ECE 4213/5213
Test 2

Thursday, December 3, 2009
12:00 PM - 1:15 PM

Fall 2009 Name: SOLUTION

Dr. Havlicek Student Num:__________

Directions: This test is open book. You may also use a calculator, a clean copy of the course notes, and a clean copy of the formula sheet from the course web site. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

Students enrolled for undergraduate credit: work any four problems. Each problem counts 25 points.

Students enrolled for graduate credit: work all five problems. Each problem counts 20 points.

SHOW ALL OF YOUR WORK for maximum partial credit! **GOOD LUCK!**

SCORE:

1. (25/20) ________

2. (25/20) ________

3. (25/20) ________

4. (25/20) ________

5. (25/20) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:__________ Date:__________

1. **25/20 pts.** An LTI digital filter H has transfer function

$$H(z) = \frac{(1-5z^{-1})(1-3z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}, \quad |z| > \tfrac{1}{2}.$$

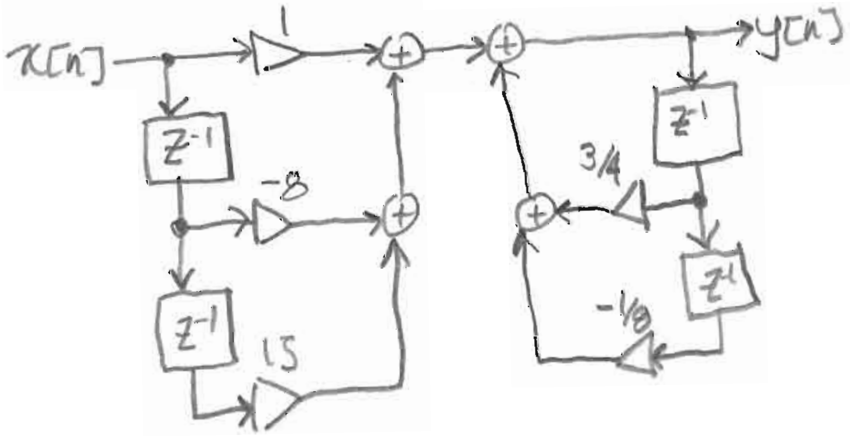
Give Direct Form I and Direct Form II realizations of $H(z)$.

$$H(z) = \frac{1 - 8z^{-1} + 15z^{-2}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

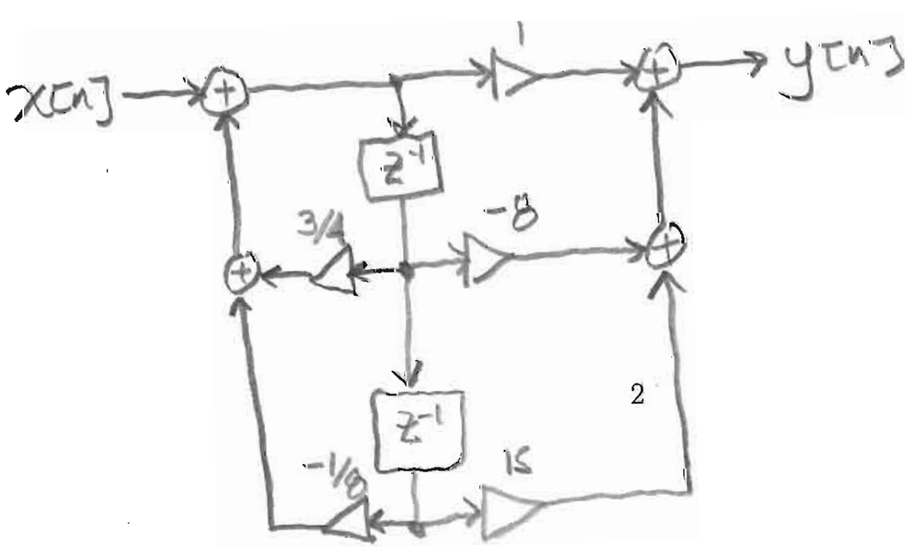
TEXT (8.23): $p_0 = 1$ $\quad p_1 = -8$ $\quad p_2 = 15$

$d_1 = -3/4$ $\quad d_2 = 1/8$

DIRECT FORM I
[Fig 8.13(a)]



DIRECT FORM II [Fig 8.14(a)]



2. **25/20 pts**. An IIR digital filter H has input $x[n]$ and output $y[n]$ related by the difference equation

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] - 8x[n-1] + 15x[n-2].$$

The system H is LTI, causal, and BIBO stable. But it is **not** minimum phase.

