ECE 5273
Test 1

Wednesday, March 3, 2004
5:00 PM - 6:15 PM

Spring 2004

Dr. Havlicek

Name: SOLUTION

Student Num: __________

Directions: This is an open book, open notes test. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________

2. (20) ________

3. (20) ________

4. (20) ________

5. (20) ________

TOTAL (100):

1. **20 pts.** The gray scale image $\mathbb{I}_1$ shown below has 8-bit pixels. This image was thresholded to obtain the binary image $\mathbb{I}_2$, which is also shown below. In $\mathbb{I}_2$, the pixel value 255 (WHITE) represents LOGIC_ONE and the pixel value zero (BLACK) represents LOGIC_ZERO.

$\mathbb{I}_1$

$\mathbb{I}_2$

Binary morphological OPEN and CLOSE operations where performed on the image $\mathbb{I}_2$ using a 5×5 diamond-shaped structuring element. The resulting images are shown as $\mathbb{J}_1$ and $\mathbb{J}_2$ below.

$\mathbb{J}_1$

$\mathbb{J}_2$

Determine which image is the result of the OPEN operation and which is the result of the CLOSE operation. Explain your answer.

Workspace for Problem 1...

$\mathbb{J}_1$: OPEN

$\mathbb{J}_2$: CLOSE

- OPEN is Erode first, Dilate second. So zeros first fill in small "white" structures; the Dilate then returns larger "white" objects to original size, but the smaller "white" objects are gone forever. We see in $\mathbb{J}_1$ that the small logical one features of the feather and Lena's eyes are removed by OPEN, but her nose and mouth are not significantly affected.

- CLOSE is Dilate first, Erode second. Ones first fill in the small "black" structures; Erode then returns larger "black" objects to original size, but the smaller "black" features are removed. In $\mathbb{J}_2$, we see that the small "white" features in the feather remain. But Lena's nose and mouth are gone.

2. **20 pts.** Consider the 4×4 image $\mathbb{I}$ shown below, where the allowable range of gray levels is $0 \leq I(i,j) \leq 15$:

$$\mathbb{I} = \begin{array}{|c|c|c|c|} \hline 14 & 11 & 5 & 8 \\ \hline 14 & 2 & 8 & 8 \\ \hline 14 & 5 & 14 & 11 \\ \hline 14 & 14 & 11 & 11 \\ \hline \end{array}$$

Construct a new image $\mathbb{J}$ by applying the histogram flattening algorithm to $\mathbb{I}$. Show the new image $\mathbb{J}$ and its histogram $H_{\mathbb{J}}$ in the spaces provided below:

$$\mathbb{J} = \begin{array}{|c|c|c|c|} \hline 15 & 9 & 3 & 6 \\ \hline 15 & 1 & 6 & 6 \\ \hline 15 & 3 & 15 & 9 \\ \hline 15 & 15 & 9 & 9 \\ \hline \end{array}$$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H_{\mathbb{J}}(k)$	0	1	0	2	0	0	3	0	0	4	0	0	0	0	0	6

$J(i,j) = \text{INT}[15 J_1(i,j) + 0.5]$

J_1	J
16/16	15
10/16	9
6/16	6
3/16	3
1/16	1

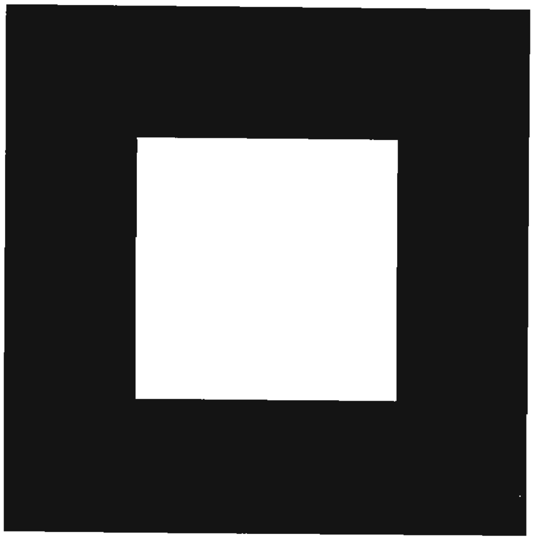
Work Space:

$$J_1 = \begin{array}{|c|c|c|c|} \hline 16/16 & 10/16 & 3/16 & 6/16 \\ \hline 16/16 & 1/16 & 6/16 & 6/16 \\ \hline 16/16 & 3/16 & 16/16 & 10/16 \\ \hline 16/16 & 16/16 & 10/16 & 10/16 \\ \hline \end{array}$$

