1(a). The following graphs were obtained by running Program P2.2 with the three specified frequencies:

\[ f = 0.05 \]
1(b). The output signals depend on the frequencies of the input signal according to:
\[ y[n] = \sin^2(\omega) = \sin^2(2\pi f). \]

**Mathematical Verification:**

Let \( \omega = 2\pi f \) and let the input signal be \( x[n] = \cos(\omega n) \). Then (2.14) on page 21 of the course Laboratory Manual becomes

\[
y[n] = x^2[n] - x[n - 1]x[n + 1] = \cos^2(\omega n) - \cos[\omega(n - 1)]\cos[\omega(n + 1)]. \tag{1}
\]

We have \( \cos[\omega(n - 1)] = \cos(\omega n - \omega) \) and \( \cos[\omega(n + 1)] = \cos(\omega n + \omega) \), so

\[
x[n - 1]x[n + 1] = \cos(\omega n - \omega)\cos(\omega n + \omega). \tag{2}
\]

Applying the trigonometric identity \( \cos(A + B)\cos(A - B) = \frac{1}{2}\cos(2A) + \frac{1}{2}\cos(2B) \) to (2), we obtain

\[
x[n - 1]x[n + 1] = \frac{1}{2}\cos(2\omega n) + \frac{1}{2}\cos(2\omega). \tag{3}
\]

Upon applying the trigonometric identity \( \cos(2A) = 2\cos^2(A) - 1 \) to the first term on the right-hand side of (3), there results

\[
x[n - 1]x[n + 1] = \cos^2(\omega n) - \frac{1}{2} + \frac{1}{2}\cos(2\omega). \tag{4}
\]

Applying the trigonometric identity \( \cos(2A) = 1 - 2\sin^2(A) \) to the second term on the right-hand side of (4) then gives us

\[
x[n - 1]x[n + 1] = \cos^2(\omega n) - \frac{1}{2} + \frac{1}{2} - \sin^2(\omega) = \cos^2(\omega n) - \sin^2(\omega). \tag{5}
\]

Substituting (5) into (1), we then obtain

\[
y[n] = \cos^2(\omega n) - \cos^2(\omega n) + \sin^2(\omega) = \sin^2(\omega), \tag{6}
\]

which verifies the desired result.
% Program Q2_6
% Modified version of P2_2 to make the input a cosine plus a constant.
cf;
 n = 0:200;
 K = 0.5;
  f = 0.1;
arg = 2*pi*f*n;
x = cos(arg) + K;
% Compute the output signal
x1 = [x 0 0];  x1[n] = x[n+1]
x2 = [0 x 0];  x2[n] = x[n]
x3 = [0 0 x];  x3[n] = x[n-1]
y = x2.*x2-x1.*x3;
y = y(2:202);
% Plot the input and output signals
subplot(2,1,1)
plot(n, x)
xlabel('Time index n'); ylabel('Amplitude');
title('Input Signal')
subplot(2,1,2)
plot(n,y)
xlabel('Time index n'); ylabel('Amplitude');
title('Output signal');
Graphs obtained by running the modified program:

\[ f = 0.1, \ K = 0.5 \]

2(b). From Eq. (3) on page 366 of the paper by Restrepo and Quiroga, we obtained

\[ y[n] = TK[\cos(\omega_0 n) + K] = \sin^2(\omega_0) + 2K[1 - \cos(\omega_0)]\cos(\omega_0 n). \]  \hspace{1cm} (7)

I will select choice (ii) and verify the correctness of (7) by starting with Eq. (2.14) on page 21 of the course Laboratory Manual.

Let \( x[n] = \cos(\omega_0 n) + K \). Eq. (2.14) of the course Laboratory Manual says that

\[ y[n] = x^2[n] - x[n - 1]x[n + 1]. \]  \hspace{1cm} (8)

We have that

\[ x^2[n] = \cos^2(\omega_0 n) + 2K\cos(\omega_0 n) + K^2. \]  \hspace{1cm} (9)

Working on the second term on the right-hand side of (8), we obtain

\[ x[n - 1] = \cos[\omega_0(n - 1)] + K = \cos(\omega_0 n - \omega_0) + K \]
and

\[ x[n + 1] = \cos[\omega_0(n + 1)] + K = \cos(\omega_0 n + \omega_0) + K, \]

so

\[ x[n - 1]x[n + 1] = \cos(\omega_0 n - \omega_0)c(\omega_0 n + \omega_0) + K[\cos(\omega_0 n - \omega_0) + \cos(\omega_0 n + \omega_0)] + K^2. \quad (10) \]

