ECE 4213/5213
Homework 3

Fall 2023 Dr. Havlicek

The goal of this assignment is to provide you with an introduction to the scholarly literature, how to read it, and how it can be helpful to you both in your education and in your engineering career. This assignment will also fulfill an ABET evaluation criterion for ECE 4213 to demonstrate that students have an ability to engage in independent learning.

Preliminaries

Conference papers and journal papers are an important part of the engineering scholarly literature. Reading them is something that seems difficult to most people at first. One of the main reasons for this is that, like riding a bicycle, reading engineering conference papers and journal papers is an *acquired skill;* it takes practice and most of us are not born already knowing how to do it.

One of the keys to success is to realize that you do not read an engineering paper in the same way that you read a popular fiction book. If you want to read *Harry Potter and the Sorcerer's Stone*, then you pick it up and you read it through. You expect to understand the details of what is written on the first reading – without having to read each paragraph over and over again and without having to come back and read it again on a different day.

But engineering papers are different. If you approach them in the same way as a popular fiction book and you expect to read them through and understand all of the details on the first reading, then that first reading may take a very long time. In many cases, it might take more than one year. So, to be effective, you need to take a different approach when you set out to read an engineering paper.

In most cases, your first read of an engineering paper should be a quick *scan* read. Your goals should be to figure out if the paper can help you, if it has anything in it that you are looking for, and what the big ideas are. If the paper seems like it can help you, then you should also make mental note (or written note) of where in the paper the details for each big idea are explained. On the first read, it is important for you to *not get bogged down* in the details! Because that will generally make the first read take *too long.*

If it turns out that the paper does have information in it that is helpful to you, then you will generally come back and spot read for the details on the second and subsequent readings.

Often, you will read an engineering paper because you are looking for answers to specific questions. On the first reading, you will make note of which sections of the paper seem to have something to do with your questions. Then, on the second reading, you will focus on just those sections. You will read them in great detail while largely ignoring the rest of the paper.

Introduction

In Homework 2, we skipped Project 2.2 in the course Laboratory Manual. That was because Project 2.2 involves some ideas that are not found in textbooks. To understand them, most people will need to read some papers. Therefore, we will return to Project 2.2 now.

In answering the questions, you will need to do some math that would take a lot of time to typeset in a word processor. You should use Matlab and Microsoft WORD for the computer parts, but the mathematics and discussion parts can be written by hand and then scanned or photographed (you can also typeset them if you want to). However, make sure to assemble your final turn-in version into one file that has all the pages. You can use Adobe Acrobat DC/Pro to insert scans of hand written pages into your PDF turn-in file. Alternatively, you can insert crops from you scanned pages into your WORD file as pictures.

The Assignment

1. Read Project 2.2 on pages 21 and 22 of the course Laboratory Manual and work question Q2.5 (the m-file for program P2_2 can be found in the handouts section of the course web site). The last paragraph above Program P2_2 on p. 21 is a misprint; it should read "The following MATLAB program can be used to generate a sinusoidal input signal $x[n]$ and simulate the nonlinear system of Eq. (2.14) to generate $y[n]$."

   (a) In Program P2_2, the input signal is given by $x[n] = \cos(\omega n)$, where $\omega = 2\pi f$. Run the program three times with input frequencies `f=0.05`, `f=0.10`, and `f=0.25`. For each case, show your graphs of the input and output signals.

   (b) To answer the question *How do the output signals depend on the frequencies of the input signal?*, obtain and read the paper

      J.F. Kaiser, "On a simple algorithm to calculate the 'energy' of a signal," in *Proc. IEEE Int'l. Conf. Acoust., Speech, Signal Proc.*, Albuquerque, NM, Apr. 3-6, 1990, pp. 381-384.

   Here is some help on how to find this paper:

   - Open a web browser, go to `libraries.ou.edu`, and log in with your OU 4×4.
   - Under *Quick Links* in the center of the page, click on *Databases & E-Reference*.
   - In the *Search By Title* box, type "IEEE Xplore" and hit return (don't type the double quote marks... I just wrote those to make it clear what you are supposed to type!). This will return a link to IEEE Xplore under the listing of the alphabet located just below the *Search By Title* box. Click the link to go to IEEE Xplore.
   - By default, IEEE Xplore comes up in basic search mode. That will be fine for our purposes here. In the search box, type "kaiser on simple algorithm calculate energy signal" and hit return (again, don't type the double quote

marks... I wrote them here to delimit the string that you are actually supposed to type). This should return just one paper... the one we are looking for!