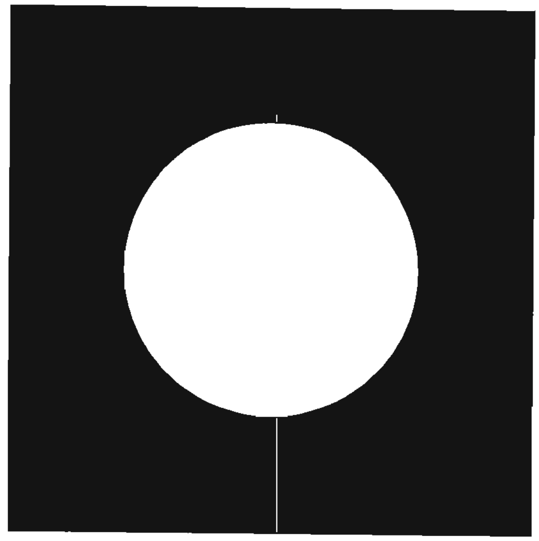
k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H(k)$	0	0	1	0	0	2	0	0	3	0	0	4	0	0	6	0
$p(k)$	0/16	0/16	1/16	0/16	0/16	2/16	0/16	0/16	3/16	0/16	0/16	4/16	0/16	0/16	6/16	0/16

3. **20 pts.** Consider the two 256×256 binary images shown in (a) and (b) below. For these images, WHITE = 255 = LOGIC_ONE and BLACK = 0 = LOGIC_ZERO. Sketch the Medial Axis Transform (morphological skeleton) of each image.

In your sketches, use WHITE or "NO INK" for LOGIC_ZERO and BLACK or "INK" for LOGIC_ONE.

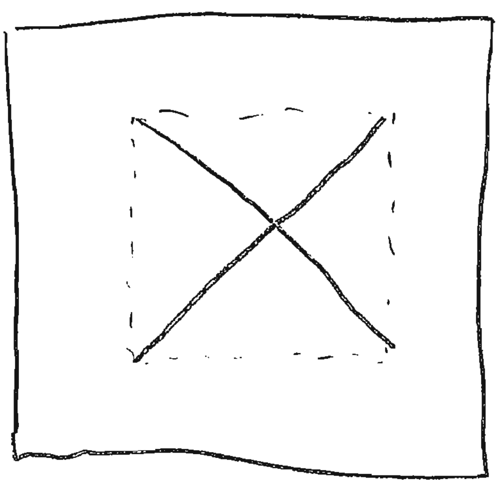


(a)

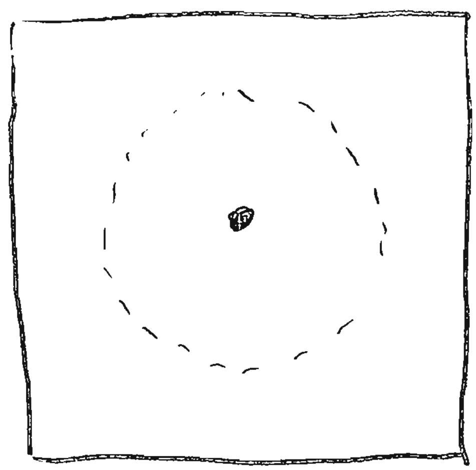


(b)

Hint: Use the "Praire Fire" concept to find the answers qualitatively without having to actually perform any erosions.



