

ECE 4213/5213

Test 2

Wednesday, December 2, 2015
4:30 PM - 5:45 PM

Fall 2015

Name: SOLUTION

Dr. Havlicek

Student Num: _____

Directions: This test is open book. You may also use a calculator, a clean copy of the course notes, and a clean copy of the formula sheet from the course web site. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

Students enrolled for undergraduate credit: work problem 5 and select any three out of problems 1-4. Each problem counts 25 points. Below, circle the numbers of the three problems you wish to have graded out of problems 1-4.

Students enrolled for graduate credit: work all five problems. Each problem counts 20 points.

SHOW ALL OF YOUR WORK for maximum partial credit! **GOOD LUCK!**

SCORE:

1. (25/20) _____

2. (25/20) _____

3. (25/20) _____

4. (25/20) _____

5. (25/20) _____

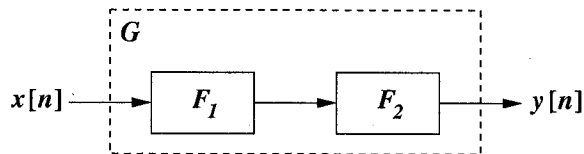
TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: _____

Date: _____

1. 25/20 pts. The causal discrete-time LTI system G shown below



is a series connection of two causal discrete-time LTI systems F_1 and F_2 . The input-output relation for system F_1 is given by

$$F_1: x[n] - 2x[n-1] + \frac{3}{4}x[n-2] = y[n] - \frac{1}{9}y[n-2].$$

The input-output relation for system F_2 is given by

$$F_2: x[n] + \frac{1}{2}x[n-1] = y[n] - \frac{4}{3}y[n-1].$$

(a) 8/7 pts. Find the transfer function $G(z)$. Be sure to specify the ROC.

$$F_1: X(z) [1 - 2z^{-1} + \frac{3}{4}z^{-2}] = Y(z) [1 - \frac{1}{9}z^{-2}]$$

$$F_1(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1} + \frac{3}{4}z^{-2}}{1 - \frac{1}{9}z^{-2}}$$

$$= \frac{(1 - \frac{3}{2}z^{-1})(1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})}$$

$$F_2: X(z) [1 + \frac{1}{2}z^{-1}] = Y(z) [1 - \frac{4}{3}z^{-1}]$$

$$F_2(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2}z^{-1}}{1 - \frac{4}{3}z^{-1}}$$

$$G(z) = F_1(z)F_2(z) = \frac{(1 - \frac{3}{2}z^{-1})(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})(1 - \frac{4}{3}z^{-1})}$$

poles: $z = \frac{1}{3}, -\frac{1}{3}, \frac{4}{3}$ zeros: $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}$

Causal \mapsto ROC: $|z| > \frac{4}{3}$

Problem 1, cont...

- (b) 9/7 pts. The system G does **not** have minimum phase. Find a new discrete-time LTI system H with the same spectral magnitude as G such that H is **causal**, **stable**, and **minimum phase** and $|H(e^{j\omega})| = |G(e^{j\omega})| \forall \omega$.

Give the transfer function $H(z)$. Be sure to specify the ROC.

$G(z)$ is not minimum phase because the zero at $z = \frac{3}{2}$ is outside the unit circle.

$G(z)$ is not stable because the pole at $z = \frac{4}{3}$ is outside the unit circle.

$$G(z) = \frac{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})} (1 - \frac{3}{2}z^{-1}) \frac{z^{-1} - \frac{3}{2}}{z^{-1} - \frac{3}{2}} \frac{1}{(1 - \frac{4}{3}z^{-1})} \frac{z^{-1} - \frac{4}{3}}{z^{-1} - \frac{4}{3}}$$

$$= \underbrace{\frac{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})(z^{-1} - \frac{3}{2})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})(z^{-1} - \frac{4}{3})}}_{H(z)} \cdot \underbrace{\frac{1 - \frac{3}{2}z^{-1}}{z^{-1} - \frac{3}{2}}}_{\text{all pass}} \cdot \underbrace{\frac{z^{-1} - \frac{4}{3}}{1 - \frac{4}{3}z^{-1}}}_{\text{all pass}}$$