- Near the bottom of the returned record, right click on the PDF icon and open the link in a new window. (Note: in some browsers, if you left click the icon instead you may be left with a blank page. This seems to be an issue with IEEE Xplore).

Hint: remember to just scan the paper on your first reading and don't get bogged down in details! Then, for your second reading, come back and read in detail just the parts that will help you answer the question *How do the output signals depend on the frequencies of the input signal?* Your answer should be short... one sentence should be enough.

Note: in our textbook and lecture notes for this course, we use Ω for continuous-time frequency and ω for discrete-time frequency. But in writing about this subject, Kaiser and some other authors often do it backwards from that: they often use Ω for discrete-time frequency and they also sometimes use ω for continuous-time frequency. Don't be confused by this!

Note: In this paper, Kaiser writes x_n to refer to a discrete-time signal. When he writes x_n, x_{n+1}, and x_{n-1}, it means the same thing as when we write $x[n]$, $x[n+1]$, and $x[n-1]$ in our course textbook and lecture notes.

Now, you will need to write some math to answer the question *Can you verify your observation mathematically?* Let $\omega = 2\pi f$ and start with $x[n] = \cos(\omega n)$. Then solve for $y[n]$ in Eq. (2.14) on page 21 of the course Laboratory Manual. You can follow Kaiser's development in Section III of his paper. You may find the trigonometric identities given on page 1 of the formula sheet from our course web site helpful.

2. Work question Q2.6 of the Lab Manual.

   Note: There is a misprint in question Q2.6. It should read "Use sinusoidal signals of the form $x[n] = \cos(\omega_0 n) + K$ as the input..." So make $x[n]$ a *cosine* as in question Q2.5 (*not* a sine) and remember that $\omega = 2\pi f$.

   (a) This question will require you to make a small modification to program P2_2 in order to change the form of the input signal to include the DC value K. Run the modified program for $f = 0.1$ ($\omega_0 = 0.2\pi$) and $K = 0.5$. Show a listing of your modified program and your graphs of the input and output signals.

   (b) To answer the question *How does the output signal $y[n]$ depend on the DC value K?*, it would be reasonable for you to search for papers on IEEE Xplore and on the Internet. You could use search terms like "teager energy sinusoid plus constant" and "teager operator properties." But not much has been written on this subject, so you would have to scan a large number of papers before you would find anything useful.

So let's not do that. Instead, obtain and read this paper that I found:

> A. Restrepo and J. Quiroga, "Root and pre-constant signals of the 1D Teager-Kaiser operator," *Signal, Image, and Video Process.*, vol. 5, no. 3, Sep. 2011, pp. 363-378.

Here is some help on how to find it:

- Open a web browser, go to `libraries.ou.edu`, and log in with your OU 4×4.
- Under *Quick Links* in the center of the page, click on *Databases & E-Reference*.
- In the *Search By Title* box, type "SpringerLink" and hit return (don't type the double quote marks... I just wrote those to make it clear what you are supposed to type! Also notice that there is *no space* between "Springer" and "Link"... it is all just one word). This will return a link to SpringerLink under the listing of the alphabet. Click it to go to SpringerLink.
- In the search box, type "Signal, Image, and Video Processing" and hit return (again, don't type the double quote marks... I wrote them here to delimit the string that you are actually supposed to type).
- The first returned record should be the journal we are looking for. Click it to go to the web page for the journal.
- A little ways down the page, it says "Latest issue" and there is a picture of the cover of the most recent issue. At the lower-right corner of that picture, there is a link that says "View all volumes and issues >"... click it and scroll down to *Volume 5*.
- Click on *September 2011, issue 3* to go to the issue.
- Scroll down and click on the paper.
- In the upper right corner of the page, click the *Download PDF* button.

