ECE 5273
Test 1

Wednesday, March 28, 2007
5:00 PM - 6:15 PM

Spring 2007 Name: SOLUTION

Dr. Havlicek Student Num:____________________

Directions: This is an open notes test. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________

2. (25) ________

3. (25) ________

4. (25) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:____________________ Date:____________________

1. ~~20~~ 25 **pts**. The connected components labeling algorithm (blob coloring algorithm) given in the notes is a *4-connected* algorithm. This means that any two LOGIC ONE pixels that are 4-neighbors will always become part of the same connected component (blob).

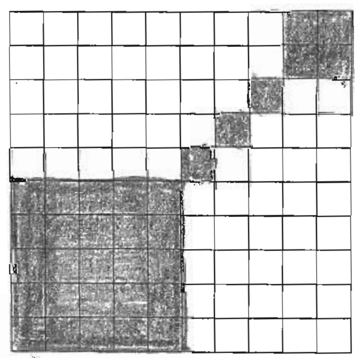
It is also possible to define an 8-connected algorithm, so that any two LOGIC ONE pixels that are 8-neighbors will always end up in the same blob.

Consider the application of connected components labeling with minor region removal. Use the blank 10×10 grid below to construct an image $\mathbb{I}$ such that the results will be different depending on whether a 4-connected or an 8-connected blob coloring algorithm is used.

Indicate LOGIC ONE pixels with the numeral "1" or by "coloring in" (shading). Indicate LOGIC ZERO pixels with the numeral "0" or by "not coloring in."

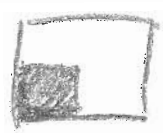
Explain your answer in the space provided below the image.

With the 4-connected algorithm, this image contains 5 blobs:

$\mathbb{I} =$ 

- lower left 5x5 square
- upper right 2x2 square
- Three small 1x1 squares.

⇒ None of them is connected to each other in the 4-connected algorithm, so after labeling, counting, and minor region removal, only the 5x5 square will remain;

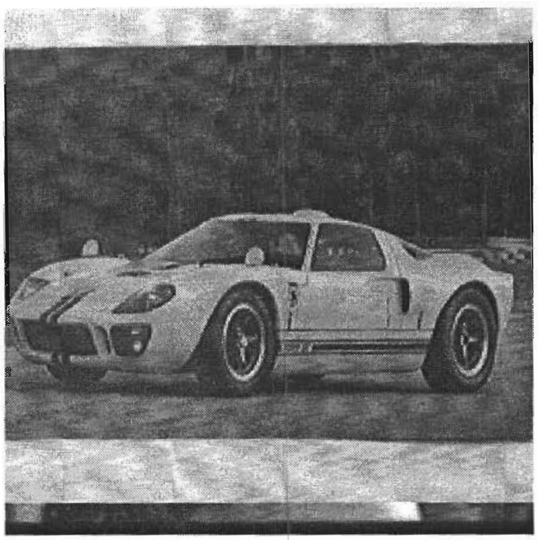


With the 8-connected algorithm, there is only one blob (or "connected component"). It contains all 32 LOGIC-ONE pixels. In this case, the image $\mathbb{I}$ is not changed by the labeling, counting, and minor region removal. In other words, the resulting image is the same as $\mathbb{I}$ shown above.

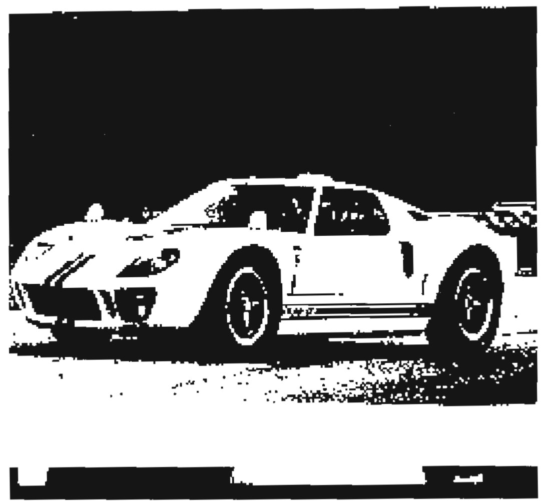
25

2. ~~20~~ **pts**. The gray scale image $\mathbb{I}_1$ shown below has 8-bit pixels. This image was thresholded to obtain the binary image $\mathbb{I}_2$, which is also shown below. In $\mathbb{I}_2$, the pixel value 255 (WHITE) represents LOGIC_ONE and the pixel value zero (BLACK) represents LOGIC_ZERO.

