ECE 4213/5213

DSP

HW 4 SOLUTION

HAVLICEK

2.1c) $T\{x[n]\} = \sum_{k=n-n_0}^{n+n_0} x[k]$ $n_0 \in \mathbb{Z}$, $n_0 > 0$

$\boxed{2.1\text{ c)}-1}$

(1) STABLE: Let $x[n]$ be a bounded input signal.

Then $\exists B \in \mathbb{R}, B>0$, s.t. $|x[n]| \leq B \ \forall n \in \mathbb{Z}$.

Then $|y[n]| = |T\{x[n]\}| = \left|\sum_{k=n-n_0}^{n+n_0} x[k]\right|$

$$\leq \sum_{k=n-n_0}^{n+n_0} |x[k]| \leq \sum_{k=n-n_0}^{n+n_0} B$$

$$= (2n_0+1)B < \infty$$

→ Every bounded input signal produces a bounded output signal

→ The system T is stable.

(2) Causal: Let $n = n_0 = 1$.

Then $y[1] = \sum_{k=0}^{2} x[k] = x[0] + x[1] + x[2]$.

Since $y[1]$ depends on the future value of the input signal $x[2]$, the system T is not causal.

Alternatively, note that

$$y[n] = x[n-n_0] + \cdots + x[n] + \cdots + x[n+n_0]$$

- It follows almost immediately that the system T is an LTI system with impulse response

$$t[n] = \sum_{k=n-n_0}^{n+n_0} \delta[n-k].$$

- For any integer $n_0 > 0$, the impulse response contains the term $\delta[n+n_0]$ which is <u>nonzero</u> for the <u>negative</u> time value $n = -n_0$.

- Since it's <u>not</u> true that $t[n] = 0 \;\forall n < 0$, the system T is <u>not</u> <u>causal</u>.

$\boxed{2.1c)-3}$

(3) LINEAR : Let $x_1[n]$ and $x_2[n]$ be input signals and let $a, b \in \mathbb{C}$ be constants.

Then $y_1[n] = T\{x_1[n]\} = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

and $y_2[n] = T\{x_2[n]\} = \sum_{k=n-n_0}^{n+n_0} x_2[k]$.

Now let $x_3[n] = ax_1[n] + bx_2[n]$.

Then $y_3[n] = T\{x_3[n]\} = \sum_{k=n-n_0}^{n+n_0} x_3[k]$

$$= \sum_{k=n-n_0}^{n+n_0} ax_1[k] + bx_2[k]$$

$$= a\sum_{k=n-n_0}^{n+n_0} x_1[k] + b\sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$= ay_1[n] + by_2[n] \checkmark$$

$\longrightarrow$ The system T is linear.

(4) TIME INVARIANT

Let $x_1[n]$ be an input signal and let $m_0 \in \mathbb{Z}$ be a shift amount.

Then $y_1[n] = T\{x_1[n]\} = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

and $y_1[n-m_0] = \sum_{k=n-m_0-n_0}^{n-m_0+n_0} x_1[k]$.

Now let $x_2[n] = x_1[n-m_0]$.

Then $y_2[n] = T\{x_2[n]\} = \sum_{k=n-n_0}^{n+n_0} x_2[k]$

$$= \sum_{k=n-n_0}^{n+n_0} x_1[k-m_0] \qquad (*)$$

Let $l = k - m_0 \longrightarrow k = l + m_0$

when $k = n - n_0$, $l = n - n_0 - m_0 = n - m_0 - n_0$

when $k = n + n_0$, $l = n + n_0 - m_0 = n - m_0 + n_0$

Then $y_2[n] = (*) = \sum_{l=n-m_0-n_0}^{n-m_0+n_0} x_1[l] = y_1[n-m_0]$ ✓

$\rightarrow$ The system T <u>is</u> <u>time</u> <u>invariant</u>.

(5) MEMORYLESS

Let $n = n_0 = 1$.

Then $y[1] = \sum_{k=0}^{2} x[k] = x[0] + x[1] + x[2]$.

Since $y[1]$, the value of the output signal at $n=1$, depends on $x[0]$ and $x[2]$, which are values of the input signal from <u>other</u> times,

$\rightarrow$ The system T is <u>not</u> <u>memoryless</u>.