(a)



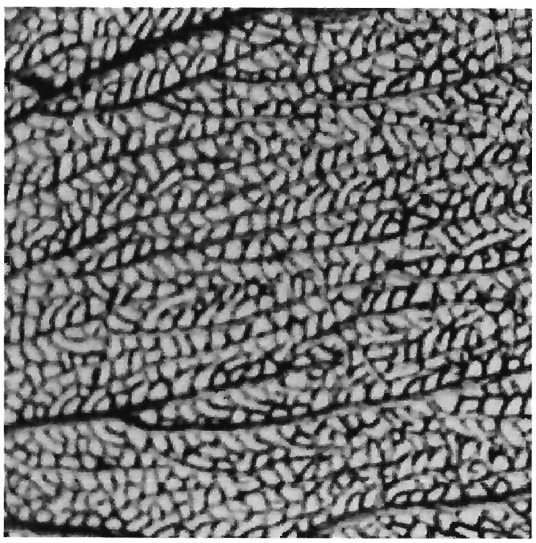
(b)

4. **20 pts**. Match the images $\mathbb{I}_1$ - $\mathbb{I}_4$ shown below with their centered log-magnitude DFT's $\widetilde{\mathbb{I}}_1$ - $\widetilde{\mathbb{I}}_4$ shown on the next page.

$\mathbb{I}_1$

$\mathbb{I}_2$

$\mathbb{I}_3$



$\mathbb{I}_4$

(a) **5 pts**. DFT[$\mathbb{I}_1$] = $\widetilde{\mathbb{I}}_2$

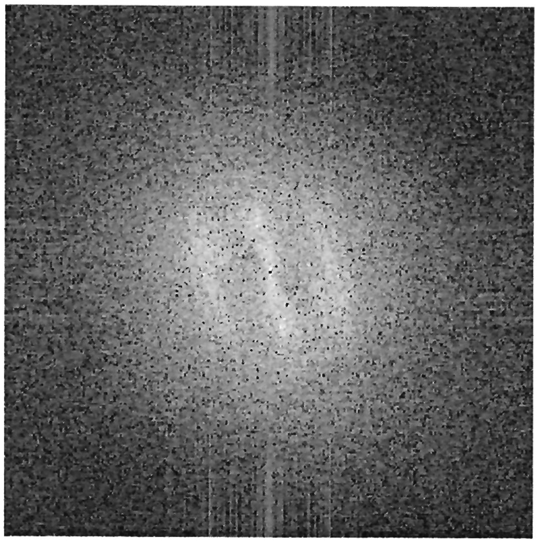
(b) **5 pts**. DFT[$\mathbb{I}_2$] = $\widetilde{\mathbb{I}}_4$

(c) **5 pts**. DFT[$\mathbb{I}_3$] = $\widetilde{\mathbb{I}}_1$

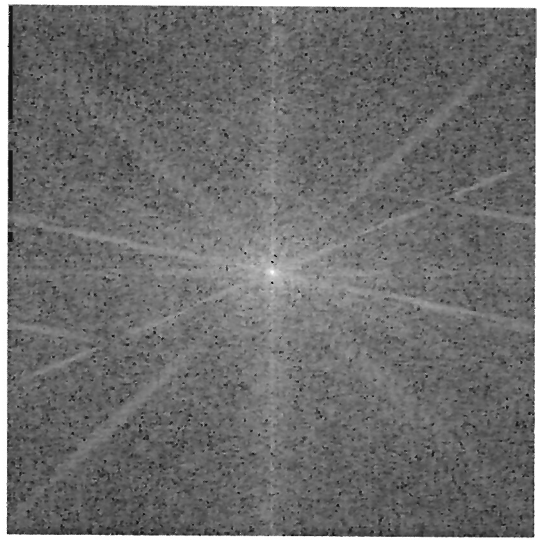
(d) **5 pts**. DFT[$\mathbb{I}_4$] = $\widetilde{\mathbb{I}}_3$

