n-5}-2^4}{1-2}\right] = \frac{1}{4}\left(\frac{1}{2}\right)^n 2^4 - \frac{1}{4}\left(\frac{1}{2}\right)^n 2^n \left(\frac{1}{2}\right)^5$$

$$= \frac{16}{4}\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^7 = 4\left(\frac{1}{2}\right)^n - \frac{1}{128}$$

Case V : $n \geq 9$:



$h[k]x[n-k]=0$

$$y[n] = \sum_{k=-\infty}^{\infty} 0 = 0$$

ALL TOGETHER:

$$y[n] = \begin{cases} 0, & n<0 \\ \frac{1}{2} - \left(\frac{1}{2}\right)^{n+2}, & 0 \leq n < 3 \\ 15\left(\frac{1}{2}\right)^{n+2}, & 3 \leq n < 5 \\ 4\left(\frac{1}{2}\right)^n - \frac{1}{128}, & 5 \leq n < 9 \\ 0, & n \geq 9 \end{cases}$$

2. **25/20 pts**. The input $x[n]$ and output $y[n]$ of a discrete-time LTI system H are related by

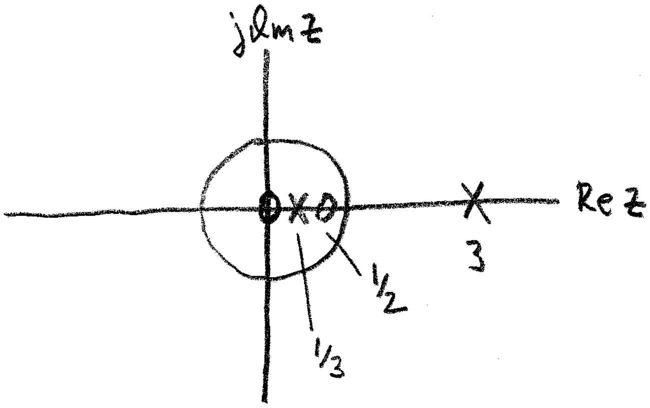
$$y[n] - \frac{10}{3}y[n-1] + y[n-2] = x[n] - \frac{1}{2}x[n-1].$$

(a) **5/4 pts**. Find the transfer function $H(z)$ and give a pole zero plot.

$$\mathcal{Z}: \; Y(z)\left[1 - \frac{10}{3}z^{-1} + z^{-2}\right] = X(z)\left[1 - \frac{1}{2}z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{10}{3}z^{-1}+z^{-2}} = \underline{\underline{\frac{1-\frac{1}{2}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-3z^{-1})}}}$$

one zero @ $z = \frac{1}{2}$; two poles @ $z = \frac{1}{3}$, $z = 3$



$$H(z)\frac{z^2}{z^2} = \frac{z(z-\frac{1}{2})}{(z-\frac{1}{3})(z-3)}$$

$\Rightarrow$ There is also another zero at $z = 0$.

$$H(\theta^{-1}) = \frac{1-\frac{1}{2}\theta}{(1-\frac{1}{3}\theta)(1-3\theta)} = \frac{A}{1-\frac{1}{3}\theta} + \frac{B}{1-3\theta}$$

$$A = \left.\frac{1-\frac{1}{2}\theta}{1-3\theta}\right|_{\theta=3} = \frac{1-\frac{3}{2}}{1-9} = \frac{-1/2}{-8} = \frac{1}{8}\cdot\frac{1}{2} = \frac{1}{16}$$

$$B = \left.\frac{1-\frac{1}{2}\theta}{1-\frac{1}{3}\theta}\right|_{\theta=\frac{1}{3}} = \frac{1-\frac{1}{6}}{1-\frac{1}{9}} = \frac{5/6}{8/9} = \frac{9}{8}\cdot\frac{5}{6} = \frac{3\cdot 5}{8\cdot 2} = \frac{15}{16}$$

$$H(z) = \frac{1/16}{1-\frac{1}{3}z^{-1}} + \frac{15/16}{1-3z^{-1}}$$

(b) **10/8 pts**. Assume that the system H is causal. Specify the ROC of $H(z)$ and find the impulse response $h[n]$. For this assumption, is the system H BIBO stable?