2.1f) $T\{x[n]\} = ax[n] + b$

(1) STABLE

Let $x[n]$ be a bounded input signal. Then $\exists B \in \mathbb{R}$ with $B > 0$ such that $|x[n]| \leq B \ \forall n \in \mathbb{Z}$.

Then
$$|y[n]| = |T\{x[n]\}| = |ax[n] + b|$$
$$\leq |ax[n]| + |b|$$
$$\leq |a|B + |b| < \infty .$$

→ Since every bounded input signal produces a bounded output signal, the system T <u>is</u> <u>stable</u>.

(2) CAUSAL

$$y[n] = ax[n] + b .$$

Since $y[n]$, the value of the output signal at time n, depends on $x[n]$, the value of the input signal at time n,

but <u>not</u> on any values of the input signal from times greater than n (which would be <u>future</u> values of the input signal),

→ The system T <u>is</u> <u>causal</u>.

(3) LINEAR

2.1 f) - 2

Let $x_1[n]$ and $x_2[n]$ be two input signals and let $\alpha, \beta \in \mathbb{C}$ be constants.

Then $y_1[n] = T\{x_1[n]\} = a x_1[n] + b$

and $y_2[n] = T\{x_2[n]\} = a x_2[n] + b$.

Let $x_3[n] = \alpha x_1[n] + \beta x_2[n]$.

Then $y_3[n] = T\{x_3[n]\} = a x_3[n] + b$

$$= a\left(\alpha x_1[n] + \beta x_2[n]\right) + b$$

$$= a\alpha x_1[n] + a\beta x_2[n] + b.$$

If $a = b = \alpha = \beta = 1$, then

$$y_3[n] = x_1[n] + x_2[n] + \underline{1}.$$

But $\alpha y_1[n] + \beta y_2[n] = a\alpha x_1[n] + \alpha b + a\beta x_2[n] + \beta b$

$$= x_1[n] + b + x_2[n] + b$$

$$= x_1[n] + x_2[n] + \underline{\underline{2}}.$$

→ Since $y_3[n] \neq \alpha y_1[n] + \beta y_2[n]$, the system T is <u>not</u> <u>linear</u>.

(4) TIME INVARIANT

Let $x_1[n]$ be an input signal and let $n_0 \in \mathbb{Z}$ be a shift amount.

Then $y_1[n] = T\{x_1[n]\} = a x_1[n] + b$.

Then $y_1[n-n_0] = a x_1[n-n_0] + b$.

Now let $x_2[n] = x_1[n-n_0]$,

Then
$$\begin{aligned} y_2[n] = T\{x_2[n]\} &= a x_2[n] + b \\ &= a x_1[n-n_0] + b \\ &= y_1[n-n_0] \checkmark \end{aligned}$$

→ The system T <u>is</u> <u>time</u> <u>invariant</u>.

(5) MEMORYLESS

Since $y[n] = T\{x[n]\} = a x[n] + b$ depends on the current value $x[n]$ of the input signal, but not on any values of the input signal from times other than n,

→ The system T <u>is</u> <u>memoryless</u>.

2.1g) $T\{x[n]\} = x[-n]$

$\boxed{2.1g) - 1}$

(1) STABLE

Let $x[n]$ be a bounded input signal. Then $\exists B \in \mathbb{R}$ with $B > 0$ such that $|x[n]| \leq B \ \forall n \in \mathbb{Z}$.

Then $|y[n]| = |T\{x[n]\}| = |x[-n]| \leq B < \infty$.

→ Since every bounded input signal produces a bounded output signal, the system T <u>is</u> <u>stable</u>.

(2) CAUSAL

Let $n = -1$.

Then $y[-1] = x[+1]$, which depends on a <u>future</u> value of the input signal.

→ The system T is <u>not</u> <u>causal</u>.

(3) LINEAR

Let $x_1[n]$ and $x_2[n]$ be two input signals and let $a, b \in \mathbb{C}$ be two constants. Then

$y_1[n] = T\{x_1[n]\} = x_1[-n]$ and $y_2[n] = T\{x_2[n]\} = x_2[-n]$.

Let $x_3[n] = a x_1[n] + b x_2[n]$.

Then $y_3[n] = T\{x_3[n]\} = x_3[-n] = a x_1[-n] + b x_2[-n]$

$= a y_1[n] + b y_2[n]$ ✓

→ The system T <u>is</u> <u>linear</u>.