Find a new LTI digital system G such that:

(a) $G(z)$ has minimum phase.

(b) G is both causal and BIBO stable.

(c) $|G(e^{j\omega})| = |H(e^{j\omega})|\ \forall\ \omega \in \mathbb{R}$.

$$Y(z)\left[1-\tfrac{3}{4}z^{-1}+\tfrac{1}{8}z^{-2}\right] = X(z)\left[1-8z^{-1}+15z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-8z^{-1}+15z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{(1-3z^{-1})(1-5z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}, \quad |z| > \frac{1}{2}$$

The two zeros at $z=3$ and $z=5$ are both outside the unit circle.
$\Rightarrow$ Reflect them inside by factoring out a pair of allpass sections.

$$H(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}(1-3z^{-1})\frac{3-z^{-1}}{3-z^{-1}}(1-5z^{-1})\frac{5-z^{-1}}{5-z^{-1}}$$

$$= \frac{(3-z^{-1})(5-z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}\;\frac{1-3z^{-1}}{3-z^{-1}}\;\frac{1-5z^{-1}}{5-z^{-1}}$$

$$= \underbrace{\frac{15(1-\frac{1}{3}z^{-1})(1-\frac{1}{5}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}}_{G(z)}\;\underbrace{\frac{1-3z^{-1}}{3-z^{-1}}}_{\text{all pass}}\;\underbrace{\frac{1-5z^{-1}}{5-z^{-1}}}_{\text{all pass}}$$

$$\boxed{G(z) = \frac{15(1-\frac{1}{3}z^{-1})(1-\frac{1}{5}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}}$$

3. **25/20 pts**. An FIR digital filter H has impulse response

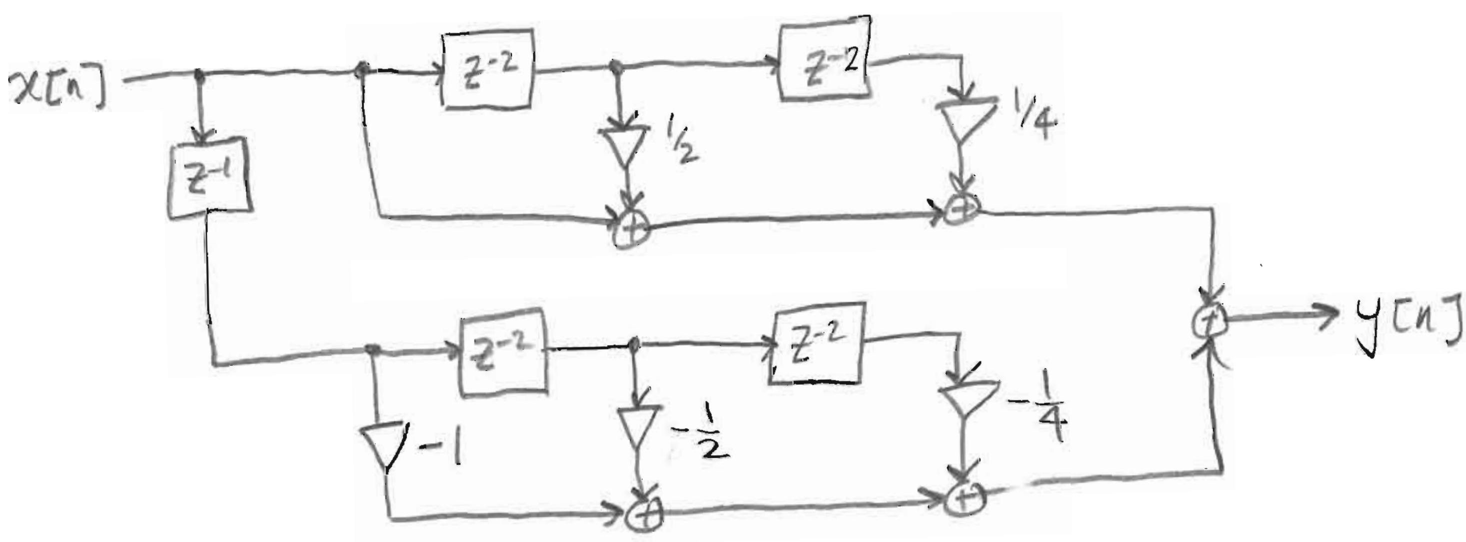
$$h[n] = \delta[n] - \delta[n-1] + \frac{1}{2}\delta[n-2] - \frac{1}{2}\delta[n-3] + \frac{1}{4}\delta[n-4] - \frac{1}{4}\delta[n-5].$$

Show the block diagram for a polyphase realization of $H(z)$ using $L = 2$ branches (e.g., two polyphases).

$$H(z) = 1 - z^{-1} + \frac{1}{2}z^{-2} - \frac{1}{2}z^{-3} + \frac{1}{4}z^{-4} - \frac{1}{4}z^{-5}$$

$$= \underbrace{\left(1 + \frac{1}{2}z^{-2} + \frac{1}{4}z^{-4}\right)}_{E_0(z^2)} - z^{-1}\underbrace{\left(1 + \frac{1}{2}z^{-2} + \frac{1}{4}z^{-4}\right)}_{-E_1(z^2)} \quad \text{From (8.18).}$$

Realize as in Fig. 8.7(c) with the basic FIR structure of Fig. 8.5(a):



4. **25/20 pts.** Design an **analog** Type I Chebyshev low pass filter to meet the following **analog** specification:

passband edge freq.	$\Omega_p = \pi$ rad/sec
stopband edge freq.	$\Omega_s = 2\pi$ rad/sec
max. stopband ripple	$1/A = 1/\pi$
passband equiripple	$1/\sqrt{1+\varepsilon^2} = 0.9$

$$\frac{1}{\sqrt{1+\varepsilon^2}} = \frac{9}{10}$$

$$\sqrt{1+\varepsilon^2} = 10/9$$

$$1+\varepsilon^2 = 100/81$$

$$\varepsilon^2 = \frac{100-81}{81} = \frac{19}{81}$$

$$\varepsilon = 0.48432 \ \boxed{00}$$

calculator register

Give the analog filter transfer function $H_a(s)$.

Hint: The design formulas for the analog Type I Chebyshev filter are given on pages 191 and 192 of the text.

(4.43): $$N = \left\lceil \frac{\cosh^{-1}(\sqrt{A^2-1}/\varepsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)} \right\rceil = \left\lceil \frac{\overset{\boxed{01}}{2.50279}}{\underset{\boxed{02}}{1.31696}} \right\rceil = \lceil 1.90043 \rceil = 2$$

(4.45b): $$\gamma = \left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{1/2} = 2.08780 \ \boxed{03} \qquad \zeta = \frac{\gamma^2+1}{2\gamma} = 1.28339 \ \boxed{04}$$

$$\xi = \frac{\gamma^2-1}{2\gamma} = 0.804412 \ \boxed{05}$$

(4.45a): $$\sigma_1 = -\pi\xi \sin\frac{\pi}{4} = -1.78695 \ \boxed{06}$$

$$\Omega_1 = \pi\zeta\cos\frac{\pi}{4} = 2.85097 \ \boxed{07}$$

$$\sigma_2 = -\pi\xi\sin\frac{3\pi}{4} = -1.78695 \ \boxed{06} = \sigma_1$$

$$\Omega_2 = \pi\zeta\cos\frac{3\pi}{4} = -2.85097 \ \boxed{08} = -\Omega_1$$

(4.44): $$p_1 = \sigma_1 + j\Omega_1 = -1.78695 + j2.85097$$

$$p_2 = \sigma_2 + j\Omega_2 = -1.78695 - j2.85097$$

The sentence above (4.44) Tells you that $H_a(s)$ is given by (4.36):