H causal $\rightarrow$ $h[n]$ right sided $\rightarrow$ ROC exterior to largest pole

$$\text{ROC}: |z| > 3$$

ROC does not include unit circle $\rightarrow$ H is UNSTABLE

$$H(z) = \underbrace{\frac{1/16}{1-\frac{1}{3}z^{-1}}}_{|z|>\frac{1}{3}} + \underbrace{\frac{15/16}{1-3z^{-1}}}_{|z|>3}, \quad |z|>3$$

Table: $h[n] = \frac{1}{16}\left(\frac{1}{3}\right)^n u[n] + \frac{15}{16} 3^n u[n]$

Problem 2, cont...

(c) **10/8 pts**. Now assume instead that the system H is BIBO stable. Specify the ROC of $H(z)$ and find the impulse response $h[n]$. For this assumption, is the system H causal?

H is BIBO stable $\rightarrow$ ROC includes unit circle

$\rightarrow$ ROC: $\frac{1}{3} < |z| < 3$

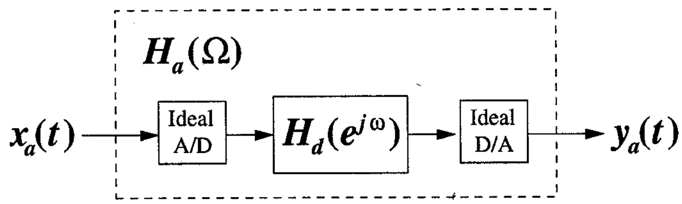
ROC is annulus $\rightarrow$ $h[n]$ is two sided

$\rightarrow$ NOT CAUSAL

$$H(z) = \underbrace{\frac{1/16}{1-\frac{1}{3}z^{-1}}}_{|z|>\frac{1}{3}} + \underbrace{\frac{15/16}{1-3z^{-1}}}_{|z|<3}, \quad \frac{1}{3} < |z| < 3$$

Table: $h[n] = \frac{1}{16}\left(\frac{1}{3}\right)^n u[n] - \frac{15}{16} 3^n u[-n-1]$

3. **25/20 pts.** The continuous-time LTI system $H_a(\Omega)$ shown in the figure below is implemented using an ideal A/D converter, a discrete-time LTI system $H_d(e^{j\omega})$, and an ideal D/A converter.



The sampling frequency of the A/D and D/A converters is $F_T = 2$ kHz.

The analog input signal is given by $x_a(t) = \sin(1000\pi t)$. $\quad \Omega_T = 2\pi F_T = 4000\pi \ \frac{\text{rad}}{\text{sec}}$

The input/output relation of the digital filter H_d is given by $\quad T = \frac{1}{F_T} = \frac{1}{2000}$ sec

$$y_d[n] - \frac{1}{3}y_d[n-1] = \frac{2}{3}x_d[n] - 2x_d[n-1].$$

Freq. of input $= 1000\pi$

Find the analog output signal $y_a(t)$.

$\frac{\Omega_T}{2} = 2000\pi > 1000\pi$

$\Rightarrow$ No aliasing ✓

DTFT: $Y_d(e^{j\omega})\left[1 - \frac{1}{3}e^{-j\omega}\right] = X(e^{j\omega})\left[\frac{2}{3} - 2e^{-j\omega}\right]$

$$H_d(e^{j\omega}) = \frac{Y_d(e^{j\omega})}{X_d(e^{j\omega})} = \frac{2\left(\frac{1}{3} - e^{-j\omega}\right)}{1 - \frac{1}{3}e^{-j\omega}}$$

$$H_a(\Omega) = H_d(e^{j\Omega T}) = \frac{2\left(\frac{1}{3} - e^{-j\Omega T}\right)}{1 - \frac{1}{3}e^{-j\Omega T}} = \frac{2\left(\frac{1}{3} - e^{-j\Omega/2000}\right)}{1 - \frac{1}{3}e^{-j\Omega/2000}}, \quad |\Omega| < 2000\pi$$

$$e^{-j\Omega/2000} = \cos\frac{\Omega}{2000} - j\sin\frac{\Omega}{2000}$$

$\rightarrow$ $H_a(\Omega)$ is conjugate symmetric

$\rightarrow$ $h_a(t)$ is <u>real</u>

$\rightarrow$ $\cos\Omega t$ and $\sin\Omega t$ are eigenfunctions

$\rightarrow$ The input $x_a(t)$ 7 is an eigenfunction

$\longrightarrow$

More Workspace for Problem 3...