(4) TIME INVARIANT

Let $x_1[n]$ be an input signal and let $n_0 = 1$ be a shift amount.

Then $y_1[n] = T\{x_1[n]\} = x_1[-n]$

and $y_1[n-n_0] = x_1[-(n-n_0)] = x_1[-n+n_0] = x_1[-n+1]$.

Now let $x_2[n] = x_1[n-n_0] = x_1[n-1]$.

Then $y_2[n] = x_2[-n] = x_1[-n-1] \neq y_1[n-n_0]$.

→ The system T is <u>not</u> <u>time</u> <u>invariant</u>.

(5) MEMORYLESS

Let $n=1$. Then $y[1] = x[-1]$, which depends on the value of the input signal from a <u>different</u> time.

→ The system T is <u>not</u> <u>memoryless</u>.

2.3) The impulse response is given as

$$h[n] = a^{-n} u[-n], \quad 0 < a < 1.$$

→ This implies $a \in \mathbb{R}$.

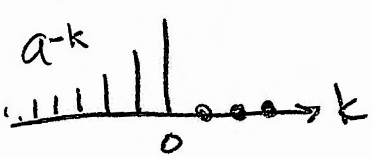
The step response $s[n]$ is the output when the input is $x[n] = u[n]$.

→ The fact that we are given an impulse response implies that the system is LTI. So the output is given by linear convolution. We have

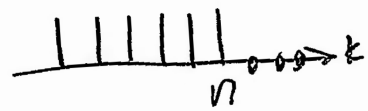
$$s[n] = x[n] * h[n] = u[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$$

For $0 < a < 1$, $a^{-n} u[-n]$ is a decaying exponential to the left that does not alternate. For example, if $a = \frac{1}{2}$, then we've got 2^{n} on the negative n's, which decays to the left as $n \to -\infty$.

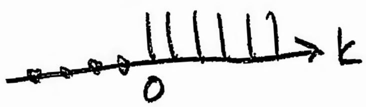
$h[k]$



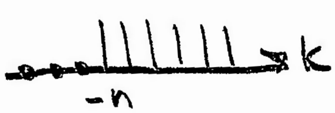
$x[-k - -n] = x[n-k]$



$x[k]$

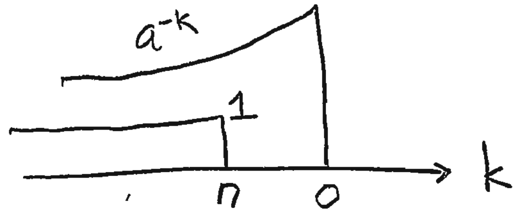


$x[k - -n] = x[n+k]$



2,3)-2

Case I) $n \leq 0$:



The product graph is nonzero from $k=-\infty$ to $k=n$.

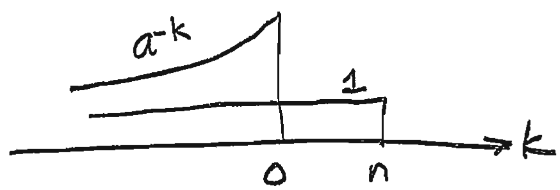
$$s[n] = \sum_{k=-\infty}^{n} h[k]\,x[n-k] = \sum_{k=-\infty}^{n} a^{-k} \cdot 1 = \sum_{k=-\infty}^{n} a^{-k}$$

$$= \sum_{k=-\infty}^{n} \left(\frac{1}{a}\right)^k = \lim_{N \to -\infty} \sum_{k=N}^{n} \left(\frac{1}{a}\right)^k = \lim_{N \to -\infty} \frac{\left(\frac{1}{a}\right)^N - \left(\frac{1}{a}\right)^{n+1}}{1 - \frac{1}{a}}$$

$$= \frac{0 - \left(\frac{1}{a}\right)\left(\frac{1}{a}\right)^n}{1 - \frac{1}{a}} \cdot \frac{-1}{-1} = \frac{\frac{1}{a}\left(\frac{1}{a}\right)^n}{\frac{1}{a} - 1} \cdot \frac{a}{a}$$

$$= \frac{\left(\frac{1}{a}\right)^n}{1-a} = \frac{a^{-n}}{1-a}.$$

Case II) $n > 0$:



$$s[n] = \sum_{k=-\infty}^{0} a^{-k} \cdot 1$$

$$= \sum_{k=-\infty}^{0} a^{-k} = \lim_{N \to -\infty} \sum_{k=N}^{0} \left(\frac{1}{a}\right)^k = \lim_{N \to -\infty} \frac{\left(\frac{1}{a}\right)^N - \frac{1}{a}}{1 - \frac{1}{a}}$$

$$= \frac{0 - \frac{1}{a}}{1 - \frac{1}{a}} \cdot \frac{-1}{-1} = \frac{\frac{1}{a}}{\frac{1}{a} - 1} \cdot \frac{a}{a} = \frac{1}{1-a}$$

