ECE 4213/5213

DSP

HW 5 SOLUTION

HAVLICEK

1) $h(t) = (2\pi\sigma^2)^{-\frac{1}{2}} \exp\left[-(t-\mu)^2/(2\sigma^2)\right]$

$$H(\Omega) = \mathcal{F}\{h(t)\} = \int_{\mathbb{R}} h(t)\, e^{-j\Omega t}\, dt$$

$$= (2\pi\sigma^2)^{-\frac{1}{2}} \int_{\mathbb{R}} e^{-(t-\mu)^2/(2\sigma^2)}\, e^{-j\Omega t}\, dt$$

$$= (2\pi\sigma^2)^{-1/2} \int_{\mathbb{R}} \exp\left[\frac{-(t-\mu)^2}{2\sigma^2} - j\Omega t\right] dt$$

$$= (2\pi\sigma^2)^{-1/2} \int_{\mathbb{R}} \exp\left[-\frac{t^2 - 2t\mu + \mu^2}{2\sigma^2} - j\Omega t\right] dt$$

$$= (2\pi\sigma^2)^{-1/2} \int_{\mathbb{R}} \exp\left[-\frac{1}{2\sigma^2}t^2 + \frac{\mu}{\sigma^2}t - \frac{\mu^2}{2\sigma^2} - j\Omega t\right] dt$$

$$= (2\pi\sigma^2)^{-1/2} \int_{\mathbb{R}} \exp\left[-\frac{1}{2\sigma^2}t^2 + \left(\frac{\mu}{\sigma^2} - j\Omega\right)t - \frac{\mu^2}{2\sigma^2}\right] dt \quad (*)$$

Let $a = \frac{1}{2\sigma^2}$, $b = \left(j\Omega - \frac{\mu}{\sigma^2}\right)$, $c = \frac{\mu^2}{2\sigma^2}$

Then $(*) = (2\pi\sigma^2)^{-1/2} \int_{\mathbb{R}} e^{-(at^2 + bt + c)}\, dt \quad (**)$

$\Rightarrow$ My math handbook (Schaum's) says:

$$\int_{-\infty}^{\infty} e^{-(ax^2 + bx + c)}\, dx = \sqrt{\frac{\pi}{a}}\, e^{(b^2 - 4ac)/4a}$$

$\Rightarrow$ Applying this to $(**)$, we get

$$H(\Omega) = (2\pi\sigma^2)^{-1/2} (\pi)^{1/2} (2\sigma^2)^{1/2} \exp\left[\left\{\left(j\Omega - \frac{\mu}{\sigma^2}\right)^2 - 4(2\sigma^2)^{-1}\left(\frac{\mu^2}{2\sigma^2}\right)\right\} \frac{1}{4}(2\sigma^2)\right]$$

$\longrightarrow$

1-2

$$= \exp\left[\left(j\Omega - \frac{\mu}{\sigma^2}\right)^2 \left(\frac{1}{2}\sigma^2\right) - \frac{\mu^2}{2\sigma^2} \right]$$

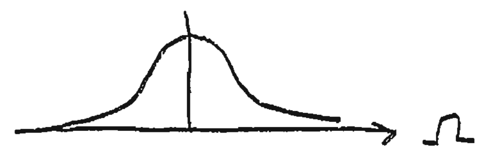
$$= \exp\left[\left\{ (j\Omega)^2 - 2j\frac{\Omega\mu}{\sigma^2} + \frac{\mu^2}{\sigma^4} \right\} \frac{1}{2}\sigma^2 - \frac{\mu^2}{2\sigma^2} \right]$$

$$= \exp\left[-\Omega^2 \frac{1}{2}\sigma^2 - j\Omega\mu + \frac{\mu^2}{2\sigma^2} - \frac{\mu^2}{2\sigma^2} \right]$$

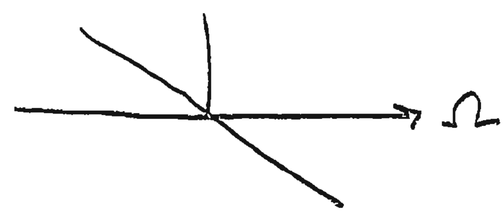
$$\boxed{H(\Omega) = e^{-\frac{1}{2}\sigma^2\Omega^2} e^{-j\mu\Omega}}$$

b) Comparing $H(\Omega) = A(\Omega)\, e^{j\theta(\Omega)}$ directly to the result in part (a), we have

$$A(\Omega) = e^{-\frac{1}{2}\sigma^2\Omega^2}$$



$$\theta(\Omega) = -\mu\Omega \quad //$$



c) $$\tau(\Omega) = -\frac{d}{d\Omega}\theta(\Omega)$$

$$= -\frac{d}{d\Omega}\left[-\mu\Omega\right] = \mu \quad //$$

→ Note that the "mean" μ, which shifts $h(t)$ in the time domain, comes out as a linear phase term in the Fourier transform... which it <u>must</u> according to the F.T. time shift property!!