$$y_a(t) = x_a(t)\, H_a(1000\pi)$$

$$H_a(1000\pi) = \frac{2\left(\frac{1}{3} - e^{-j\pi/2}\right)}{1 - \frac{1}{3}e^{-j\pi/2}} = 2\,\frac{\frac{1}{3}+j}{1+\frac{1}{3}j} \cdot \frac{1-\frac{1}{3}j}{1-\frac{1}{3}j}$$

$$= 2\,\frac{\frac{1}{3} - \frac{1}{9}j + j + \frac{1}{3}}{1 - \frac{1}{3}j + \frac{1}{3}j + \frac{1}{9}} = 2\,\frac{\frac{2}{3} + j\,\frac{8}{9}}{10/9} = 2\,\frac{9}{10}\left(\frac{2}{3} + j\,\frac{8}{9}\right)$$

$$= \frac{36}{30} + j\,\frac{16}{10} = \frac{6}{5} + j\,\frac{8}{5}$$

$$|H_a(1000\pi)| = \sqrt{\frac{36}{25} + \frac{64}{25}} = \sqrt{\frac{100}{25}} = \sqrt{4} = 2$$

$$\arg H_a(1000\pi) = \arctan\frac{8/5}{6/5} = \arctan\frac{8}{6} = \arctan\frac{4}{3}$$

$$y_a(t) = H_a(1000\pi)\sin(1000\pi t)$$

$$= |H_a(1000\pi t)|\sin(1000\pi t + \arg H_a(1000\pi))$$

$$= 2\sin(1000\pi t + \arctan 4/3)$$

$$\underline{\underline{y_a(t) = 2\sin(1000\pi t + \arctan 4/3)}}$$

4. **25/20 pts.** A discrete-time LTI system H has impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

$$Y(z) = X(z)H(z)$$

The system input is given by

$$x[n] = \cos\left(\frac{\pi}{3}n\right)u[n].$$ Table: $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$

Use the z-transform to find the system output $y[n]$.

Table: $X(z) = \dfrac{1-\cos(\frac{\pi}{3})z^{-1}}{1-2\cos(\frac{\pi}{3})z^{-1}+z^{-2}} = \dfrac{1-\frac{1}{2}z^{-1}}{1-z^{-1}+z^{-2}}, \quad |z|>1$

$$Y(z) = X(z)H(z) = \frac{1}{1-z^{-1}+z^{-2}}, \quad |z|>1$$

Table: $\sin(\omega_0 n)u[n] \overset{Z}{\longleftrightarrow} \dfrac{\sin(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}, \quad |z|>1$

Time Shift: $\sin[\omega_0(n+1)]u[n+1] \overset{Z}{\longleftrightarrow} \dfrac{\sin\omega_0}{1-2\cos(\omega_0)z^{-1}+z^{-2}}, \quad |z|>1$

$$\left.\begin{array}{l} \omega_0 = \frac{\pi}{3} \\ \cos\omega_0 = \frac{1}{2} \\ \sin\omega_0 = \sqrt{3}/2 \end{array}\right\} \quad \sin\left[\frac{\pi}{3}(n+1)\right]u[n+1] \overset{Z}{\longleftrightarrow} \frac{\sqrt{3}/2}{1-z^{-1}+z^{-2}}, \quad |z|>1$$

$$y[n] = Z^{-1}\left\{\frac{1}{1-z^{-1}+z^{-2}}\right\} = \underline{\underline{\frac{2}{\sqrt{3}}\sin\left[\frac{\pi}{3}(n+1)\right]u[n+1]}}$$

ALTERNATE INVERSION BY PFE: $Y(z) = \frac{1}{1-z^{-1}+z^{-2}}$, $|z|>1$

More Workspace for Problem 4... Quadratic Formula: $a=1$, $b=-1$, $c=1$

$$\text{poles} = \text{roots} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm \frac{\sqrt{-3}}{2} = \frac{1}{2} \pm \sqrt{\frac{-3}{4}}$$

$$= \frac{1}{2} \pm j\sqrt{\frac{3}{4}} = re^{j\theta}$$

$$r = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1; \quad \theta = \arctan \frac{\sqrt{3}/2}{1/2} = \arctan\sqrt{3} = \pi/3$$

$$\text{poles} = \text{roots} = e^{\pm j\pi/3}$$

$$Y(z) = \frac{1}{(1-e^{j\pi/3}z^{-1})(1-e^{-j\pi/3}z^{-1})} = \frac{A}{1-e^{j\pi/3}z^{-1}} + \frac{A^*}{1-e^{-j\pi/3}z^{-1}}$$

$$A = \frac{1}{1-e^{-j\pi/3}z^{-1}}\bigg|_{z=e^{j\pi/3}} = \frac{1}{1-e^{-j\pi/3}e^{-j\pi/3}} = \frac{1}{1-e^{-j2\pi/3}}$$

$$= \frac{1}{1-[\cos(-\frac{2\pi}{3})+j\sin(-\frac{2\pi}{3})]} = \frac{1}{1-\cos\frac{2\pi}{3}+j\sin\frac{2\pi}{3}} = \frac{1}{1+\frac{1}{2}+j\frac{1}{2}\sqrt{3}}$$

$$= \frac{1}{\frac{3}{2}+j\sqrt{3/4}} \cdot \frac{3/2-j\sqrt{3/4}}{3/2-j\sqrt{3/4}} = \frac{3/2-j\sqrt{3/4}}{9/4+3/4} = \frac{3/2-j\sqrt{3/4}}{12/4} = \frac{3/2-j\sqrt{3/4}}{3}$$

$$= \frac{1}{2}-j\sqrt{3/36} = \frac{1}{2}-j\sqrt{1/12}.$$

$$|A| = \sqrt{\frac{1}{4}+\frac{1}{12}} = \sqrt{4/12} = \sqrt{1/3} = \frac{1}{\sqrt{3}}. \quad \text{Let } \alpha = |A| = \frac{1}{\sqrt{3}}$$

$$\arg A = \arctan\left(\frac{-\sqrt{1/12}}{1/2}\right) = \arctan\left(-2\sqrt{1/12}\right) = \arctan\left(-\sqrt{4/12}\right)$$

$$= \arctan\left(-1/\sqrt{3}\right) = \arctan\left(-\frac{1}{3}\sqrt{3}\right) = \frac{11\pi}{6}. \quad \text{Let } \varphi = \arg A = \frac{11\pi}{6}$$

$$\Rightarrow A = \frac{1}{2}-j\sqrt{1/12} = \alpha e^{j\varphi}. \quad A^* = \frac{1}{2}+j\sqrt{1/12} = \alpha e^{-j\varphi}.$$

$$Y(z) = \frac{A}{1-e^{j\pi/3}z^{-1}} + \frac{A^*}{1-e^{-j\pi/3}z^{-1}} = \frac{\alpha e^{j\varphi}}{1-e^{j\pi/3}z^{-1}} + \frac{\alpha e^{-j\varphi}}{1-e^{-j\pi/3}z^{-1}}, \quad |z|>1$$

Table: $y[n] = \alpha e^{j\varphi}e^{j\frac{\pi}{3}n}u[n] + \alpha e^{-j\varphi}e^{-j\frac{\pi}{3}n}u[n]$

$$= \alpha\left[e^{j(\pi/3\, n+\varphi)} + e^{-j(\pi/3\, n+\varphi)}\right]u[n] = 2\alpha\cos\left(\frac{\pi}{3}n+\varphi\right)u[n]$$

$$= \frac{2}{\sqrt{3}}\cos\left(\frac{\pi}{3}n+\frac{11\pi}{6}\right)u[n] = \frac{2}{\sqrt{3}}\cos\left(\frac{\pi}{3}n-\frac{\pi}{6}+\underbrace{\frac{12\pi}{6}}_{2\pi}\right)u[n]$$

$$\underline{y[n] = \frac{2}{\sqrt{3}}\cos\left(\frac{\pi}{3}n-\frac{\pi}{6}\right)u[n]}$$