Now, it was already shown in (2)-(5) above that

\[ \cos(\omega_0 n - \omega_0) \cos(\omega_0 n + \omega_0) = \cos^2(\omega_0 n) - \sin^2(\omega_0). \quad (11) \]

Substituting this into (10) and applying the trigonometric identity \( \cos(A + B) + \cos(A - B) = 2 \cos(A) \cos(B) \) to the term in square brackets, we get

\[ x[n - 1]x[n + 1] = \cos^2(\omega_0 n) - \sin^2(\omega_0) + 2K \cos(\omega_0 n) \cos(\omega_0) + K^2. \quad (12) \]

Substituting (9) and (12) into (8) then produces the final result

\[
y[n] = \sin^2(\omega_0) + 2K \cos(\omega_0 n) - 2K \cos(\omega_0 n) \cos(\omega_0) \\
= \sin^2(\omega_0) + 2K[1 - \cos(\omega_0)] \cos(\omega_0 n),
\]

thereby verifying the correctness of (7).

Equations (7) and (13) provide the answer to how the output signal \( y[n] \) depends on the DC value \( K \). Compared to the case where \( x[n] = \cos(\omega_0 n) \), there is an extra term added to \( y[n] \). This extra term is a cosine having the same frequency as the input and having an amplitude of \( 2K[1 - \cos(\omega_0)] \). This term adds a ripple to \( y[n] \) which may be plainly seen in the graph on the preceding page. The amplitude of the ripple is proportional to \( K \).
3(a). I selected the paper:


Another good one is:


3(b). Listing of Matlab program to implement DESA-1:

```matlab
% Program DESA1
% Implement the DESA-1 discrete energy separation algorithm
clear;
clf;
% Generate the input signal
n = 0:250;
A = 5.0;
f = 0.1;
w = 2*pi*f;
x = A*cos(w*n);
x2 = [0 x 0]; % zero pad x[n]
y = [0 x 0] - [0 0 x]; % y[n] = x[n] - x[n-1]
z = [x 0 0] - [0 x 0]; % z[n] = x[n+1] - x[n]

% TK operator on y[n]
ynp1 = [y 0 0]; % y[n+1]
yn = [0 y 0]; % y[n]
ynm1 = [0 0 y]; % y[n-1]
Psi_y = yn.*yn-ynm1.*ynp1;

% TK operator on z[n]
zn = [0 z 0]; % z[n]
znm1 = [0 0 z]; % z[n-1]
Psi_z = zn.*zn-znm1.*znp1;

% TK operator on x[n]
x2np1 = [x2 0 0]; % x2[n+1]
```
\[ x_{2n} = [0 \ x2 \ 0]; \quad \% \ x2[n] \]
\[ x_{2n-1} = [0 \ 0 \ x2]; \quad \% \ x2[n-1] \]
\[ \Psi_{x2} = x_{2n} \times x_{2n} - x_{2n-1} \times x_{2n+1}; \]

\% Resize TK outputs to original size of \( x[n] \)
\[ \Psi_y = \Psi_y(3:length(\Psi_y)-2); \]
\[ \Psi_z = \Psi_z(3:length(\Psi_z)-2); \]
\[ \Psi_{x2} = \Psi_{x2}(3:length(\Psi_{x2})-2); \]

\% Estimate the IA and IF
\[ \text{IFhat} = \text{acos}(1 - (\Psi_y + \Psi_z)/(4*\Psi_{x2})/(2*pi)); \]
\[ \text{IAhat} = \sqrt{\Psi_{x2}/((1 - (\Psi_y + \Psi_z)/(4*\Psi_{x2}))^2));} \]

\% Plot the input and output signals
\[ \text{figure}(1); \]
\[ \text{plot}(n,x); \]
\[ \text{axis}([0 \ 250 \ -6 \ 6]); \]
\[ \text{xlabel}('Time index n'); \text{ylabel}('Amplitude'); \]
\[ \text{title}('Input Signal'); \]
\[ \text{plot}(n,\text{IAhat}); \]
\[ \text{xlabel}('Time index n'); \text{ylabel}('IA a[n]'); \]
\[ \text{title}('Instantaneous Amplitude (IA)'); \]
\[ \text{plot}(n,\text{IFhat}); \]
\[ \text{xlabel}('Time index n'); \text{ylabel}('IF \ \omega[n] / (2\pi)'); \]
\[ \text{title}('Instantaneous Frequency (IF)'); \]
Plots obtained by running the program with the input signal $x[n] = 5 \cos[2\pi(0.1)n]$: 
3(c). Plots obtained by running the program on an AM-FM chirp signal: