1.1 GENERATION OF SEQUENCES

Project 1.1  Unit sample and unit step sequences

A copy of Program P1_1 is given below.

% Program P1_1
% Generation of a Unit Sample Sequence
clear;
% Generate a vector from -10 to 20
n = -10:20;
% Generate the unit sample sequence
u = [zeros(1,10) 1 zeros(1,20)];
% Plot the unit sample sequence
stem(n,u);
xlabel('Time index n'); ylabel('Amplitude');
title('Unit Sample Sequence');
axis([-10 20 0 1.2]);

Answers:

Q1.1  The unit sample sequence \( u[n] \) generated by running Program P1_1 is shown below:
Q1.2  The purpose of `clf` command is – clear the current figure

The purpose of `axis` command is – control axis scaling and appearance

The purpose of `title` command is – add a title to a graph or an axis and specify text properties

The purpose of `xlabel` command is – add a label to the x-axis and specify text properties

The purpose of `ylabel` command is – add a label to the y-axis and specify the text properties

Q1.3  The modified Program P1_1 to generate a delayed unit sample sequence \( u_d[n] \) with a delay of 11 samples is given below along with the sequence generated by running this program.

```matlab
% Program P1_1, MODIFIED for Q1.3
% Generation of a DELAYED Unit Sample Sequence
clf;
% Generate a vector from -10 to 20
n = -10:20;
% Generate the DELAYED unit sample sequence
u = [zeros(1,21) 1 zeros(1,9)];
% Plot the DELAYED unit sample sequence
stem(n,u);
xlabel('Time index n'); ylabel('Amplitude');
title('DELAYED Unit Sample Sequence');
axis([-10 20 0 1.2]);
```

![DELAYED Unit Sample Sequence](image)
Q1.4 The modified Program P1_1 to generate a unit step sequence $s[n]$ is given below along with the sequence generated by running this program.

```
% Program Q1_4
% Generation of a Unit Step Sequence
clf;
% Generate a vector from -10 to 20
n = -10:20;
% Generate the unit step sequence
s = [zeros(1,10) ones(1,21)];
% Plot the unit step sequence
stem(n,s);
xlabel('Time index n');ylabel('Amplitude');
title('Unit Step Sequence');
axis([-10 20 0 1.2]);
```

![Unit Step Sequence](image)

Q1.5 The modified Program P1_1 to generate a unit step sequence $sd[n]$ with an advance of 7 samples is given below along with the sequence generated by running this program.

```
% Program Q1_5
% Generation of an ADVANCED Unit Step Sequence
clf;
% Generate a vector from -10 to 20
n = -10:20;
% Generate the ADVANCED unit step sequence
sd = [zeros(1,3) ones(1,28)];
% Plot the ADVANCED unit step sequence
stem(n,sd);
xlabel('Time index n');ylabel('Amplitude');
title('ADVANCED Unit Step Sequence');
axis([-10 20 0 1.2]);
```
Project 1.2 Exponential signals

A copy of Programs P1_2 and P1_3 are given below.

% Program P1_2
% Generation of a complex exponential sequence
clf;
c = -(1/12)+(pi/6)*i;
K = 2;
n = 0:40;
x = K*exp(c*n);
subplot(2,1,1);
stem(n,real(x));
xlabel('Time index n');ylabel('Amplitude');
title('Real part');
subplot(2,1,2);
stem(n,imag(x));
xlabel('Time index n');ylabel('Amplitude');
title('Imaginary part');

% Program P1_3
% Generation of a real exponential sequence
clf;
n = 0:35; a = 1.2; K = 0.2;
x = K*a.^n;
stem(n,x);
xlabel('Time index n');ylabel('Amplitude');
Answers:

Q1.6 The complex-valued exponential sequence generated by running Program P1_2 is shown below:

Q1.7 The parameter controlling the rate of growth or decay of this sequence is – the real part of \( c \).

The parameter controlling the amplitude of this sequence is - \( K \).

Q1.8 The result of changing the parameter \( c \) to \( (1/12) + (\pi/6) \cdot i \) is – since \( \exp(-1/12) \) is less than one and \( \exp(1/12) \) is greater than one, this change means that the envelope of the signal will grow with \( n \) instead of decay with \( n \).

Q1.9 The purpose of the operator \texttt{real} is – to extract the real part of a Matlab vector.

The purpose of the operator \texttt{imag} is – to extract the imaginary part of a Matlab vector.