All Together: $$s[n] = \begin{cases} \dfrac{a^{-n}}{1-a}, & n \leq 0 \\ \dfrac{1}{1-a}, & n > 0 \end{cases}.$$

//

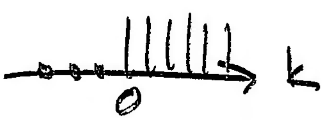
Now, for illustrative purposes, let's work this convolution the "other way" and show that you get the same thing.

$$h[n] = a^{-n} u[-n], \quad 0 < a < 1$$

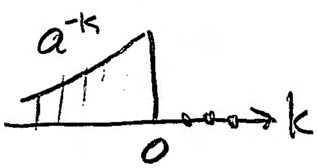
$$x[n] = u[n]$$

$$s[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

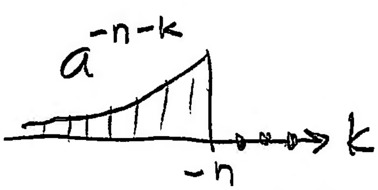
$x[k]$



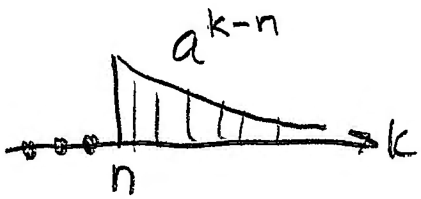
$h[k]$



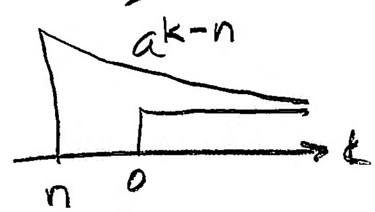
$h[k - -n] = h[n+k]$



$h[-k - -n] = h[n-k]$



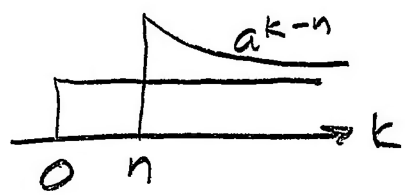
Case I) $n \leq 0$



$$s[n] = \sum_{k=0}^{\infty} a^{k-n} = a^{-n} \sum_{k=0}^{\infty} a^{k}$$

$$= a^{-n} \frac{1}{1-a} = \frac{a^{-n}}{1-a}$$

Case II) $n > 0$:



$$s[n] = \sum_{k=n}^{\infty} a^{k-n}$$

$$= \lim_{N\to\infty} a^{-n} \sum_{k=n}^{N} a^{k} = \lim_{N\to\infty} a^{-n} \frac{a^{n} - a^{N+1}}{1-a}$$

$$= a^{-n} \frac{a^{n} - 0}{1-a} = a^{-n} \frac{a^{n}}{1-a} = \frac{1}{1-a}$$

All Together:

$$s[n] = \begin{cases} \dfrac{a^{-n}}{1-a}, & n \leq 0 \\[2ex] \dfrac{1}{1-a}, & n > 0 \end{cases} \quad //$$

2.7a) $x[n] = e^{j\pi n/6} = e^{j\frac{\pi}{6}n}$ $\boxed{2.7a)-1}$

$$\omega_0 = \frac{\pi}{6}$$

$$\frac{\omega_0}{2\pi} = \frac{\pi/6}{2\pi} = \frac{1}{2\cdot 6} = \frac{1}{12} = \frac{m}{N}$$

$\rightarrow$ periodic, since $\frac{\omega_0}{2\pi} \in \mathbb{Q}$.

Fundamental period $= N = \underline{\underline{12}}$.