$$H(z) = \frac{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})(-\frac{3}{2})(1 - \frac{2}{3}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})(-\frac{4}{3})(1 - \frac{3}{4}z^{-1})}$$

$$= \frac{(\frac{3}{4})(\frac{3}{2})(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})(1 - \frac{2}{3}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})(1 - \frac{3}{4}z^{-1})} = \frac{9/8 (1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})(1 - \frac{2}{3}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})(1 - \frac{3}{4}z^{-1})}$$

causal \mapsto ROC: $|z| > 3/4$

All zeros inside unit circle: minimum phase ✓
 All poles inside unit circle: stable ✓

Problem 1, cont...

(c) 8/6 pts. Give a Direct Form I realization of $H(z)$.

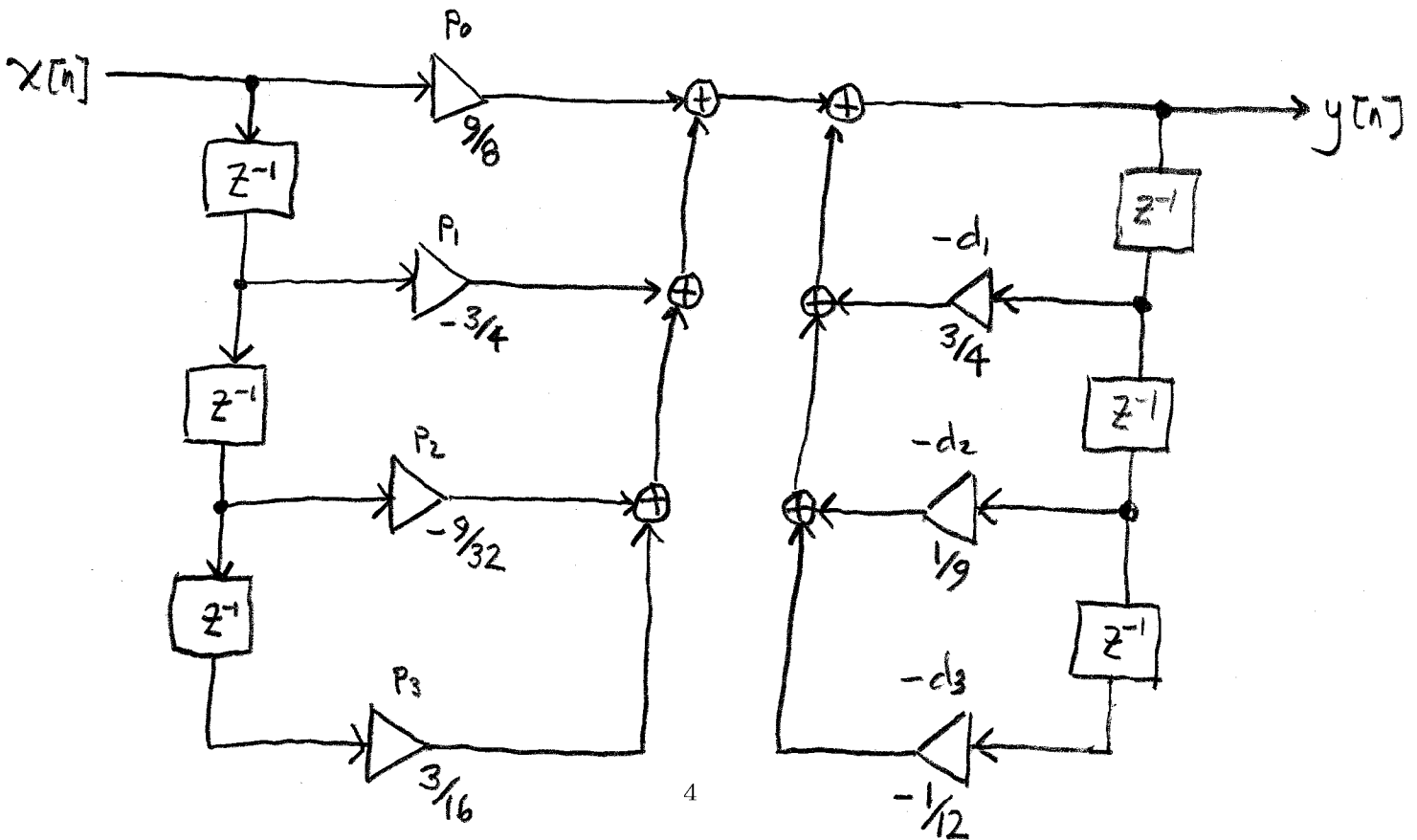
$$H(z) = \frac{\frac{9}{8} (1 - \frac{1}{4} z^{-2}) (1 - \frac{2}{3} z^{-1})}{(1 - \frac{1}{9} z^{-2}) (1 - \frac{3}{4} z^{-1})} = \frac{\frac{9}{8} (1 - \frac{2}{3} z^{-1} - \frac{1}{4} z^{-2} + \frac{1}{6} z^{-3})}{1 - \frac{3}{4} z^{-1} - \frac{1}{9} z^{-2} + \frac{1}{12} z^{-3}}$$