Q1.10 The purpose of the command \texttt{subplot} is – to plot more than one graph in the same Matlab figure.
Q1.11 The real-valued exponential sequence generated by running Program P1_3 is shown below:

Q1.12 The parameter controlling the rate of growth or decay of this sequence is - a

The parameter controlling the amplitude of this sequence is - K

Q1.13 The difference between the arithmetic operators ^ and .^ is — “^” raises a square matrix to a power using matrix multiplication. “. ^” raises the elements of a matrix or vector to a power; this is a “pointwise” operation.
Q1.14  The sequence generated by running Program P1_3 with the parameter $a$ changed to 0.9 and the parameter $K$ changed to 20 is shown below:

![Graph of sequence generated by Program P1_3 with parameters $a=0.9$ and $K=20$.](image)

Q1.15  The length of this sequence is 36.

It is controlled by the following MATLAB command line:

```matlab
n = 0:35;
```

It can be changed to generate sequences with different lengths as follows (give an example command line and the corresponding length):

```
n = 0:99;  % makes the length 100.
```

Q1.16  The energies of the real-valued exponential sequences $x[n]$ generated in Q1.11 and Q1.14 and computed using the command `sum` are $4.5673e+004$ and $2.1042e+003$. 
Project 1.3  Sinusoidal sequences

A copy of Program P1_4 is given below.

```matlab
% Program P1_4
% Generation of a sinusoidal sequence
n = 0:40;
f = 0.1;
phase = 0;
A = 1.5;
arg = 2*pi*f*n - phase;
x = A*cos(arg);
clf;  % Clear old graph
stem(n,x);  % Plot the generated sequence
axis([0 40 -2 2]);
grid;
title('Sinusoidal Sequence');
xlabel('Time index n');
ylabel('Amplitude');
axis;
```

Answers:

Q1.17  The sinusoidal sequence generated by running Program P1_4 is displayed below.
Q1.18 The frequency of this sequence is \( f = 0.1 \) cycles/sample. It is controlled by the following MATLAB command line: \( f = 0.1; \)

A sequence with new frequency 0.2 can be generated by the following command line:

\( f = 0.2; \)

The parameter controlling the phase of this sequence is \( \text{phase} \)

The parameter controlling the amplitude of this sequence is \( A \)

The period of this sequence is \( 2\pi/\omega = 1/f = 10 \)

Q1.19 The length of this sequence is - 41

It is controlled by the following MATLAB command line: \( n = 0:40; \)

A sequence with new length __81__ can be generated by the following command line:

\( n = 0:80; \)

Q1.20 The average power of the generated sinusoidal sequence is –

\[ \text{sum}(x(1:10).*x(1:10))/10 = 1.1250 \]

Q1.21 The purpose of \texttt{axis} command is – to set the range of the x-axis to \([0,40]\) and the range of the y-axis to \([-2,2]\).

The purpose of \texttt{grid} command is – to turn on the drawing of grid lines on the graph.

Q1.22 The modified Program P1_4 to generate a sinusoidal sequence of frequency 0.9 is given below along with the sequence generated by running it.

```matlab
% Program Q1_22A
% Generation of a sinusoidal sequence
n = 0:40;
f = 0.9;
phase = 0;
A = 1.5;
arg = 2*pi*f*n - phase;
x = A*cos(arg);
cif; % Clear old graph
stem(n,x); % Plot the generated sequence
axis([0 40 -2 2]);
grid;
title('Sinusoidal Sequence');
xlabel('Time index n');
ylabel('Amplitude');
axis;
```
A comparison of this new sequence with the one generated in Question Q1.17 shows that the two graphs are identical. This is because, in the modified program, we have $\omega = 0.9 \times 2\pi$. This generates the same graph as a cosine with angular frequency $\omega - 2\pi = -0.1 \times 2\pi$. Because cosine is an even function, this is the same as a cosine with angular frequency $+0.1 \times 2\pi$, which was the value used in P1_4.m in Q1.17.

In terms of Hertzian frequency, we have for P1_4.m in Q1.17 that $f = 0.1$ Hz/sample. For the modified program in Q1.22, we have $f = 0.9$ Hz/sample, which generates the same graph as $f = 0.9 - 1 = -0.1$. Again because cosine is even, this makes a graph that is identical to the one we got in Q1.17 with $f = +0.1$ Hz/sample.
A sinusoidal sequence of frequency 1.1 generated by modifying Program P1_4 is shown below.

A comparison of this new sequence with the one generated in Question Q1.17 shows - the graph here is again identical to the one in Q1.17. This is because a cosine of frequency \( f = 1.1 \) Hz/sample is identical to one with frequency \( f = 1.1 - 1 = 0.1 \) Hz/sample, which was the frequency used in Q1.17.
The sinusoidal sequence of length 50, frequency 0.08, amplitude 2.5, and phase shift of 90 degrees generated by modifying Program P1_4 is displayed below.

NOTE: for this program, it is necessary to convert the phase of 90 deg to radians and account for the minus sign that appears in the statement “\texttt{arg} = 2*\pi*f*n - \texttt{phase};” as opposed to the plus sign shown in eq. (1.12) of the lab manual. The correct statement to generate the phase is “\texttt{phase} = -90*\pi/180;”. It is also necessary to modify the \texttt{axis} command to account for the new length and amplitude of the signal. The correct \texttt{axis} statement is “\texttt{axis([0 50 -3 3]);}”.

The period of this sequence is \(- \frac{2\pi}{\omega} = \frac{1}{f} = \frac{1}{0.08} = \frac{1}{(8/100)} = \frac{100}{8} = \frac{25}{2}\). Therefore, the fundamental period is 25 and the graph has the “appearance” of going through 2 cycles of a cosine waveform during each period.
Q1.24 By replacing the `stem` command in Program P1_4 with the `plot` command, the plot obtained is as shown below:

The difference between the new plot and the one generated in Question Q1.17 is – instead of drawing stems from the x-axis to the points on the curve, the “plot” command connects the points with straight line segments, which approximates the graph of a continuous-time cosine signal.

Q1.25 By replacing the `stem` command in Program P1_4 with the `stairs` command the plot obtained is as shown below:
The difference between the new plot and those generated in Questions Q1.17 and Q1.24 is—the “stairs” command produces a stairstep plot as opposed to the stem graph that was generated in Q1.17 and the straight-line interpolation plot that was generated in Q1.24.

**Project 1.4 Random signals**

**Answers:**

**Q1.26** The MATLAB program to generate and display a random signal of length 100 with elements uniformly distributed in the interval \([-2, 2]\) is given below along with the plot of the random sequence generated by running the program:

```matlab
% Program Q1_26
% Generation of a uniform random sequence
n = 0:99;
A = 2;
rand('state',sum(100*clock)); % "seed" the generator
% rand(1,100) is uniform in [0,1]
% rand(1,100)-0.5 is uniform in [-0.5,0.5]
% 4*(rand(1,100)-0.5) is uniform in [-2,2]
x = 2*A*(rand(1,length(n))-0.5);
clear; % Clear old graph
stem(n,x); % Plot the generated sequence
axis([0 length(n) -round(2*(A+0.5))/2 round(2*(A+0.5))/2]);
grid;
title('Uniform Random Sequence');
xlabel('Time index n');
ylabel('Amplitude');
axis;
```
Q1.27  The MATLAB program to generate and display a Gaussian random signal of length 75 with elements normally distributed with zero mean and a variance of 3 is given below along with the plot of the random sequence generated by running the program:

```matlab
% Program Q1_27
% Generation of a Gaussian random sequence
% NOTE: if X is a random variable with zero mean and
% unity variance, then (aX + b) is a random variable
% with mean b and variance a^2. This follows from
% basic probability theory.
n = 0:74;
xmean = 0;      % mean of x
xstd = sqrt(3); % standard deviation of x
randn('state',sum(100*clock)); % "seed" the generator
% generate the sequence
x = xstd*randn(1,length(n)) + xmean;
% setup the graph and plot
cif;            % Clear old graph
stem(n,x);      % Plot the generated sequence
xmax = max(abs(x));
Ylim = round(2*(xmax+0.5))/2;
axis([0 length(n) -Ylim Ylim]);
grid;
title('Gaussian Random Sequence');
xlabel('Time index n');
ylabel('Amplitude');
axis;
```
The MATLAB program to generate and display five sample sequences of a random sinusoidal signal of length 31

\[ \{X[n]\} = \{A \cdot \cos(\omega_0 n + \phi)\} \]

where the amplitude \(A\) and the phase \(\phi\) are statistically independent random variables with uniform probability distribution in the range \(0 \leq A \leq 4\) for the amplitude and in the range \(0 \leq \phi \leq 2\pi\) for the phase is given below. Also shown are five sample sequences generated by running this program five different times.

```matlab
% Generates the "deterministic stochastic process" % called for in Q1.28.

n = 0:30;
f = 0.1;
Amax = 4;
phimax = 2*pi;
rand('state',sum(100*clock)); % "seed" the generator
A = Amax*rand;
% NOTE: successive calls to rand without arguments % return a random sequence of scalars. Since this % random sequence is "white" (uncorrelated), it is % not necessary to re-seed the generator for phi.
phi = phimax*rand;
% generate the sequence
arg = 2*pi*f*n + phi;
x = A*cos(arg);
% plot
cif; % Clear old graph
stem(n,x); % Plot the generated sequence
Ylim = round(2*(Amax+0.5))/2;
axis([0 length(n) -Ylim Ylim]);
grid;
title('Sinusoidal Sequence with Random Amplitude and Phase');
xlabel('Time index n');
ylabel('Amplitude');
axis;
Sinusoidal Sequence with Random Amplitude and Phase

Amplitude

Time index n
1.2 SIMPLE OPERATIONS ON SEQUENCES

Project 1.5 Signal Smoothing

A copy of Program P1_5 is given below.