$\boxed{2.7d)-1}$

2.7d) $x[n] = e^{j\pi n/\sqrt{2}} = e^{j\frac{\pi}{\sqrt{2}}n}$

$$\omega_0 = \frac{\pi}{\sqrt{2}}$$

$$\frac{\omega_0}{2\pi} = \frac{\pi/\sqrt{2}}{2\pi} = \frac{1}{2\sqrt{2}} \notin \mathbb{Q}.$$

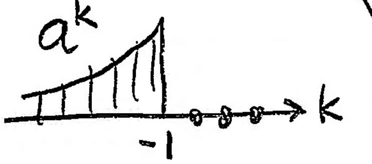
<u>Not</u> <u>Periodic</u> since $\frac{\omega_0}{2\pi} \notin \mathbb{Q}$.

(see Notes pp. 1.45 - 1.47)

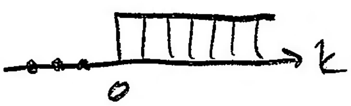
2.10a) $x[n] = u[n] \qquad h[n] = a^n u[-n-1], \; a > 1.$ $\boxed{2.10a)-1}$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

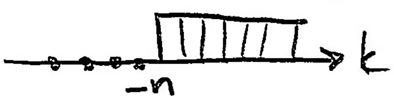
$h[k]$



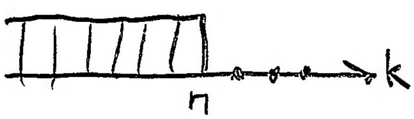
$x[k]$



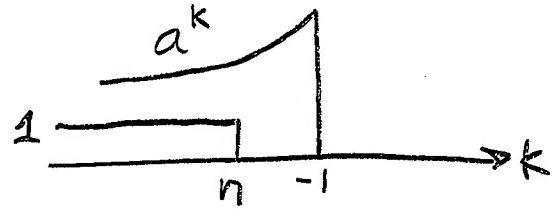
$x[k--n] = x[n+k]$



$x[-k--n] = x[n-k]$



case I) $n \leq -1$:

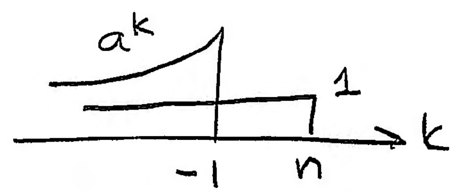


$$y[n] = \sum_{k=-\infty}^{n} a^k$$

$$= \lim_{N \to -\infty} \sum_{k=N}^{n} a^k = \lim_{N \to -\infty} \frac{a^N - a^{n+1}}{1-a}$$

$$= \frac{0 - a^{n+1}}{1-a} = \frac{a a^n}{a-1} \cdot \frac{\frac{1}{a}}{\frac{1}{a}} = \frac{a^n}{1 - \frac{1}{a}}$$

case II) $n > -1$:



$$y[n] = \sum_{k=-\infty}^{-1} a^k$$

$$= \lim_{N \to -\infty} \sum_{k=N}^{-1} a^k = \lim_{N \to -\infty} \frac{a^N - a^0}{1-a} = \frac{0-1}{1-a}$$

$$= \frac{1}{a-1}$$

All Together:

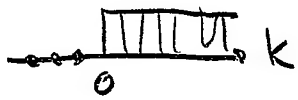
$$y[n] = \begin{cases} \dfrac{a^n}{1-\frac{1}{a}}, & n \leq -1 \\ \dfrac{1}{a-1}, & n > -1 \end{cases}$$

OTHER WAY:

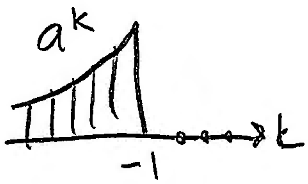
$x[n] = u[n] \qquad h[n] = a^n u[-n-1], \; a > 1.$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

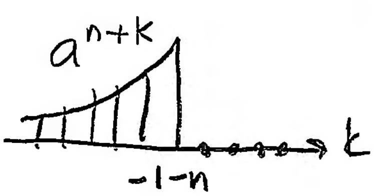
$x[k]$



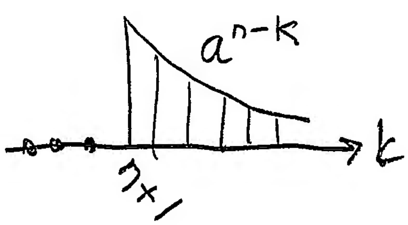
$h[k]$



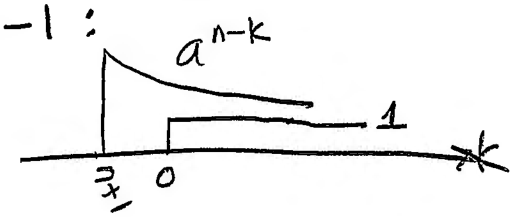
$h[k--n] = h[n+k]$



$h[-k--n] = h[n-k]$



case I) $n+1 \le 0$: $n \le -1$:

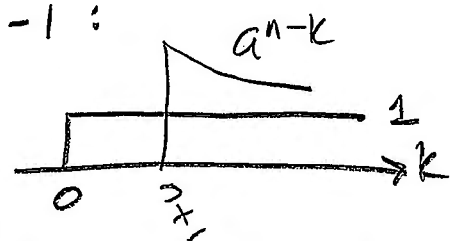


$$y[n] = \sum_{k=0}^{\infty} a^{n-k}$$

$$= a^n \sum_{k=0}^{\infty} a^{-k} = a^n \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^k$$

$$= a^n \frac{1}{1-\frac{1}{a}} = \frac{a^n}{1-\frac{1}{a}}$$

Case II) $n+1 > 0$: $n > -1$:



$$y[n] = \sum_{k=n+1}^{\infty} a^{n-k}$$

$$= a^n \sum_{k=n+1}^{\infty} a^{-k} = a^n \sum_{k=n+1}^{\infty} \left(\frac{1}{a}\right)^k = \lim_{N\to\infty} a^n \sum_{k=n+1}^{N} \left(\frac{1}{a}\right)^k$$

$$= \lim_{N\to\infty} a^n \frac{\left(\frac{1}{a}\right)^{n+1} - \left(\frac{1}{a}\right)^{N+1}}{1-\frac{1}{a}} = a^n \frac{\frac{1}{a}\left(\frac{1}{a}\right)^n}{1-\frac{1}{a}}$$

$$= \frac{\frac{1}{a}}{1-\frac{1}{a}} \cdot \frac{a}{a} = \frac{1}{a-1}$$

All Together:

$$y[n] = \begin{cases} \dfrac{a^n}{1-\frac{1}{a}}, & n \le -1 \\[2ex] \dfrac{1}{a-1}, & n > -1 \end{cases}$$

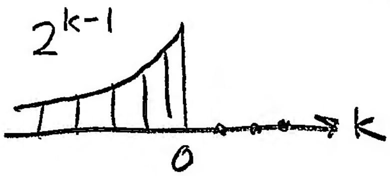
2.10 c) $x[n] = u[n]$

$$h[n] = \left(\tfrac{1}{2}\right)2^{n}u[-n] = 2^{n-1}u[-n]$$

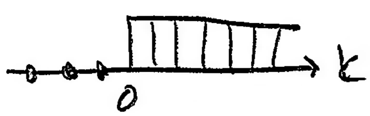
→ Ignore the comment at the end of problem 2.10 and just work the convolution using the steps on pages 2.52 – 2.53 of the notes.

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

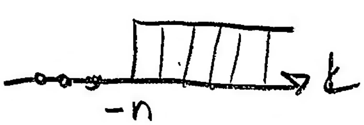
$h[k]$



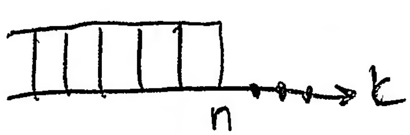
$x[k]$



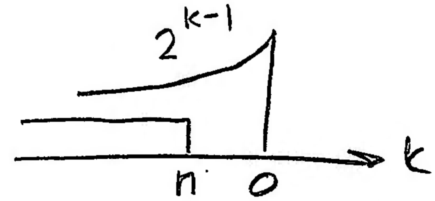
$x[k--n] = x[n+k]$



$x[-k--n] = x[n-k]$



case I) $n \le 0$:

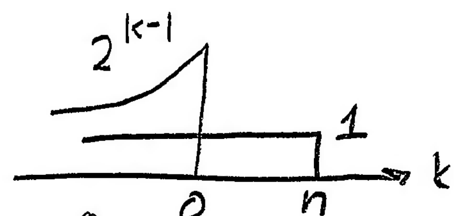


$$y[n] = \sum_{k=-\infty}^{n} 2^{k-1} = \frac{1}{2}\sum_{k=-\infty}^{n} 2^{k} = \lim_{N\to-\infty} \frac{1}{2}\sum_{k=N}^{n} 2^{k}$$

$$= \lim_{N\to-\infty} \frac{1}{2}\,\frac{2^{N} - 2^{n+1}}{1-2} = \frac{1}{2}\,\frac{0 - 2\cdot 2^{n}}{1-2}$$

$$= \frac{1}{2}\,\frac{-2\cdot 2^{n}}{-1} = 2^{n}$$

case II) $n > 0$:



$$y[n] = \sum_{k=-\infty}^{0} 2^{k-1} = \frac{1}{2}\lim_{N\to-\infty}\sum_{k=N}^{0} 2^{k} = \frac{1}{2}\lim_{N\to-\infty}\frac{2^{N}-2^{1}}{1-2}$$

$$= \frac{1}{2}\,\frac{0-2}{-1} = \frac{1}{2}\cdot 2 = 1$$

All Together:

$$y[n] = \begin{cases} 2^{n}, & n \le 0 \\ 1, & n > 0 \end{cases}$$

$$= 2^{n}u[-n] \quad //$$

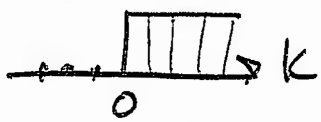
"OTHER WAY"

$x[n] = u[n]$

$h[n] = 2^{n-1}u[-n]$

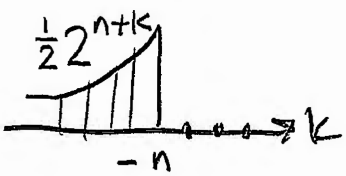
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$x[k]$

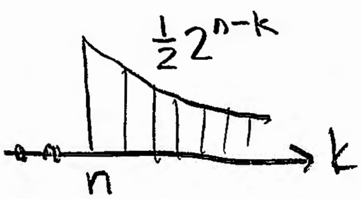


$h[k]$

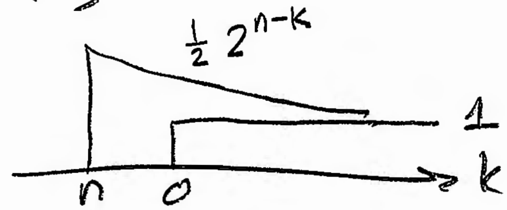
$h[k--n] = h[n+k]$



$h[-k--n] = h[n-k]$



case I) $n \le 0$:



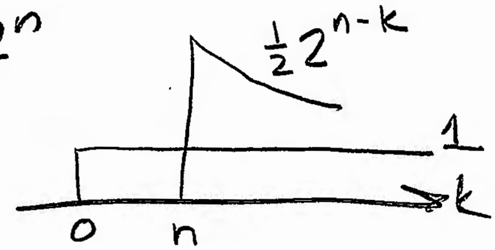
$$y[n] = \sum_{k=0}^{\infty} \frac{1}{2}2^{n-k}$$

$$= \frac{1}{2}2^n \sum_{k=0}^{\infty} 2^{-k} = 2^{n-1}\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k$$

$$= 2^{n-1}\frac{1}{1-\frac{1}{2}} = 2^{n-1}\frac{1}{\frac{1}{2}}$$

$$= 2^{n-1} \cdot 2 = 2^n$$

case II) $n > 0$



$$y[n] = \sum_{k=n}^{\infty} \frac{1}{2} \cdot 2^{n-k}$$

$$= \frac{1}{2} \cdot 2^n \sum_{k=n}^{\infty} 2^{-k} = 2^{n-1} \lim_{N \to \infty} \sum_{k=n}^{N} \left(\frac{1}{2}\right)^k$$

$$= 2^{n-1} \lim_{N \to \infty} \frac{\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^{N+1}}{1-\frac{1}{2}} = 2^{n-1}\frac{\left(\frac{1}{2}\right)^n - 0}{\frac{1}{2}}$$

$$= 2 \cdot 2^{n-1} \cdot \left(\frac{1}{2}\right)^n = 2^n\left(\frac{1}{2}\right)^n = 1$$

All Together:

$$y[n] = \begin{cases} 2^n, & n \le 0 \\ 1, & n > 0 \end{cases} = \underline{\underline{2^n u[-n]}}$$

2.18d)

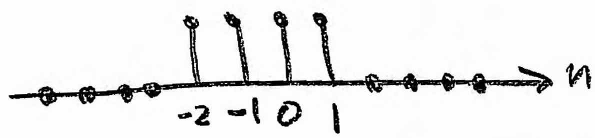
$h[n] = u[n+2] - u[n-2]$

→ $u[n+2]$ turns on at $n=-2$

→ $u[n-2]$ turns on at $n=2$

→ So $h[n]$ goes from $n=-2$ to $n=1$

$h[n]$



→ A discrete-time LTI system is causal iff $h[n]=0 \;\; \forall \; n<0$.