$$= \frac{\frac{9}{8} - \frac{3}{4} z^{-1} - \frac{9}{32} z^{-2} + \frac{3}{16} z^{-3}}{1 - \frac{3}{4} z^{-1} - \frac{1}{9} z^{-2} + \frac{1}{12} z^{-3}}$$

$$p_0 = \frac{9}{8} \quad p_1 = -\frac{3}{4} \quad p_2 = -\frac{9}{32} \quad p_3 = \frac{3}{16}$$

$$d_1 = -\frac{3}{4} \quad d_2 = -\frac{1}{9} \quad d_3 = \frac{1}{12}$$

NOTES p. 8-21:



2. 25/20 pts. H is a causal, stable 4th-order linear phase FIR filter ($N = 4$). The transfer function $H(z)$ has a zero at $z = 1 + j$. The DC gain of the filter is given by

$$H(e^{j0}) = H(1) = 0.5.$$

(a) 9/7 pts. Find the transfer function $H(z)$ and give a pole-zero plot.

We are given that there is a zero at $z_1 = 1 + j$. Because H is a real coefficients linear phase FIR filter with order $N=4$, there must be three more zeros at

$$z_2 = z_1^* = 1 - j$$

$$z_3 = 1/z_1 = \frac{1}{1+j} = \frac{1-j}{1-j} = \frac{1-j}{2} = \frac{1}{2} - j\frac{1}{2}; \quad z_4 = z_3^* = \frac{1}{2} + j\frac{1}{2}$$

So, for some constant C_0 ,

$$\begin{aligned} H(z) &= C_0 (1 - z_1 z^{-1})(1 - z_2 z^{-1})(1 - z_3 z^{-1})(1 - z_4 z^{-1}) \\ &= C_0 [1 - (1+j)z^{-1}][1 - (1-j)z^{-1}][1 - (\frac{1}{2} - j\frac{1}{2})z^{-1}][1 - (\frac{1}{2} + j\frac{1}{2})z^{-1}] \\ &= C_0 [1 - (1-j)z^{-1} - (1+j)z^{-1} + 2z^{-2}][1 - (\frac{1}{2} - j\frac{1}{2})z^{-1} - (\frac{1}{2} + j\frac{1}{2})z^{-1} + \frac{1}{2}z^{-2}] \\ &= C_0 [1 - 2z^{-1} + 2z^{-2}][1 - z^{-1} + \frac{1}{2}z^{-2}] \\ &= C_0 [1 - z^{-1} + \frac{1}{2}z^{-2} - 2z^{-1} + 2z^{-2} - z^{-3} + 2z^{-2} - 2z^{-3} + z^{-4}] \\ &= C_0 [1 - 3z^{-1} + \frac{9}{2}z^{-2} - 3z^{-3} + z^{-4}] \end{aligned}$$

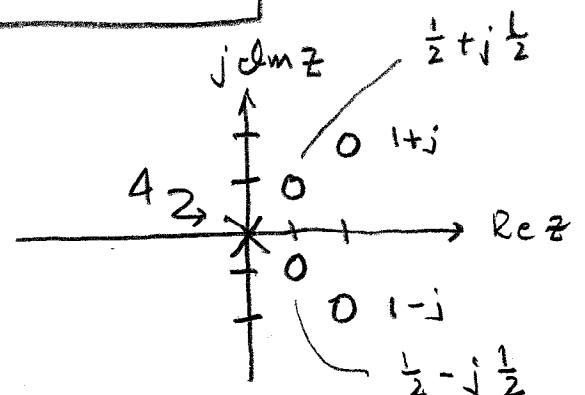
$$\text{@ } z=1, \quad H(1) = C_0 [1 - 3 + \frac{9}{2} - 3 + 1] = \frac{1}{2} C_0 = \frac{1}{2} \Rightarrow C_0 = 1$$

$$H(z) = 1 - 3z^{-1} + \frac{9}{2}z^{-2} - 3z^{-3} + z^{-4}, \quad |z| > 0$$

Note: $H(z) = \frac{1 - 3z + \frac{9}{2}z^2 - 3z^3 + z^4}{z^4}$

(multiplied by z^4/z^4)

\Rightarrow 4th order pole at $z=0$.



Problem 2, cont...

- (b) 8/6 pts. Find the impulse response $h[n]$. Is H a Type 1, Type 2, Type 3, or Type 4 linear phase FIR filter?

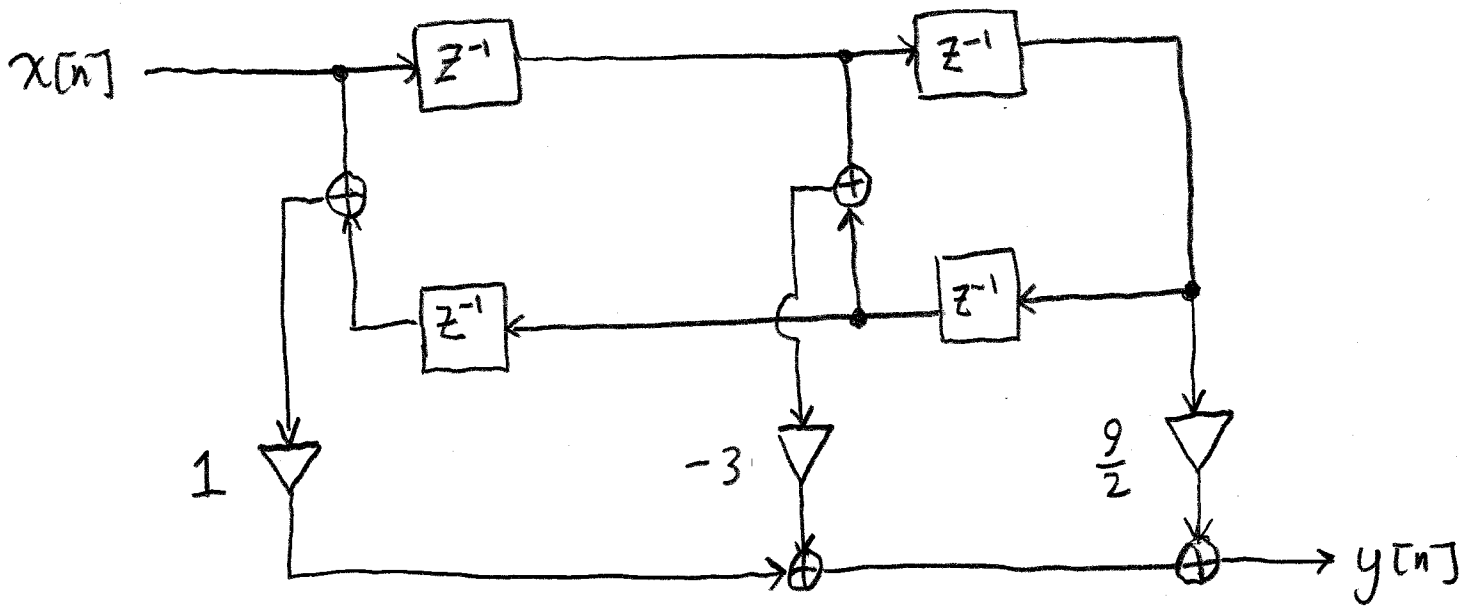
Directly from $H(z)$, we have

$$h[n] = \delta[n] - 3\delta[n-1] + \frac{9}{2}\delta[n-2] - 3\delta[n-3] + \delta[n-4]$$

$$= [1 \quad -3 \quad \frac{9}{2} \quad -3 \quad 1]$$

$h[0] = h[4] = 1$ $h[1] = h[3] = -3$ $h[2] = -\frac{9}{2}$	}	$N=4$ length=5 Even symmetry	}	Type - 1 FIR
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- (c) 8/7 pts. Give a linear-phase FIR structure (block diagram) for the filter H using the minimum number of multipliers.



3. 25/20 pts. Use the window design method with an appropriate fixed window from Table 10.2 (p. 540 of the text) to design a causal lowpass FIR digital filter that meets the following specifications:

Passband Edge Freq.	$\omega_p = 0.2\pi$ rad/sample
Stopband Edge Freq.	$\omega_s = 0.9\pi$ rad/sample
Max. Passband Ripple	$\delta_p = 0.005$
Max. Stopband Ripple	$\delta_s = 0.005$

Give the filter impulse response $h[n]$.

$$\alpha_s = -20 \log_{10} \delta_s = -20 \log_{10} 0.005 = 46.0206 \text{ dB}$$

Table 10.2: Hamming and Blackman can meet the spec.

$$\Delta\omega = \omega_s - \omega_p = 0.9\pi - 0.2\pi = 0.7\pi$$

$$\text{Hamming: } \Delta\omega = \frac{3.32\pi}{M} : M = \left\lceil \frac{3.32\pi}{\Delta\omega} \right\rceil = \left\lceil \frac{3.32\pi}{0.7\pi} \right\rceil = \lceil 4.74 \rceil = 5$$

$$\text{Blackman: } \Delta\omega = \frac{5.56\pi}{M} : M = \left\lceil \frac{5.56\pi}{\Delta\omega} \right\rceil = \left\lceil \frac{5.56\pi}{0.7\pi} \right\rceil = \lceil 7.94 \rceil = 8$$

\Rightarrow Hamming meets the spec with a lower order \rightarrow use Hamming.

$$M=5. \text{ Order} = N = 2M = 10. \text{ Length} = 2M+1 = 11.$$

$$\omega_c = \frac{\omega_p + \omega_s}{2} = \frac{0.2\pi + 0.9\pi}{2} = \frac{1.1\pi}{2} = 0.55\pi$$

$$h_{LP}[n] = \frac{\sin \omega_c n}{\pi n} = \frac{\sin 0.55\pi n}{\pi n}$$

$$(10.34) : W[n] = 0.54 + 0.46 \cos\left(\frac{\pi}{5}n\right) = 0.54 + 0.46 \cos\left(\frac{\pi}{5}n\right)$$

$$W[n]h_{LP}[n] = \left\{ 0.54 + 0.46 \cos\left(\frac{\pi}{5}n\right) \right\} \frac{\sin(0.55\pi n)}{\pi n}, \quad -5 \leq n \leq 5$$

Shift to make causal:

$$h[n] = \frac{\sin[0.55\pi(n-5)]}{\pi(n-5)} \left\{ 0.54 + 0.46 \cos\left[\frac{\pi}{5}(n-5)\right] \right\} \quad 0 \leq n \leq 10$$

4. 25/20 pts. Design an analog Type 2 Chebyshev low pass filter to meet the following analog specification:

passband edge freq.	$\Omega_p = \pi$ rad/sec
stopband edge freq.	$\Omega_s = 2\pi$ rad/sec
stopband equiripple	$1/A = 1/\pi$
max. passband attenuation	$1/\sqrt{1 + \epsilon^2} = 0.9$

$$A = \pi$$

$$\frac{1}{(0.9)^2} = 1 + \epsilon^2$$

$$\epsilon^2 = \frac{1}{0.81} - 1$$

$$= 19/81$$

Give the analog filter transfer function $H_a(s)$.

$$\epsilon = \sqrt{\frac{19}{81}} = 0.484322$$

Hint: The design formulas for the analog Type 2 Chebyshev filter are given on pages A-8 and A-9 of the notes file ECE5213NotesAnalogFilterDesign.pdf and in Appendix A.3 of the text on page 869.

$$(A.17): N = \left\lceil \frac{\cosh^{-1}(\sqrt{A^2 - 1}/\epsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)} \right\rceil = \left\lceil \frac{\cosh^{-1} 6.14919}{\cosh^{-1} 2} \right\rceil = \left\lceil \frac{2.50279}{1.31696} \right\rceil$$

$$= \lceil 1.90043 \rceil = 2$$

$$(A.22): z_1 = \frac{j 2\pi}{\cos\left[\frac{(2-1)\pi}{4}\right]} = \frac{j 2\pi}{\cos \pi/4} = j \frac{2\pi}{0.707107} = j 8.88577$$

$$z_2 = \frac{j 2\pi}{\cos 3\pi/4} = j \frac{2\pi}{-0.707107} = -j 8.88577$$

$$(A.24c): \gamma = [A + \sqrt{A^2 - 1}]^{1/2} = \sqrt{\pi + \sqrt{\pi^2 - 1}} = 2.47382$$

$$\xi = \frac{\gamma^2 + 1}{2\gamma} = 1.43903; \quad \zeta = \frac{\gamma^2 - 1}{2\gamma} = 1.03479$$

(A.24b):

$$\alpha_1 = -\Omega_p \zeta \sin \pi/4 = -2.29873$$

$$\alpha_2 = -\Omega_p \zeta \sin 3\pi/4 = -2.29873$$

$$\beta_1 = \Omega_p \xi \cos \pi/4 = 3.19671$$

$$\beta_2 = \Omega_p \xi \cos 3\pi/4 = -3.19671$$

(A.24a):

$$\sigma_1 = \frac{\Omega_s \alpha_1}{\alpha_1^2 + \beta_1^2} = -0.931641$$

$$\Omega_1 = \frac{-\Omega_s \beta_1}{\alpha_1^2 + \beta_1^2} = -1.29558$$

$$\sigma_2 = \frac{\Omega_s \alpha_2}{\alpha_2^2 + \beta_2^2} = -0.931641$$

$$\Omega_2 = \frac{-\Omega_s \beta_2}{\alpha_2^2 + \beta_2^2} = 1.29558$$

More Workspace for Problem 4...

$$(A.23): p_1 = \sigma_1 + j\omega_1 = -0.931641 - j1.29558$$

$$p_2 = \sigma_2 + j\omega_2 = -0.931641 + j1.29558$$

$$(A.21): H_a(s) = C_0 \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

$$H_a(0) = C_0 \frac{(-z_1)(-z_2)}{(-p_1)(-p_2)} = C_0 \frac{z_1 z_2}{p_1 p_2} = 1$$

$$\Rightarrow C_0 = \frac{p_1 p_2}{z_1 z_2} = \frac{p_1 p_1^*}{z_1 z_1^*} = \frac{|p_1|^2}{|z_1|^2} = 0.0322515$$