```matlab
% Program P1_5
% Signal Smoothing by Averaging
clf;
R = 51;
d = 0.8*(rand(R,1) - 0.5); % Generate random noise
m = 0:R-1;
s = 2*m.*(0.9.^m); % Generate uncorrupted signal
x = s + d'; % Generate noise corrupted signal
subplot(2,1,1);
plot(m,d','r-',m,s,'g--',m,x,'b-.');
xlabel('Time index n');ylabel('Amplitude');
legend('d[n] ','s[n] ','x[n] ');

x1 = [0 0 x];x2 = [0 x 0];x3 = [x 0 0];
y = (x1 + x2 + x3)/3;
subplot(2,1,2);
plot(m,y(2:R+1),'r-',m,s,'g--');
legend( 'y[n] ','s[n] ');
```
Answers:

Q1.29 The signals generated by running Program P1_5 are displayed below:

Q1.30 The uncorrupted signal \( s[n] \) is the product of a linear growth with a slowly decaying real exponential.

The additive noise \( d[n] \) is a random sequence uniformly distributed between -0.4 and +0.4.

Q1.31 The statement \( x = s + d \) CANNOT be used to generate the noise corrupted signal because \( d \) is a column vector, whereas \( s \) is a row vector; it is necessary to transpose one of these vectors before adding them.

Q1.32 The relations between the signals \( x_1, x_2, \) and \( x_3, \) and the signal \( x \) are – all three signals \( x_1, x_2, \) and \( x_3 \) are extended versions of \( x \), with one additional sample appended at the left and one additional sample appended to the right. The signal \( x_1 \) is a delayed version of \( x \), shifted one sample to the right with zero padding on the left. The signal \( x_2 \) is equal to \( x \), with equal zero padding on both the left and right to account for the extended length. Finally, \( x_3 \) is a time advanced version of \( x \), shifted one sample to the left with zero padding on the right.
Q1.33 The purpose of the `legend` command is to create a legend for the graphs. In P1_5, the signals are plotted using different colors and line types; the legend provides information about which color and line type is associated with each signal.

Project 1.6 Generation of Complex Signals

A copy of Program P1_6 is given below.

```matlab
% Program P1_6
% Generation of amplitude modulated sequence
clf;
N = 0:100;
m = 0.4; fH = 0.1; fL = 0.01;
xH = sin(2*pi*fH*N);
xL = sin(2*pi*fL*N);
y = (1+m*xL).*xH;
stem(N,y);grid;
xlabel('Time index n');ylabel('Amplitude');
```

Answers:

Q1.34 The amplitude modulated signals \( y[n] \) generated by running Program P1_6 for various values of the frequencies of the carrier signal \( xH[n] \) and the modulating signal \( xL[n] \), and various values of the modulation index \( m \) are shown below:

\[
m=0.4; \quad fH=0.1; \quad fL=0.01:
\]
$m=0.9; f_H=0.1; f_L=0.1$:

$\begin{array}{c}
\text{Amplitude} \\
\text{Time index } n
\end{array}$

$\begin{array}{c}
\text{Amplitude} \\
\text{Time index } n
\end{array}$

$m=0.4; f_H=0.1; f_L=0.005$:
\[ m = 0.4; \ f_H = 0.25; \ f_L = 0.01: \]
Q1.35 The difference between the arithmetic operators * and .* is – “*” multiplies two conformable matrices or vectors using matrix multiplication. “.*” takes the pointwise products of the elements of two matrices or vectors that have the same dimensions.

A copy of Program P1_7 is given below.