Now, this paper is quite a bit harder to read than our last one. So let me walk you through the process of reading it step-by-step:

- First, we quickly scan the whole paper (you should do that now). The only thing we find that seems really useful for answering our question is Eq. (3) on page 366.
- So, on our second time through the paper we come back immediately to Eq. (3) and start reading for the details. The line above the equation tells us that $\mathbf{x}$ is a signal and c is a constant. That seems good, but we still need to know what TK and $\mathbf{h}$ mean in Eq. (3).
- Scanning backwards from Eq. (3), we find that the notation TK is defined in Eq. (1) on page 364. But Eq. (1) has "funny" notation ${}_1\mathbf{x}$ and ${}_{-1}\mathbf{x}$ in it. We find the definition for this notation in the paragraph right above Eq. (1). It verifies that TK($\mathbf{x}$) in this paper is exactly the same as $y[n]$ in Eq. (2.14) on page 21 of our course Lab Manual. That's good!

- It means that Eq. (3) of this paper is just what we are looking for: it is the output of our nonlinear system in Project 2.2 when the input is a signal plus a constant.
- So we return to Eq. (3) on page 366 of this paper, but we still need to know what **h** means on the right side. We find the answer in the two lines immediately below Eq. (3).
- It seems a little bit complicated, but immediately below those two lines we find an even more useful version of Eq. (3). It says:

$$TK(\mathbf{x} + \{c\}) = TK(\mathbf{x}) + c(-\{x_{n-1}\} + 2\{x_n\} - \{x_{n+1}\}).$$

  For question Q2.6 (with the misprints corrected as I indicated in the **Note** above), we have $\mathbf{x} = \cos(\omega_0 n)$ and $c = K$. So, translating Eq. (3) of this paper into our notation from the course Lab Manual and substituting it in for the right side of Eq. (2.14) of the course Lab Manual, we get

$$\begin{aligned} y[n] &= TK[\cos(\omega_0 n) + K] \\ &= TK[\cos(\omega_0 n)] + K\{-\cos[\omega_0(n-1)] + 2\cos(\omega_0 n) - \cos[\omega_0(n+1)]\}. \quad (1) \end{aligned}$$

- This is really starting to look like it's just what we need to answer the question in Q2.6 about how the output signal depends on K!

  Applying the trigonometric identity for $\cos(A \pm B)$ from page 1 of the course formula sheet to the cosine terms on the right side of (1) above, we get

$$\cos[\omega_0(n-1)] = \cos(\omega_0 n - \omega_0) = \cos(\omega_0 n)\cos(\omega_0) + \sin(\omega_0 n)\sin(\omega_0) \quad (2)$$

  and

$$\cos[\omega_0(n+1)] = \cos(\omega_0 n + \omega_0) = \cos(\omega_0 n)\cos(\omega_0) - \sin(\omega_0 n)\sin(\omega_0). \quad (3)$$

  So

$$\begin{aligned} &K\{-\cos[\omega_0(n-1)] + 2\cos(\omega_0 n) - \cos[\omega_0(n+1)]\} \\ &\quad = K[-\cos(\omega_0 n)\cos(\omega_0) - \sin(\omega_0 n)\sin(\omega_0) + 2\cos(\omega_0 n) \\ &\quad\qquad - \cos(\omega_0 n)\cos(\omega_0) + \sin(\omega_0 n)\sin(\omega_0)] \\ &\quad = K[2\cos(\omega_0 n) - 2\cos(\omega_0 n)\cos(\omega_0)] \\ &\quad = 2K[1 - \cos(\omega_0)]\cos(\omega_0 n). \quad (4) \end{aligned}$$

  Moreover, in answering question Q2.5 you should have gotten that

$$\begin{aligned} TK[\cos(\omega_0 n)] &= \cos^2(\omega_0 n) - \cos[\omega_0(n-1)]\cos[\omega_0(n+1)] \\ &= \sin^2(\omega_0). \quad (5) \end{aligned}$$

  Plugging (4) and (5) into (1) above, we have

$$y[n] = TK[\cos(\omega_0 n) + K] = \sin^2(\omega_0) + 2K[1 - \cos(\omega_0)]\cos(\omega_0 n). \quad (6)$$

Great! This appears to be the answer to our question! For Q2.6, it says that the output signal depends on the DC value K as follows: you get the same answer as you got without K, but added to it there is a cosine wave at the same frequency as the input signal. The amplitude of this added cosine wave is given by the constant term $2K[1 - \cos(\omega_0)]$.

And this seems to agree with the graph of $y[n]$ that we got in Q2.6 when we ran the modified version of Program P2_2!

So we are done – provided that we can *verify* that (6) above is actually *correct.*

It is a **mistake** to blindly trust results from published papers unless you can independently verify them yourself! The reason is that these papers sometimes contain errors ranging from simple typos to full-blown theoretical screw ups.