Problem 4, cont…

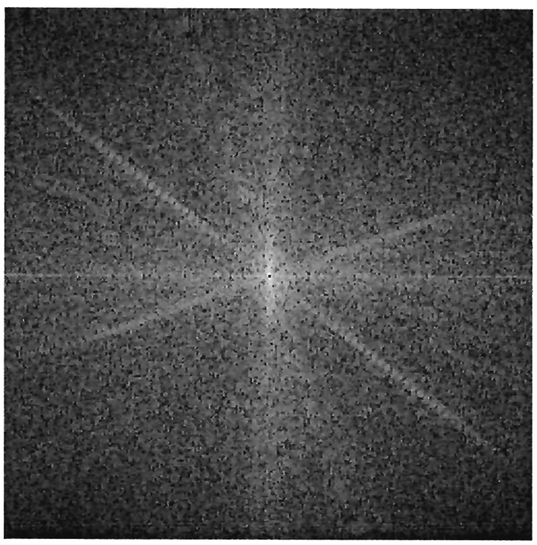
$\widetilde{\mathbb{I}}_1$



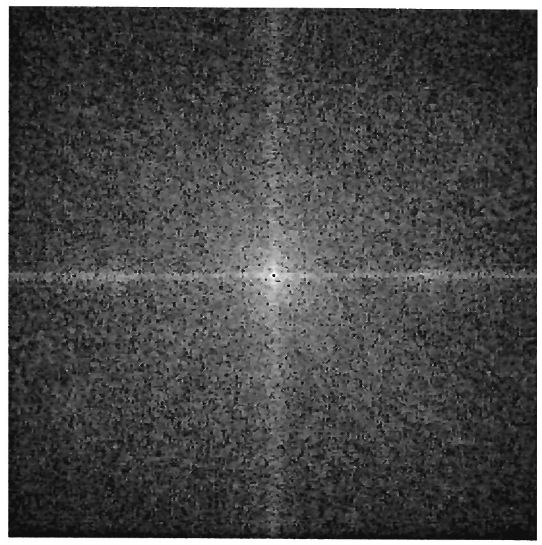
$\widetilde{\mathbb{I}}_2$



$\widetilde{\mathbb{I}}_3$



$\widetilde{\mathbb{I}}_4$



5. **20 pts.** A continuous optical image $I_C(x,y)$ given by

$$I_C(x,y) = e^{-3x-2y}\mu(x)\mu(y)$$

is incident on the focal plane array of an ideal pinhole digital camera, where $\mu(t)$ is the unit step function defined by

$$\mu(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

and where x and y are specified in units of meters.

(a) **15 pts.** Find the continuous Fourier transform $\widetilde{I}_C(\omega_x, \omega_y)$.

(b) **5 pts.** Can the image $I_C(x,y)$ be sampled (converted into a digital image) without aliasing? If so, what are the maximum horizontal and vertical pixel spacings X and Y that will guarantee that no aliasing occurs?

a)

$$\widetilde{I}_c(\omega_x,\omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_c(x,y)\, e^{-2\pi\sqrt{-1}\,(x\omega_x + y\omega_y)}\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-3x}e^{-2y}\mu(x)\mu(y)\,e^{-2\pi\sqrt{-1}\,x\omega_x}\,e^{-2\pi\sqrt{-1}\,y\omega_y}\,dx\,dy$$

$$= \left[\int_0^{\infty} e^{-3x}e^{-2\pi\sqrt{-1}\,x\omega_x}\,dx\right]\left[\int_0^{\infty} e^{-2y}e^{-2\pi\sqrt{-1}\,y\omega_y}\,dy\right]$$

$$= \left[\int_0^{\infty} \exp\left[x(-3-2\pi\sqrt{-1}\,\omega_x)\right]dx\right]\left[\int_0^{\infty} \exp\left[y(-2-2\pi\sqrt{-1}\,\omega_y)\right]dy\right]$$

$$= \frac{1}{-3-2\pi\sqrt{-1}\,\omega_x}\left[e^{x(-3-2\pi\sqrt{-1}\,\omega_x)}\right]_{x=0}^{\infty} \frac{1}{-2-2\pi\sqrt{-1}\,\omega_y}\left[e^{y(-2-2\pi\sqrt{-1}\,\omega_y)}\right]_{y=0}^{\infty}$$

$$= \boxed{\frac{1}{(3+2\pi\sqrt{-1}\,\omega_x)(2+2\pi\sqrt{-1}\,\omega_y)}}$$

b) NO. $\widetilde{I}_c(\omega_x,\omega_y)$ is nonzero $\forall$ finite (ω_x,ω_y). So $I_c(x,y)$ is not bandlimited and aliasing will occur