Z-transform Quickstart for LTI systems

$$x[n] \rightarrow \boxed{H} \rightarrow y[n]$$

$$y[n] = x[n] * h[n]$$
$$Y(z) = X(z)H(z)$$
$$H(z) = \frac{Y(z)}{X(z)}$$

FACT: if $x[n] \overset{Z}{\longleftrightarrow} X(z)$, then $x[n-1] \overset{Z}{\longleftrightarrow} z^{-1}X(z)$ and $x[n-n_0] \overset{Z}{\longleftrightarrow} z^{-n_0}X(z)$ $\forall n_0 \in \mathbb{Z}$.

Pure Delay: let $h[n] = \delta[n-1]$

Then $x[n] \rightarrow \boxed{H} \rightarrow y[n] = x[n] * \delta[n-1] = x[n-1]$.

This is usually drawn like this:

$$x[n] \rightarrow \boxed{z^{-1}} \rightarrow x[n-1].$$

Suppose a system H with I/O relation

$$y[n] - 2y[n-1] + y[n-2] = x[n] + 3x[n-1]$$

Take z-transform on both sides:

$$Y(z) - 2z^{-1}Y(z) + z^{-2}Y(z) = X(z) + 3z^{-1}X(z)$$

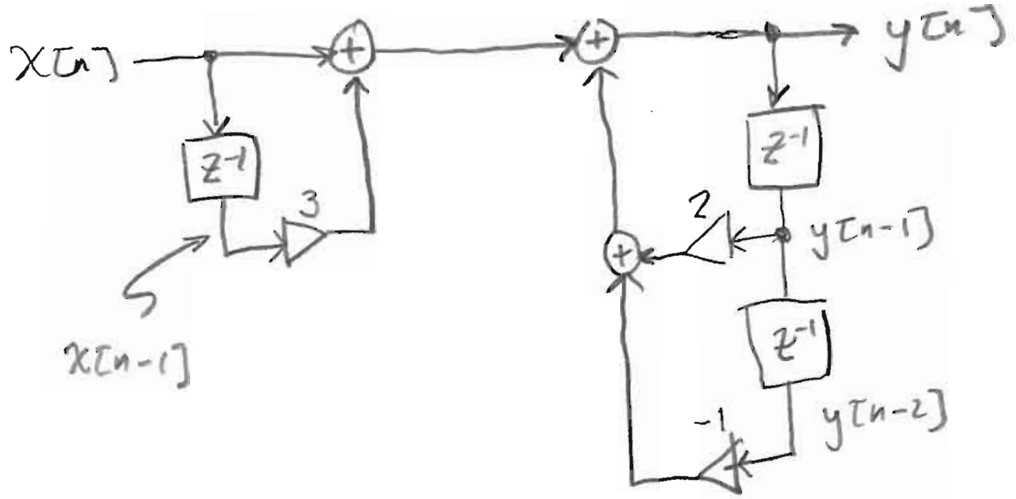
$$Y(z)\left[1 - 2z^{-1} + z^{-2}\right] = X(z)\left[1 + 3z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1}}{1 - 2z^{-1} + z^{-2}},$$ and $h[n] = \mathcal{Z}^{-1}\{H(z)\}$.

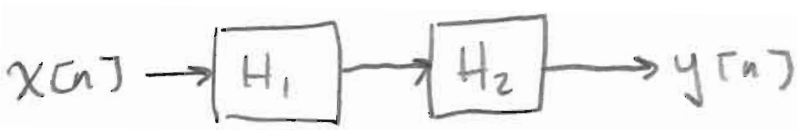
Solve the difference equation for $y[n]$:

$$y[n] = x[n] + 3x[n-1] + 2y[n-1] - y[n-2]$$

block diagram:



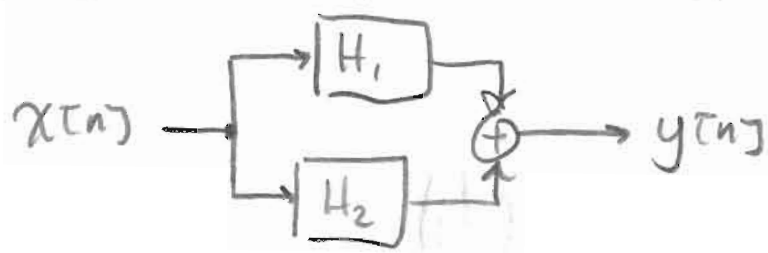
Series connection of two LTI systems:



$$h[n] = h_1[n] * h_2[n]$$

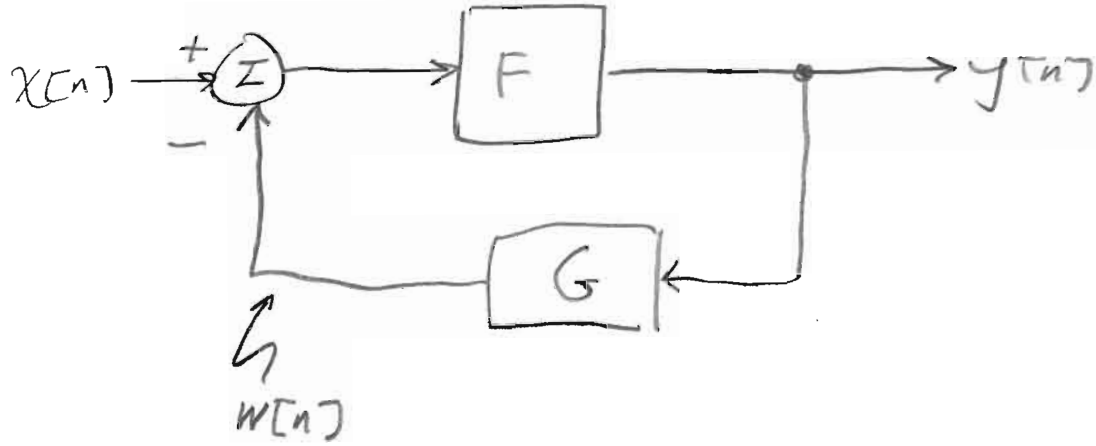
$$H(z) = H_1(z)\,H_2(z)$$

Parallel connection of two LTI systems:



$$h[n] = h_1[n] + h_2[n] \; ; \quad H(z) = H_1(z) + H_2(z)$$

Feedback Connection:



$$W(z) = G(z)Y(z) \qquad (*)$$

$$Y(z) = F(z)\left[X(z) - W(z)\right]$$

$$= F(z)X(z) - F(z)W(z)$$

plug in (*): $Y(z) = F(z)X(z) - F(z)G(z)Y(z)$

$$Y(z) + F(z)G(z)Y(z) = F(z)X(z)$$

$$Y(z)\left[1 + F(z)G(z)\right] = X(z)F(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{F(z)}{1 + F(z)G(z)}$$