$$H_a(s) = \frac{C}{(s-p_1)(s-p_2)} = \frac{C}{s^2 - (p_1+p_2)s + p_1p_2}$$

$$= \frac{C}{s^2 - 2\sigma_1 s + \sigma_1^2 + \Omega_1^2} \quad (*) \longrightarrow$$

More Workspace for Problem 4...

$$H_a(\Omega) = H_a(s)\Big|_{s=j\Omega} = \frac{C}{-\Omega^2 - 2\sigma_1 j\Omega + \sigma_1^2 + \Omega_1^2}$$

To solve C, note that the peak of $H_a(\Omega)$ occurs at $\Omega = 0$ where $H_a(0) = 1$:

$$1 = H_a(0) = \frac{C}{\sigma_1^2 + \Omega_1^2} \Rightarrow C = \sigma_1^2 + \Omega_1^2 = 11.3212 \quad \boxed{p9}$$

From (*) on p. 5,

$$H_a(s) = \frac{\sigma_1^2 + \Omega_1^2}{s^2 - 2\sigma_1 s + \sigma_1^2 + \Omega_1^2} = \frac{11.3212}{s^2 + 3.57391s + 11.3212}$$

$$\boxed{H_a(s) = \frac{11.3212}{s^2 + 3.57391s + 11.3212}}$$

$$= \frac{11.3212}{[s-(-1.78695+j2.85097)][s-(-1.78695-j2.85097)]}$$

5. **25/20 pts.** Use the bilinear transform with $T = 2$ and **Steps 1-5** of the first approach given at the top of page 501 of the text to design a highpass digital Butterworth filter meeting the following specification:

passband edge freq.	$\hat{\omega}_p = 0.75\pi$ rad/sample
stopband edge freq.	$\hat{\omega}_s = 0.25\pi$ rad/sample
max. passband ripple	$\varepsilon = 0.75$
max. stopband ripple	$A = 4$

Let $H_D(e^{j\hat{\omega}})$ and $H_D(z)$ be the frequency response and transfer function of the desired digital highpass filter.

Let $\hat{\omega}_p$ and $\hat{\omega}_s$ be the critical frequencies of the desired digital highpass filter given in the table above.

Let $\hat{\Omega}_p$ and $\hat{\Omega}_s$ be the critical frequencies of the equivalent analog highpass filter. Call the transfer function of this filter $H_D(\hat{s})$.

Let Ω_p and Ω_s be the critical frequencies of the prototype analog lowpass filter. Call the transfer function of this filter $H_a(s)$.

(a) **3/3 pts. Step 1:** use (9.18) with $T = 2$ to prewarp the digital critical frequencies $\hat{\omega}_p$ and $\hat{\omega}_s$ and obtain the analog critical frequencies $\hat{\Omega}_p$ and $\hat{\Omega}_s$ of the equivalent analog highpass filter.

$$\hat{\Omega}_p = \tan\left(\frac{\hat{\omega}_p}{2}\right) = \tan\left(\frac{3\pi}{8}\right) = 2.41421 \text{ rad/sec} \quad \boxed{00}$$

$$\hat{\Omega}_s = \tan\left(\frac{\hat{\omega}_s}{2}\right) = \tan\left(\frac{\pi}{8}\right) = 0.414214 \text{ rad/sec} \quad \boxed{01}$$

(b) **4/3 pts. Step 2:** obtain the critical frequencies of the prototype analog lowpass filter by letting $\Omega_p = 1$ and using the transformation (4.63) to solve for Ω_s. **Hint:** (4.63) gives you the stopband edge frequency that is located on the negative frequency axis. Since the magnitude is even, you can multiply this by -1 to get the positive stopband edge frequency.

$\Omega_p \equiv 1.$

$$\Omega_s = \frac{-\Omega_p \hat{\Omega}_p}{\hat{\Omega}_s} = \frac{-\hat{\Omega}_p}{\hat{\Omega}_s} = -\frac{2.41421}{0.414214} = -5.82843 \text{ rad/sec}$$

$\Rightarrow$ use the positive edge frequency

$$\Omega_s = +5.82843 \text{ rad/sec} \quad \boxed{02}$$

Problem 5 cont....

(c) 10/7 pts. **Step 3**: design the prototype analog lowpass Butterworth filter $H_a(s)$ using the design formulas on pages 189 and 190 of the text. As suggested on page 190, use (4.34b) to solve for Ω_C after you obtain the order using (4.35).

$$(4.35) \quad N = \left\lceil \frac{1}{2} \frac{\log\left[(A^2-1)/\varepsilon^2\right]}{\log\left(\Omega_s/\Omega_p\right)} \right\rceil = \left\lceil \frac{1}{2} \frac{\log\left[15 \cdot 16/9\right]}{\log\left[5.82843\right]} \right\rceil$$

$$= \left\lceil \frac{1}{2} \frac{\overset{\boxed{03}}{1.42597}}{\underset{\boxed{04}}{0.765552}} \right\rceil = \left\lceil 0.931334 \right\rceil = \underline{\underline{1}}$$

$$(4.34b) \quad \frac{1}{A^2} = \frac{1}{16} = \frac{1}{1 + \Omega_s^2/\Omega_c^2} \Rightarrow 16 = 1 + \Omega_s^2/\Omega_c^2$$

$$15 = \Omega_s^2/\Omega_c^2$$

$$\Omega_c = \frac{\Omega_s}{\sqrt{15}} = \frac{5.82843}{\sqrt{15}} = 1.50489 \quad \boxed{05}$$

$$(4.37) \quad P_1 = \Omega_c e^{j[\pi(1+2-1)/2]} = \Omega_c e^{j\pi} = -\Omega_c = -1.50489$$

$$(4.36) \quad H_a(s) = \frac{\Omega_c}{s - p_1} = \frac{\Omega_c}{s + \Omega_c} = \underline{\underline{\frac{1.50489}{s + 1.50489}}}$$

Problem 5 cont....

(d) **4/3 pts. Step 4**: use (4.62) to convert the prototype lowpass filter $H_a(s)$ into the desired analog highpass filter $H_D(\hat{s})$.

$$H_D(\hat{s}) = H_a(s)\Big|_{s=\frac{\Omega_p\hat{\Omega}_p}{\hat{s}} = \hat{\Omega}_p/\hat{s}} = \frac{\Omega_c}{\hat{\Omega}_p/\hat{s} + \Omega_c} \cdot \frac{\hat{s}}{\hat{s}}$$

$$= \frac{\Omega_c\,\hat{s}}{\hat{\Omega}_p + \Omega_c\hat{s}} \quad \frac{1/\Omega_c}{1/\Omega_c} = \frac{\hat{s}}{\hat{s} + \hat{\Omega}_p/\Omega_c}$$

$$H_D(\hat{s}) = \frac{\hat{s}}{\hat{s} + 1.60424}$$

(e) **4/4 pts. Step 5**: use the bilinear transform (9.14) with $T = 2$ to obtain the transfer function $H_D(z)$ of the desired highpass digital filter.

$$H_D(z) = H_D(\hat{s})\Big|_{\hat{s}=\frac{1-z^{-1}}{1+z^{-1}}} = \frac{\frac{1-z^{-1}}{1+z^{-1}}}{\frac{1-z^{-1}}{1+z^{-1}} + 1.60424} \cdot \frac{1+z^{-1}}{1+z^{-1}}$$

$$= \frac{1-z^{-1}}{1-z^{-1} + 1.60424 + 1.60424z^{-1}} = \frac{1-z^{-1}}{2.60424 + 0.60424z^{-1}}$$

$$= \frac{0.383989(1-z^{-1})}{1 + 0.232022z^{-1}}$$