To see that this is the same as the previous answer, we use two facts:

FACT 1: $\sin\theta = \cos(\theta - \frac{\pi}{2})$

FACT 2: $\sin(0) = 0$

Now, from the last page we have

$$y[n] = \frac{2}{\sqrt{3}} \cos\left(\frac{\pi}{3}n - \frac{\pi}{6}\right) u[n]$$

$$= \frac{2}{\sqrt{3}} \cos\left(\frac{\pi}{3}n + \frac{2\pi}{6} - \frac{3\pi}{6}\right) u[n] = \frac{2}{\sqrt{3}} \cos\left(\frac{\pi}{3}n + \frac{\pi}{3} - \frac{\pi}{2}\right) u[n]$$

$$\overset{\text{FACT 1}}{=} \frac{2}{\sqrt{3}} \sin\left(\frac{\pi}{3}n + \frac{\pi}{3}\right) u[n] = \frac{2}{\sqrt{3}} \sin\left[\frac{\pi}{3}(n+1)\right] u[n]$$

$$\overset{\text{FACT 2}}{=} \frac{2}{\sqrt{3}} \sin\left[\frac{\pi}{3}(n+1)\right] u[n+1]$$

Because $u[n+1]$ is "turned on" at $n=-1$, but at $n=-1$ we have $\sin(\frac{\pi}{3}(1-1)) = 0$.

which is the same as the previous answer ✓

5. **25/20 pts.** The LTI FIR digital filter H has a length-3 impulse response given by

$$h[n] = [-1\ 3\ -2] = -\delta[n] + 3\delta[n-1] - 2\delta[n-2],\ 0 \le n \le 2.$$

The input is a length-4 digital signal $x[n]$ given by

$$x[n] = [1\ 2\ 3\ -1] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] - \delta[n-3],\ 0 \le n \le 3.$$

Use the DFT to find the filter output (linear convolution) $y[n]$.

N_1 = length of $h[n] = 3$. N_2 = length of $x[n] = 4$.

N = length of $y[n] = N_1 + N_2 - 1 = 3 + 4 - 1 = 6$.

$h'[n] = [-1\ 3\ -2\ 0\ 0\ 0]$; $x'[n] = [1\ 2\ 3\ -1\ 0\ 0]$

$$H'[k] = \sum_{n=0}^{5} h'[n] W_6^{nk} = -1 + 3W_6^k - 2W_6^{2k}$$

$$X'[k] = \sum_{n=0}^{5} x'[n] W_6^{nk} = 1 + 2W_6^k + 3W_6^{2k} - W_6^{3k}$$

$$\begin{aligned} Y[k] &= -X'[k] + 3W_6^k X'[k] - 2W_6^{2k} X'[k] \\ &= -1 - 2W_6^k - 3W_6^{2k} + W_6^{3k} \\ &\quad + 3W_6^k + 6W_6^{2k} + 9W_6^{3k} - 3W_6^{4k} \\ &\quad - 2W_6^{2k} - 4W_6^{3k} - 6W_6^{4k} + 2W_6^{5k} \end{aligned}$$

$$\begin{aligned} Y[k] &= -1 + W_6^k + W_6^{2k} + 6W_6^{3k} - 9W_6^{4k} + 2W_6^{5k} \\ &= \sum_{n=0}^{5} y[n] W_6^{nk} = y[0] + y[1]W_6^k + y[2]W_6^{2k} + \cdots + y[5]W_6^{5k} \end{aligned}$$

$\longrightarrow$ read off the $y[n]$'s from above:

$$\underline{\underline{y[n] = [-1\ 1\ 1\ 6\ -9\ 2]}}$$