$\boxed{1-3}$

d) if $\mu = 0$ then there is no time shift on $h(t)$ and the linear phase term in $H(\Omega)$ is reduced to $e^{-j\mu\Omega} = 1$, giving

$$H(\Omega) = e^{-\frac{1}{2}\sigma^2\Omega^2},$$ which is Gaussian.

So the required condition is:

$$\boxed{\mu = 0}$$

2) <u>Theorem:</u> if $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\Omega)$,

then $x(t-t_0) \overset{\mathcal{F}}{\longleftrightarrow} X(\Omega)e^{-j\Omega t_0}$

Note: since t_0 is a time shift, it is assumed that $t_0 \in \mathbb{R}$ is a constant.

<u>Proof</u>:

$$\mathcal{F}\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(t-t_0)e^{-j\Omega t}\,dt \qquad (*)$$

Let $u = t - t_0$.

Then $du = dt$ and $t = u + t_0$.

when $t \to \infty$, $u \to \infty$.

when $t \to -\infty$, $u \to -\infty$.

$$\text{So } (*) = \int_{-\infty}^{\infty} x(u)e^{-j\Omega(u+t_0)}\,du$$

$$= \int_{-\infty}^{\infty} x(u)\,e^{-j\Omega u}e^{-j\Omega t_0}\,du$$

$$= e^{-j\Omega t_0}\int_{-\infty}^{\infty} x(u)\,e^{-j\Omega u}\,du$$

$$= e^{-j\Omega t_0}X(\Omega).$$

<u>QED</u>

3) <u>Theorem:</u> if $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\Omega)$ $\boxed{3\text{-}1}$

then $\frac{d}{dt} x(t) \overset{\mathcal{F}}{\longleftrightarrow} j\Omega X(\Omega)$.

<u>Proof:</u> Let $x(t)$ be differentiable with $x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\Omega)$.

Then $x(t) = \mathcal{F}^{-1}\{X(\Omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t}\, d\Omega$.

Then
$$\frac{d}{dt} x(t) = \frac{d}{dt}\left\{ \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t}\, d\Omega \right\}$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega) \frac{d}{dt}\left\{e^{j\Omega t}\right\} d\Omega$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, j\Omega\, e^{j\Omega t}\, d\Omega$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left\{ j\Omega X(\Omega)\right\} e^{j\Omega t}\, d\Omega$$
$$= \mathcal{F}^{-1}\left\{ j\Omega X(\Omega)\right\}.$$

So $\frac{d}{dt} x(t) \overset{\mathcal{F}}{\longleftrightarrow} j\Omega X(\Omega)$.

<u>QED</u>

2.6 b)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j2\omega}}$$

$$Y(e^{j\omega})\left[1 + \tfrac{1}{2}e^{-j\omega} + \tfrac{3}{4}e^{-j2\omega}\right] = X(e^{j\omega})\left[1 - \tfrac{1}{2}e^{-j\omega} + e^{-j3\omega}\right]$$

$$Y(e^{j\omega}) + \tfrac{1}{2}e^{-j\omega}Y(e^{j\omega}) + \tfrac{3}{4}e^{-j2\omega}Y(e^{j\omega})$$

$$= X(e^{j\omega}) - \tfrac{1}{2}e^{-j\omega}X(e^{j\omega}) + e^{-j3\omega}X(e^{j\omega})$$

→ Apply IDTFT and the time shift property:

$$y[n] + \tfrac{1}{2}y[n-1] + \tfrac{3}{4}y[n-2] = x[n] - \tfrac{1}{2}x[n-1] + x[n-3]$$

<u>Note</u>: you are not asked to find $h[n]$. But if you <u>were</u> asked to find it, then it would be important to realize that $H(e^{j\omega})$ is an <u>improper</u> <u>fraction</u>. So you <u>cannot</u> apply partial fractions directly! You have to do long division first to clear the improper fraction, then apply partial fractions to the remainder.

☆☆ REVIEW THIS IF YOU HAVE FORGOTTEN HOW TO DO IT !!!!

2.8) $h[n] = 5\left(-\frac{1}{2}\right)^n u[n]$

$x[n] = \left(\frac{1}{3}\right)^n u[n]$

Table: $H(e^{j\omega}) = \dfrac{5}{1+\frac{1}{2}e^{-j\omega}}$

Table: $X(e^{j\omega}) = \dfrac{1}{1-\frac{1}{3}e^{-j\omega}}$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{5}{(1-\frac{1}{3}e^{-j\omega})(1+\frac{1}{2}e^{-j\omega})}$$

$$= \frac{A}{1-\frac{1}{3}e^{-j\omega}} + \frac{B}{1+\frac{1}{2}e^{-j\omega}}$$

$$A = \left.\frac{5}{1+\frac{1}{2}\theta}\right|_{\theta=3} = \frac{5}{1+3/2} = 5/\tfrac{5}{2} = 2$$

$$B = \left.\frac{5}{1-\frac{1}{3}\theta}\right|_{\theta=-2} = \frac{5}{1+2/3} = \frac{5}{5/3} = 3$$

$$Y(e^{j\omega}) = \frac{2}{1-\frac{1}{3}e^{-j\omega}} + \frac{3}{1+\frac{1}{2}e^{-j\omega}}$$

Table: $\boxed{y[n] = 2\left(\frac{1}{3}\right)^n u[n] + 3\left(-\frac{1}{2}\right)^n u[n]}$

2.9a) $\boxed{2.9a)-1}$

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = \frac{1}{3}x[n-1]$$

DTFT: $Y(e^{j\omega})\left[1 - \frac{5}{6}e^{-j\omega} + \frac{1}{6}e^{-j2\omega}\right] = \frac{1}{3}e^{-j\omega}X(e^{j\omega})$

$$\boxed{H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\frac{1}{3}e^{-j\omega}}{1 - \frac{5}{6}e^{-j\omega} + \frac{1}{6}e^{-j2\omega}}}$$

$$H(e^{j\omega}) = \frac{\frac{1}{3}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})} = \frac{A}{1-\frac{1}{3}e^{-j\omega}} + \frac{B}{1-\frac{1}{2}e^{-j\omega}}$$

$$A = \left.\frac{\frac{1}{3}\theta}{1-\frac{1}{2}\theta}\right|_{\theta=3} = \frac{\frac{1}{3}\cdot 3}{1-\frac{3}{2}} = \frac{1}{-\frac{1}{2}} = -2$$

$$B = \left.\frac{\frac{1}{3}\theta}{1-\frac{1}{3}\theta}\right|_{\theta=2} = \frac{2/3}{1-2/3} = \frac{2/3}{1/3} = 2$$

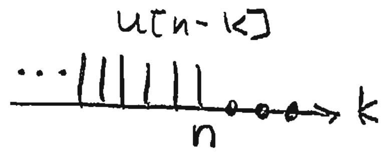
$$H(e^{j\omega}) = \frac{-2}{1-\frac{1}{3}e^{-j\omega}} + \frac{2}{1-\frac{1}{2}e^{-j\omega}}$$

Table: $\boxed{h[n] = -2\left(\frac{1}{3}\right)^n u[n] + 2\left(\frac{1}{2}\right)^n u[n]}$

$\longrightarrow$

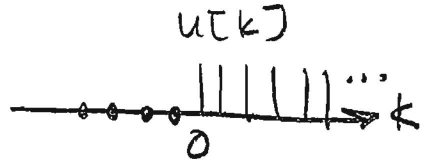
Notes p. 2-86: step response $= s[n] = u[n] * h[n]$ $\boxed{2.9a)-2}$

$$s[n] = \sum_{k=-\infty}^{\infty} h[k]\,u[n-k]$$



$$= \sum_{k=-\infty}^{n} h[k]$$

$$= \sum_{k=-\infty}^{n} -2\left(\tfrac{1}{3}\right)^k u[k] + 2\left(\tfrac{1}{2}\right)^k u[k] \quad (*)$$



→ If $n<0$, then $(*) = 0$ because there are <u>no</u> nonzero terms in the sum.

→ If $n \geq 0$, then

$$(*) = -2\sum_{k=0}^{n}\left(\tfrac{1}{3}\right)^k + 2\sum_{k=0}^{n}\left(\tfrac{1}{2}\right)^k$$

apply the sum formula $\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1-\alpha}$ (see course formula sheet)

$$= -2\,\frac{\left(\frac{1}{3}\right)^0 - \left(\frac{1}{3}\right)^{n+1}}{1 - 1/3} + 2\,\frac{\left(\frac{1}{2}\right)^0 - \left(\frac{1}{2}\right)^{n+1}}{1 - 1/2}$$

$$= -2\,\frac{1 - \frac{1}{3}\left(\frac{1}{3}\right)^n}{2/3} + 2\,\frac{1 - \frac{1}{2}\left(\frac{1}{2}\right)^n}{1/2}$$

$$= -2\left(\tfrac{3}{2}\right)\left[1 - \tfrac{1}{3}\left(\tfrac{1}{3}\right)^n\right] + 2(2)\left[1 - \tfrac{1}{2}\left(\tfrac{1}{2}\right)^n\right]$$

$$= -3\left[1 - \tfrac{1}{3}\left(\tfrac{1}{3}\right)^n\right] + 4\left[1 - \tfrac{1}{2}\left(\tfrac{1}{2}\right)^n\right]$$

$$= -3 + \left(\tfrac{1}{3}\right)^n + 4 - 2\left(\tfrac{1}{2}\right)^n = 1 + \left(\tfrac{1}{3}\right)^n - 2\left(\tfrac{1}{2}\right)^n.$$

All Together:

$$\boxed{s[n] = \left[1 + \left(\tfrac{1}{3}\right)^n - 2\left(\tfrac{1}{2}\right)^n\right] u[n]}$$

2.11) $H(e^{j\omega}) = \dfrac{1-e^{-j2\omega}}{1+\frac{1}{2}e^{-j4\omega}}$, $-\pi < \omega \le \pi$

$$x[n] = \sin\frac{\pi}{4}n = \frac{e^{j\pi/4\, n} - e^{-j\pi/4\, n}}{2j}$$

$$= \frac{1}{2j}e^{j\frac{\pi}{4}n} - \frac{1}{2j}e^{-j\pi/4\, n} \qquad (*)$$

→ The input signal is a sum of two eigenfunctions with frequencies $\omega = \frac{\pi}{4}$ and $\omega = -\pi/4$.

→ $H(e^{j\omega})$ is conjugate symmetric. This implies that $h[n]$ is real. So it follows from pages 3.113 - 3.114 of the notes that

$$y[n] = A(\tfrac{\pi}{4})\sin\left[\tfrac{\pi}{4}n + \theta(\tfrac{\pi}{4})\right] \qquad (**)$$

where $A(\frac{\pi}{4}) = \left|H(e^{j\omega})\right|_{\omega=\pi/4}$

and $\theta(\frac{\pi}{4}) = \arg H(e^{j\omega})\Big|_{\omega=\pi/4}$

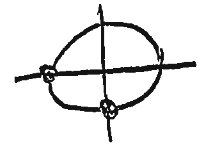
→ But let's solve it from (*) above and show that this is true.

→ First, we need to find the eigenvalues of the system for the eigenfunctions with $\omega = \pi/4$ and $\omega = -\pi/4$.

⟶

2.11-2

$$H(e^{j\pi/4}) = \frac{1-e^{-j2\pi/4}}{1+\frac{1}{2}e^{-j4\pi/4}}$$



$e^{-j\pi/2} = -j$

$e^{-j\pi} = -1$

$$= \frac{1-e^{-j\pi/2}}{1+\frac{1}{2}e^{-j\pi}}$$

$$= \frac{1+j}{1-1/2} = \frac{1+j}{1/2} = 2+2j$$

$$|H(e^{j\pi/4})| = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{2\cdot 4} = 2\sqrt{2}$$

$$\arg H(e^{j\pi/4}) = \arctan \frac{2}{2} = \arctan 1 = \pi/4$$

$\rightarrow$ So $H(e^{j\pi/4}) = 2\sqrt{2}\, e^{j\pi/4}$

$\rightarrow$ Since $H(e^{j\omega})$ is conjugate symmetric, it follows that $H(e^{-j\pi/4}) = 2\sqrt{2}\, e^{-j\pi/4}$.

$\rightarrow$ So $y[n] = H\{x[n]\} = H\{\frac{1}{2j}e^{j\pi/4 n}\} - H\{\frac{1}{2j}e^{-j\pi/4 n}\}$

$$= \frac{1}{2j} H\{e^{j\pi/4 n}\} - \frac{1}{2j} H\{e^{-j\pi/4 n}\}$$

$$= \frac{1}{2j} 2\sqrt{2}\, e^{j\pi/4} e^{j\pi/4 n} - \frac{1}{2j} 2\sqrt{2}\, e^{-j\pi/4} e^{-j\pi/4 n}$$

$$= 2\sqrt{2}\left[\frac{e^{j(\pi/4 n + \pi/4)} - e^{-j(\pi/4 n + \pi/4)}}{2j}\right]$$

$$\boxed{y[n] = 2\sqrt{2}\sin\left(\frac{\pi}{4}n + \frac{\pi}{4}\right)}$$

Note that this agrees with (**) on the previous page.

2.13) Which of the following discrete-time signals are eigenfunctions of a stable discrete-time LTI system H ?

a) $x[n] = e^{j\frac{2\pi}{3}n}$

→ It is shown on p. 3-89 of the notes that $e^{j\omega_0 n}$ is an eigenfunction $\forall\ \omega_0 \in \mathbb{R}$. Here, we have $\omega_0 = \frac{2\pi}{3}$. So this $x[n]$ is an eigenfunction.

b) $x[n] = 3^n$.

$$y[n] = H\{x[n]\} = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

$$= \sum_{k=-\infty}^{\infty} 3^{n-k} h[k] = 3^n \sum_{k=-\infty}^{\infty} 3^{-k} h[k]$$

$$= x[n] \underbrace{\sum_{k=-\infty}^{\infty} 3^{-k} h[k]}$$

a number... there's no "n" in it.

→ So this $x[n]$ is an eigenfunction.

d) $x[n] = \cos \omega_0 n = \frac{1}{2} e^{j\omega_0 n} + \frac{1}{2} e^{-j\omega_0 n}$.

Since $e^{j\omega_0 n}$ and $e^{-j\omega_0 n}$ are both eigenfunctions,

$$y[n] = \tfrac{1}{2} H(e^{j\omega_0}) e^{j\omega_0 n} + \tfrac{1}{2} H(e^{-j\omega_0}) e^{-j\omega_0 n}.$$

→ If $H(e^{j\omega})$ is even, then $H(e^{j\omega_0}) = H(e^{-j\omega_0})$ and $x[n]$ is an eigenfunction. But this is not true in general. So the answer is NO ... not an eigenfunction.

2.33) $y[n] = -2x[n] + 4x[n-1] - 2x[n-2]$

a) Since there are no shifts of $y[n]$ in the difference equation (only one AR term), we know that this is an FIR LTI filter.

From the convolution equation, we have

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$
$$= \underset{\substack{\uparrow \\ h[0]}}{-2}x[n] + \underset{\substack{\uparrow \\ h[1]}}{4}x[n-1] \underset{\substack{\uparrow \\ h[2]}}{-2}x[n-2].$$

- It follows immediately that

$$\boxed{h[n] = -2\delta[n] + 4\delta[n-1] - 2\delta[n-2]}$$

b) $H(e^{j\omega}) = \mathrm{DTFT}\{h[n]\}$

$$= -2 + 4e^{-j\omega} - 2e^{-j2\omega}$$
$$= -2\underbrace{e^{j\omega}e^{-j\omega}}_{\text{one}} + 4e^{-j\omega} - 2e^{-j2\omega}$$
$$= -2e^{-j\omega}\left[e^{j\omega} - 2 + e^{-j\omega}\right]$$

$\longrightarrow$

$$\ldots H(e^{j\omega}) = -2e^{-j\omega}\left[2\cos\omega - 2\right]$$

$$= 2e^{-j\omega}\left[2 - 2\cos\omega\right]$$

$$= 4e^{-j\omega}\left[1 - \cos\omega\right]$$

$$\left\{ \sin^2 A = \tfrac{1}{2} - \tfrac{1}{2}\cos 2A \right.$$

$$= 4e^{-j\omega}\left[2\sin^2\left(\tfrac{\omega}{2}\right)\right]$$

$$\rightarrow 2\sin^2\left(\tfrac{\omega}{2}\right) = 1 - \cos\omega$$

$$H(e^{j\omega}) = \left[8\sin^2\left(\tfrac{\omega}{2}\right)\right]e^{-j\omega}$$

$$A(\omega) = 8\sin^2\tfrac{\omega}{2}$$

$$\theta(\omega) = -\omega$$

$$n_d = \tau(\omega) = -\tfrac{d}{d\omega}\theta(\omega) = 1$$

2.34b) Using the first expression given for $H(e^{j\omega})$, we obtain $\boxed{2.34b)-1}$

$$H(e^{j\omega}) = \frac{1-1.25e^{-j\omega}}{1-0.8e^{-j\omega}}$$

$$= \frac{1}{1-0.8e^{-j\omega}} - \frac{1.25e^{-j\omega}}{1-0.8e^{-j\omega}}$$

Table: $\frac{1}{1-0.8e^{-j\omega}} \overset{DTFT}{\longleftrightarrow} (0.8)^n u[n]$

Table: $\frac{-1.25}{1-0.8e^{-j\omega}} \overset{DTFT}{\longleftrightarrow} -1.25(0.8)^n u[n]$

Time Shift property: $\frac{-1.25e^{-j\omega}}{1-0.8e^{-j\omega}} \overset{DTFT}{\longleftrightarrow} -1.25(0.8)^{n-1} u[n-1]$

So,

$$h[n] = (0.8)^n u[n] - 1.25(0.8)^{n-1} u[n-1]$$

$$= (0.8)^n \{\delta[n] + u[n-1]\} - 1.25(0.8)^{n-1} u[n-1]$$

$$= \underset{\substack{\uparrow \\ \text{this is 1 when } n=0}}{(0.8)^n}\delta[n] + (0.8)^n u[n-1] - 1.25(0.8)^{n-1} u[n-1]$$

$$= \delta[n] + 0.8(0.8)^{n-1} u[n-1] - 1.25(0.8)^{n-1} u[n-1]$$

$$= \delta[n] + [0.8 - 1.25](0.8)^{n-1} u[n-1]$$

$$\boxed{h[n] = \delta[n] - 0.45(0.8)^{n-1} u[n-1]}$$

Let us alternatively try using the second expression given for $H(e^{j\omega})$.

- First, you may be wondering where the second expression came from... try dividing the denominator into the numerator:

$$\begin{array}{r|l} & \quad 1 \qquad \leftarrow \text{quotient} = 1 \\ 1-0.8e^{-j\omega} & 1-1.25e^{-j\omega} \\ & 1-\ 0.8e^{-j\omega} \\ \hline & \quad -0.45e^{-j\omega} \qquad \leftarrow \text{remainder} \end{array}$$

- So $H(e^{j\omega}) = \text{quotient} + \dfrac{\text{remainder}}{\text{divisor}}$

$$= 1 - \frac{0.45e^{-j\omega}}{1-0.8e^{-j\omega}} \quad ✓$$

Now,

Table: $1 \overset{DTFT}{\longleftrightarrow} \delta[n]$

Table: $\dfrac{-0.45}{1-0.8e^{-j\omega}} \overset{DTFT}{\longleftrightarrow} -0.45(0.8)^n u[n]$

Time Shift Property: $\dfrac{-0.45e^{-j\omega}}{1-0.8e^{-j\omega}} \overset{DTFT}{\longleftrightarrow} -0.45(0.8)^{n-1} u[n-1]$

$$\boxed{h[n] = \delta[n] - 0.45(0.8)^{n-1} u[n-1]}$$

→ agrees with the first way ✓

11) a) $X_1(e^{j\omega}) = \dfrac{1}{1 - \frac{1}{2}e^{j\omega}}$

Let $\hat{X}_1(e^{j\omega}) = X_1(e^{-j\omega}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\omega}}$

Table: $\hat{x}_1[n] = \left(\frac{1}{2}\right)^n u[n]$

Time Reversal Property: $x_1[n] = \hat{x}_1[-n] = \left(\frac{1}{2}\right)^{-n} u[-n]$

$$\boxed{x_1[n] = 2^n u[-n]}$$

b) $X_2(e^{j\omega}) = \dfrac{-3e^{-j\omega}}{1 - 3e^{-j\omega}} \cdot \dfrac{-\frac{1}{3}e^{j\omega}}{-\frac{1}{3}e^{j\omega}} = \dfrac{1}{-\frac{1}{3}e^{j\omega} + 1}$

$= \dfrac{1}{1 - \frac{1}{3}e^{j\omega}}$

Proceeding exactly as in part (a), we get

$$\boxed{x_2[n] = 3^n u[-n]}$$