```matlab
% Program P1_7
% Generation of a swept frequency sinusoidal sequence
n = 0:100;
a = pi/2/100;
b = 0;
arg = a*n.*n + b*n;
x = cos(arg);
clf;
stem(n, x);
axis([0,100,-1.5,1.5]);
title('Swept-Frequency Sinusoidal Signal');
xlabel('Time index n');
ylabel('Amplitude');
grid; axis;
```

Answers:

Q1.36 The swept-frequency sinusoidal sequence \( x[n] \) generated by running Program P1_7 is displayed below.
Q1.37 The minimum and maximum frequencies of this signal are - The minimum occurs at \( n=0 \), where we have \( 2a_n + b = 0 \) rad/sample = 0 Hz/sample. The maximum occurs at \( n=100 \), where we have \( 2a_n + b = 200a = 200\pi(0.5)(0.01) = \pi \) rad/sample = 0.5 Hz/sample.

Q1.38 The Program 1_7 modified to generate a swept sinusoidal signal with a minimum frequency of 0.1 and a maximum frequency of 0.3 is given below:

**Note:** for a minimum frequency of 0.1 Hz/sample = \( \pi/5 \) rad/sample at \( n=0 \), we must have \( 2a(0) + b = \pi/5 \), which implies \( b=\pi/5 \). For a maximum frequency of 0.3 Hz/sample = \( 3\pi/5 \) rad/sample at \( n=100 \), we must have \( 2a(100) + \pi/5 = 3\pi/5 \), which implies \( a=\pi/500 \).

% Program Q1_38
% Generation of a swept frequency sinusoidal sequence
n = 0:100;
a = pi/500;
b = pi/5;
arg = a*n.*n + b*n;
x = cos(arg);
clf;
stem(n, x);
axis([0,100,-1.5,1.5]);
title('Swept-Frequency Sinusoidal Signal');
xlabel('Time index n');
ylabel('Amplitude');
grid; axis;

1.3 WORKSPACE INFORMATION

Q1.39 The information displayed in the command window as a result of the `who` command is – a listing of the names of the variables defined in the current workspace.

Q1.40 The information displayed in the command window as a result of the `whos` command is – a long form listing of the variables defined in the current workspace, including the variable names, their dimensions (size), the number of bytes of storage required for each variable, and the datatype of each variable. The total number of bytes of storage for the entire workspace is also displayed.
1.4 OTHER TYPES OF SIGNALS (Optional)

Project 1.8 Squarewave and Sawtooth Signals

Answer:

Q1.41 MATLAB programs to generate the square-wave and the sawtooth wave sequences of the type shown in Figures 1.1 and 1.2 are given below along with the sequences generated by running these programs:

% Program Q1_41A
% Generation of the square wave in Fig. 1.1(a)
\n n = 0:30;
f = 0.1;
phase = 0;
duty=60;
A = 2.5;
arg = 2*pi*f*n + phase;
x = A*square(arg,duty);
clf; % Clear old graph
stem(n,x); % Plot the generated sequence
axis([0 30 -3 3]);
grid;
title('Square Wave Sequence of Fig. 1.1(a)');
xlabel('Time index n');
ylabel('Amplitude');
axis;

% Program Q1_41B
% Generation of the square wave in Fig. 1.1(b)
\n n = 0:30;
f = 0.1;
phase = 0;
duty=30;
A = 2.5;
arg = 2*pi*f*n + phase;
x = A*square(arg,duty);
clf; % Clear old graph
stem(n,x); % Plot the generated sequence
axis([0 30 -3 3]);
grid;
title('Square Wave Sequence of Fig. 1.1(b)');
xlabel('Time index n');
ylabel('Amplitude');
axis;
% Program Q1_41C
% Generation of the square wave in Fig. 1.2(a)
n = 0:50;
f = 0.05;
phase = 0;
peak = 1;
A = 2.0;
arg = 2*pi*f*n + phase;
x = A*sawtooth(arg,peak);
cif;                 % Clear old graph
stem(n,x);          % Plot the generated sequence
axis([0 50 -2 2]);  
grid;
title('Sawtooth Wave Sequence of Fig. 1.2(a)');
xlabel('Time index n');
ylabel('Amplitude');
axis;

% Program Q1_41D
% Generation of the square wave in Fig. 1.2(b)
n = 0:50;
f = 0.05;
phase = 0;
peak = 0.5;
A = 2.0;
arg = 2*pi*f*n + phase;
x = A*sawtooth(arg,peak);
cif;                 % Clear old graph
stem(n,x);          % Plot the generated sequence
axis([0 50 -2 2]);  
grid;
title('Sawtooth Wave Sequence of Fig. 1.2(b)');
xlabel('Time index n');
ylabel('Amplitude');
axis;
Square Wave Sequence of Fig. 1.1(a)

Square Wave Sequence of Fig. 1.1(b)
Sawtooth Wave Sequence of Fig. 1.2(a)

Sawtooth Wave Sequence of Fig. 1.2(b)