- Since our result (6) above depends on Eq. (3) from the paper by Restrepo and Quiroga, we return once again to that paper to try and figure out where Eq. (3) came from. In the line immediately above the equation, it says that Eq. (3) results from Proposition 6.
- So we scan the paper backwards from Eq. (3) again to look for Proposition 6. We find it right away at the bottom of page 365. It is a more general result that applies to the Teager energy of the sum of two arbitrary signals. The first two terms on the right-hand side of the last equation on page 365 look pretty simple, but the third term involves another strange symbol: $\otimes$. And, unfortunately, the paper does not provide us with a proof for Proposition 6. So it seems like it will take some work to verify Eq. (3).
- I know that you are probably getting tired of this by now, but in order to complete question Q2.6 you need to do one of two things. You need to **EITHER**

  (i) Read more of the paper to figure out what the symbol $\otimes$ means, then verify that Proposition 6 is correct, and finally show how Eq. (3) follows from Proposition 6 (this will verify our result in (6) above),

  OR

  (ii) Try to independently verify our result in (6) above. To do this, you should start by letting $x[n] = \cos(\omega_0 n) + K$ as specified in question Q2.6. Then plug this expression for $x[n]$ into Eq. (2.14) on page 21 of the course Lab Manual and try to use trigonometric substitutions to show that $y[n]$ is equal to our result in (6) above.
- So pick one of these two choices and include a writeup of your solution as part of your answer to question Q2.6.
- If you pick number (ii), which is what I recommend, then you may begin to wonder what if any value you got from reading Restrepo's and Quiroga's paper in the first place. Because to answer question Q2.6 you ended up having to independently show how the output signal $y[n]$ depends on K all by yourself anyway.

Here's the answer to that: the value of reading the paper is that it told you what the result is supposed to look like, at least according to Restrepo and Quiroga. That gave you an end result to aim for in trying to simplify Eq. (2.14). And, assuming that you ultimately get the same answer that we already did in (6) above, it gives you confidence that your work is correct – which is worth something to you in the big picture, especially once you are out of school and working on problems that will not have any solutions posted.

3. Now let's suppose that you have graduated and you are working for a big company that does DSP as part of its business. Your manager calls you in. She heard that you know something about the Teager-Kaiser operator. The company needs some research on algorithms for estimating instantaneous amplitude and instantaneous frequency as part of a feasibility study for a new product that is being proposed by the avionics division.

   She continues to explain:

   "As you know, for a discrete-time sinusoidal signal with amplitude A and frequency ω, the discrete TK operator estimates the quantity $A^2 \sin^2 \omega$.

   More generally, for a discrete-time signal

$$x[n] = a[n] \cos(\varphi[n]) \tag{7}$$

   where $a[n]$ and $\varphi[n]$ are time-varying functions of n, the discrete TK operator

$$\Psi_d \{x[n]\} = x^2[n] - x[n-1]x[n+1] \tag{8}$$

   estimates the time-varying quantity $a^2[n] \sin^2(\omega[n])$ where

$$\omega[n] = \frac{d}{dn} \varphi[n]. \tag{9}$$

   The meaning of the derivative in (9) is that you differentiate the function $\varphi[n]$ with respect to n as though n was a continuous variable. The quantity $a^2[n] \sin^2(\omega[n])$ is called the *discrete Teager energy* of the signal $x[n]$.

   In (7), the function $a[n]$ is called the *instantaneous amplitude* (IA) of the signal $x[n]$. It is also called the *amplitude modulation* and sometimes the *amplitude envelope*. The function $\omega[n]$ is called the *instantaneous frequency* (IF). It is also called *frequency modulation*.

   Based on the TK operator, there are algorithms which are capable of estimating the individual IA and IF functions from the Teager energy. These are called *discrete energy separation algorithms*.

   The most popular ones are called DESA-1, DESA-1a, and DESA-2. Among these, DESA-1 is generally considered the best because it has the smallest estimation errors and it is capable of estimating the widest range of instantaneous frequencies."

Finally, your manager says that your assignment is to develop a Matlab program to implement the DESA-1 discrete energy separation algorithm.