→ In this case, we have $h[-1] = 1 \neq 0$. X

The system is <u>not</u> <u>causal</u>.

2.19a) $h[n] = 4^n u[n]$

→ A discrete-time LTI system is stable iff $h[n] \in \ell^1(\mathbb{Z})$.

That is, iff $\|h\|_{\ell^1} = \sum_{n=-\infty}^{\infty} |h[n]| < \infty$.

→ In this case, we have

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} 4^n = \lim_{N\to\infty} \sum_{n=0}^{N} 4^n$$

$$= \lim_{N\to\infty} \frac{1-4^{N+1}}{1-4} = \lim_{N\to\infty} \frac{1-4\cdot 4^N}{-3}$$

$$= \lim_{N\to\infty} \frac{4\cdot 4^N - 1}{3} = \lim_{N\to\infty} \frac{4}{3}\cdot 4^N - \frac{1}{3}$$

$$= \frac{4}{3} \lim_{N\to\infty} 4^N - \frac{1}{3} \longrightarrow \infty$$

<u>Not Stable</u>

2.23b) $T\{x[n]\} = x[n^2]$

(1) STABLE

Let $x[n]$ be a bounded input signal. Then $\exists B \in \mathbb{R}$ with $B > 0$ such that $|x[n]| \le B \quad \forall n \in \mathbb{Z}$.

Then $|y[n]| = |x[n^2]| \le B < \infty$.

The system T is stable.

(2) CAUSAL

Let $n = -1$. Then $y[-1] = x[(-1)^2] = x[1]$.

Since the value of the output signal at $n=-1$ depends on the future value of the input signal at $n=1$,

the system T is not causal.

(3) LINEAR

Let $x_1[n]$ and $x_2[n]$ be two input signals and let $a, b \in \mathbb{C}$ be constants.

Then $y_1[n] = T\{x_1[n]\} = x_1[n^2]$

and $y_2[n] = T\{x_2[n]\} = x_2[n^2]$.

Now let $x_3[n] = a x_1[n] + b x_2[n]$.

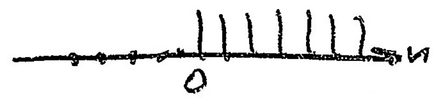
Then $y_3[n] = T\{x_3[n]\} = x_3[n^2]$

$= a x_1[n^2] + b x_2[n^2] = a y_1[n] + b y_2[n]$ ✓

The system T is linear.

(4) TIME INVARIANT

Let the input signal be $x_1[n] = u[n]$



and let the shift amount be $n_0 = 2$.

Then $y_1[n] = T\{x_1[n]\} = x_1[n^2] = u[n^2]$.

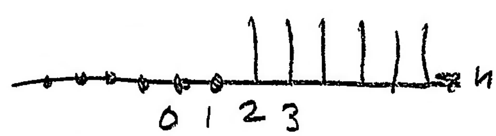
Since n^2 is always ≥ 0, this gives us

$$y_1[n] = 1 \quad \forall\ n.$$

Shifting this to the right by $n_0 = 2$ does not change the graph at all, so we get

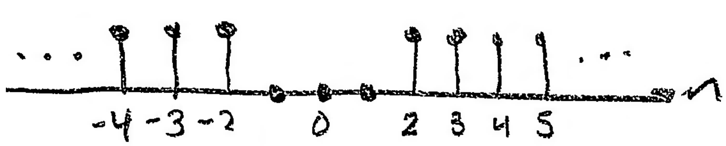
$$y_1[n-2] = 1 \quad \forall\ n.$$

Now let $x_2[n] = x_1[n-2] = u[n-2]$



Then $y_2[n] = T\{x_2[n]\} = x_2[n^2]$

=



Since $y_2[n] \neq y_1[n-n_0]$, the system is <u>not time invariant</u>.