$$\begin{aligned} H_a(s) &= \frac{p_1 p_2}{z_1 z_2} \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)} = \frac{p_1 p_1^*}{z_1 z_1^*} \frac{s^2 - (z_1 + z_2)s + z_1 z_2}{s^2 - (p_1 + p_2)s + p_1 p_2} \\ &= \frac{|p_1|^2}{|z_1|^2} \frac{s^2 - (z_1 + z_1^*)s + z_1 z_1^*}{s^2 - (p_1 + p_1^*)s + p_1 p_1^*} = \frac{|p_1|^2}{|z_1|^2} \frac{s^2 - 2\operatorname{Re}\{z_1\}s + |z_1|^2}{s^2 - 2\operatorname{Re}\{p_1\}s + |p_1|^2} \\ &= \frac{\frac{|p_1|^2}{|z_1|^2} s^2 - 2 \frac{|p_1|^2}{|z_1|^2} \operatorname{Re}\{z_1\} s + |p_1|^2}{s^2 - 2\operatorname{Re}\{p_1\}s + |p_1|^2} = \frac{\frac{|p_1|^2}{|z_1|^2} s^2 + |p_1|^2}{s^2 - 2\operatorname{Re}\{p_1\}s + |p_1|^2} \end{aligned}$$

$$H_a(s) = \frac{0.0322515 s^2 + 2.5465}{s^2 + 1.8633 s + 2.5465}, \operatorname{Re}\{s\} > -0.931641$$

5. 25/20 pts. Use the bilinear transform with $T = 2$ to design a lowpass digital Butterworth filter that meets the following specifications:

$$\frac{1}{A} = 0.2 = \frac{2}{10}$$

$$A = \frac{10}{2} = 5$$

Passband Edge Freq.	$\omega_p = \pi/8$ rad/sample
Stopband Edge Freq.	$\omega_s = 2\pi/5$ rad/sample
Max. Passband Ripple	$1/\sqrt{1+\epsilon^2} = 0.9$
Min. Stopband Atten.	$1/A = 0.2$

$$\frac{1}{\sqrt{1+\epsilon^2}} = \frac{9}{10}$$

$$\sqrt{1+\epsilon^2} = 10/9$$

$$1+\epsilon^2 = 100/81$$

$$\epsilon^2 = \frac{100}{81} - 1 = \frac{100-81}{81}$$

$$\epsilon = \sqrt{\frac{100-81}{81}} = \sqrt{19/81}$$

$$\epsilon = 0.48432$$

(a) 9/7 pts. Give an explicit expression for $H_a(s)$. Use Eq. (A.8b) on p. 866 of the text to solve for Ω_c .

(b) 8/6 pts. Give an explicit expression for $H(z)$.

(c) 8/7 pts. Give a Direct Form II realization of $H(z)$.

$$\Omega_p = \tan\left(\frac{\omega_p}{2}\right) = \tan\left(\frac{\pi}{16}\right) = 0.198912$$

$$\Omega_s = \tan\left(\frac{\omega_s}{2}\right) = \tan\left(\frac{\pi}{5}\right) = 0.726543$$

$$(A.9): N = \left\lceil \frac{\frac{1}{2} \log_{10} [(A^2-1)/\epsilon^2]}{\log_{10} (\Omega_s/\Omega_p)} \right\rceil = \left\lceil \frac{\frac{1}{2} \log_{10} [24.81/9]}{\log_{10} \left[\frac{0.726543}{0.198912}\right]} \right\rceil = \left\lceil \frac{\frac{1}{2} \log_{10} 102.316}{\log_{10} 3.65258} \right\rceil$$

$$= \left\lceil \frac{\frac{1}{2} (2.00994)}{0.562599} \right\rceil = \left\lceil \frac{1.00497}{0.562599} \right\rceil = \lceil 1.78630 \rceil = 2$$

$$(A.8b): \frac{1}{1 + (\Omega_s/\Omega_c)^{2N}} = \frac{1}{A^2}; \quad A^2 = 1 + (\Omega_s/\Omega_c)^{2N}; \quad A^2 - 1 = (\Omega_s/\Omega_c)^{2N}$$

$$\Omega_c^{2N} = \frac{\Omega_s^{2N}}{A^2 - 1}; \quad \Omega_c = \frac{\Omega_s}{(A^2 - 1)^{1/2N}} = \Omega_s (A^2 - 1)^{-\frac{1}{2N}}$$

$$= 0.726543 (24)^{-\frac{1}{4}}$$

$$= 0.726543 (0.451801)$$

$$\Omega_c = 0.328253$$

More Workspace for Problem 5...

$$(A.11): p_l = \Omega_c e^{j[\pi(N+2l-1)/2N]}, \quad l=1,2$$

$$p_1 = \Omega_c e^{j\pi(2+2-1)/4} = \Omega_c e^{j3\pi/4} = 0.328253 e^{j3\pi/4}$$

$$p_2 = \Omega_c e^{j\pi(2+4-1)/4} = \Omega_c e^{j5\pi/4} = 0.328253 e^{j5\pi/4} \\ = 0.328253 e^{-j3\pi/4}$$

$$H_a(s) = \frac{\Omega_c^N}{\prod_{l=1}^N (s-p_l)} = \frac{\Omega_c^2}{(s-p_1)(s-p_2)} = \frac{\Omega_c^2}{(s-\Omega_c e^{j3\pi/4})(s-\Omega_c e^{-j3\pi/4})}$$

$$= \frac{\Omega_c^2}{s^2 - s\Omega_c e^{-j3\pi/4} - s\Omega_c e^{j3\pi/4} + \Omega_c^2 e^{j0}}$$

$$= \frac{\Omega_c^2}{s^2 - s\Omega_c (e^{j3\pi/4} + e^{-j3\pi/4}) + \Omega_c^2}$$

$$= \frac{\Omega_c^2}{s^2 - s\Omega_c 2\cos 3\pi/4 + \Omega_c^2} = \frac{\Omega_c^2}{s^2 + s\Omega_c\sqrt{2} + \Omega_c^2}$$

$$H_a(s) = \frac{0.107750}{s^2 + 0.464219s + 0.107750}$$

More Workspace for Problem 5...

$$H(z) = H_a(s) \Big|_{s = \frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.107750}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 0.464219\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.107750}$$

$$H(z) \cdot \frac{(1+z^{-1})^2}{(1+z^{-1})^2} = \frac{0.107750(1+z^{-1})^2}{(1-z^{-1})^2 + 0.464219(1-z^{-1})(1+z^{-1}) + 0.107750(1+z^{-1})^2}$$

$$= \frac{0.107750(1+2z^{-1}+z^{-2})}{1-2z^{-1}+z^{-2} + 0.464219(1-z^{-2}) + 0.107750(1+2z^{-1}+z^{-2})}$$

$$= \frac{0.107750 + 0.215500z^{-1} + 0.107750z^{-2}}{(1+0.464219+0.107750) + (-2+0.215500)z^{-1} + (1-0.464219+0.107750)z^{-2}}$$

$$= \frac{0.107750 + 0.215500z^{-1} + 0.107750z^{-2}}{1.57197 - 1.78450z^{-1} + 0.643530z^{-2}}$$

→ divide top and bottom by 1.57197 to get a 1 in the first term of the denominator

$$H(z) = \frac{0.0685445 + 0.137089z^{-1} + 0.0685445z^{-2}}{1 - 1.13520z^{-1} + 0.409379z^{-2}}$$

More Workspace for Problem 5...

Use Fig. 8.14(a) on p. 429 of the text:

$$p_0 = 0.0685445 \quad p_1 = 0.137089 \quad p_2 = 0.0685445$$

$$d_1 = -1.13520 \quad d_2 = 0.409379$$