(a) The first thing you will need to do is find a paper or two that can help you. Go to IEEE Xplore and do a basic search with the search string "energy separation algorithm amplitude frequency modulation" (don't type the double quotes in the search box... I added them here simply to delimit the search string).

This search should return 50 results: 37 conference papers and 13 journal papers. To see them all, you will need to change the number of results displayed per page from 25 to 50. To do this, look for the pulldown labeled "Items Per Page:" in the ribbon right under the search bar. Click on that pulldown and change the number of displayed results from 25 to 50.

What you need is to find just one or two of the papers that can really help you with this assignment. Start by scanning the titles; you will be able to eliminate several of these papers based on the title alone. For example, you are not interested in music and vocal signals for this assignment. So, based on title alone, you can eliminate the paper "Music and vocal separation using multiband modulation features." Eliminate as many papers as you can by scanning the titles.

Next, scan the abstracts of the remaining papers. Try to narrow your search down to just a few papers that look like they are most likely to have the information that you need. Then do an initial read on just those few papers. Remember to *scan* for the information you need! Remember to avoid getting bogged down in details!

Your goal is to select just one or two papers that you will read more carefully to help you figure out how to implement DESA-1 in Matlab.

- To answer this part of Problem 3, show the bibliographic information for the one or two papers that you select. Use a format similar to the way that I listed the bibliographic information for the Kaiser paper and the Restrepo and Quiroga paper above on pages 2 and 4 of this homework assignment.

Hint: even for the papers that you eliminate, you might find it useful to quickly scan the bibliography. What you usually find is that most of the papers in an area will *all* cite a small set of fundamental papers. That small set includes the *really good ones* that can probably help you the most. But doing that is optional for this assignment. You should be able to find the really good papers without needing to scan all of the bibliographies.

(b) Use your selected paper(s) to help you develop a Matlab program to implement the DESA-1 discrete energy separation algorithm. Given the input signal, the output of your program should be graphs of the IA and IF as functions of the time variable n.

Note that DESA-1 naturally estimates the IF in units of radians per sample. You should divide it by 2π to convert the units to cycles per sample.

As a baseline for development and debugging purposes, make the input signal $x[n] = A\cos(\omega_0 n)$, where $0 \leq n \leq 250$, $A = 5$, $\omega_0 = 2\pi f_0$, and $f_0 = 0.1$. Similar to Program P2_2 on page 21 of the course Laboratory Manual, here are some Matlab statements to generate this input signal:

```
n = 0:250;
A = 5.0;
f = 0.1;
w = 2*pi*f;
x = A*cos(w*n);
```

Show your Matlab code as well as graphs for the input signal, the estimated IA, and the estimated IF. Except for "edge effects" near the very beginning and very end of the signal, you should get that the estimated IA is $a[n] = 5.0$ (constant for all n) and the estimated IF is $\omega[n]/(2\pi) = 0.1$ (also constant for all n).

(c) Test your program with the AM-FM input signal

$$x[n] = A\exp\left[-\frac{(n-128)^2}{2\sigma^2}\right]\cos(\varphi[n]), \tag{10}$$

where $0 \leq n \leq 250$, $A = 5.0$, $2\sigma^2 = 17,881.0$, and

$$\varphi[n] = 2\pi(0.00078)n^2 + 2\pi(0.01)n. \tag{11}$$

This is a chirp signal (quadratic phase) with Gaussian amplitude modulation. The IF rises linearly from an initial value of $\omega[n] = 2\pi(0.01)$ radians per sample ($f = 0.01$ cycles per sample) at $n = 0$ to a final value of $\omega[n] = 2\pi(0.40)$ radians per sample ($f = 0.4$ cycles per sample) at $n = 250$. The amplitude envelope is a Gaussian that starts at 2.0 when $n = 0$ and peaks at 5.0 when $n = 128$.

Here are some Matlab statements to generate this input signal:

```
n = 0:250;
Amax = 5.0;
TwoSigSq = 17881;
AM = Amax*exp(-(n - 128).^2 / TwoSigSq);
alpha = 2*pi*0.00078;
beta = 2*pi*0.01;
phi = alpha*n.^2 + beta*n;
x = AM.*cos(phi);
```

Use your Matlab program to apply DESA-1 and estimate the IA and IF of this signal. Show graphs of the input signal, the estimated IA, and the estimated IF as functions of n.

Submit this assignment electronically on Canvas.

DUE: 9/15/2023, 11:59 PM