$\mathbb{I}_1$

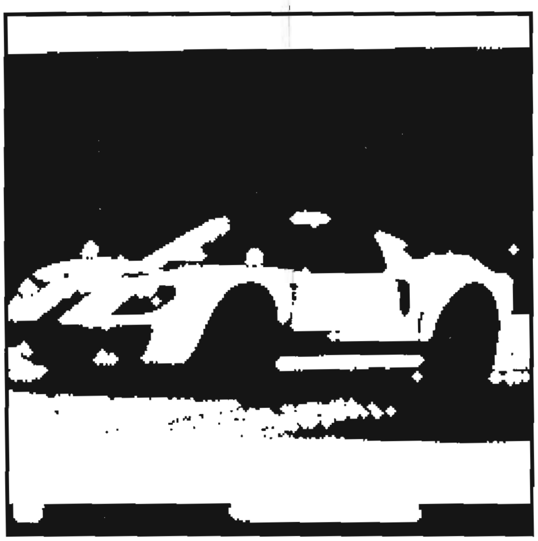


$\mathbb{I}_2$



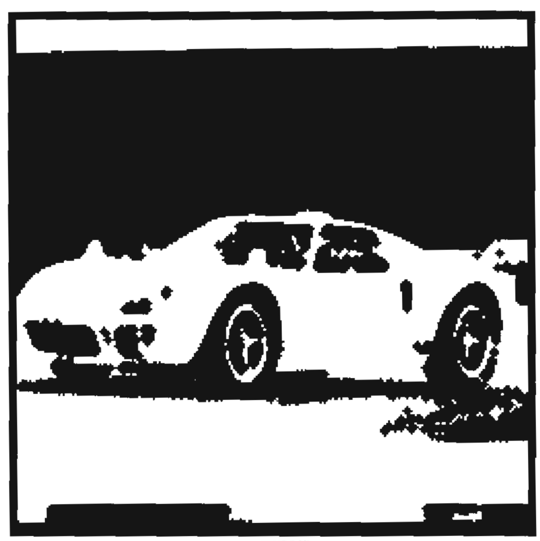
Binary morphological OPEN and CLOSE operations where performed on the image $\mathbb{I}_2$ using a 5×5 diamond-shaped structuring element. The resulting images are shown as $\mathbb{J}_1$ and $\mathbb{J}_2$ below.

$\mathbb{J}_1$



OPEN

$\mathbb{J}_2$



CLOSE

Determine which image is the result of the OPEN operation and which is the result of the CLOSE operation. Explain your answer.

Workspace for Problem 2...

J_1 is the result of "OPEN".

- open applies erode first, which has two main effects:

  ① Large white objects are eroded by about 2 to 3 pixels, becoming smaller.

  ② small white objects are completely eliminated.

- Then open applies dilation. This tends to restore the larger white objects to approximately original size. However, the smaller white objects are permenently removed from the image. This can be seen most clearly in the wheels and windows.

J_2 is the result of "CLOSE." Close applies the dilation first. This expands the white objects by approximately 2 to 3 pixels. As a result,

  ① Larger black objects become smaller by about 2 to 3 pixels.

  ② Smaller black objects are completely eliminated.

- then close applies the erosion. The larger black objects are returned to approximately their original size. However, the smaller black objects are permenently removed.

This can be seen clearly, e.g., in the stripe on the hood and the headlamps.

3. **25 pts.** Consider the 4×4 images $\mathbb{I}$ and $\mathbb{I}'$ shown below, where the allowable range of gray levels is $0 \leq I(i,j), I'(i,j) \leq 15$:

$\mathbb{I} =$

10	**3**	**2**	**1**
4	**3**	**2**	**10**
3	**4**	**9**	**9**
2	**1**	**4**	**9**

$\mathbb{I}' =$

14	**11**	**5**	**8**
14	**2**	**8**	**8**
14	**5**	**14**	**11**
14	**14**	**11**	**11**

Construct a new image $\mathbb{J}$ by applying the histogram matching algorithm to shape the histogram of image $\mathbb{I}$, where the desired shape is given by the histogram of the image $\mathbb{I}'$. Show the new image $\mathbb{J}$ and its histogram $H_{\mathbb{J}}$ in the spaces provided below. Work space is given on the next page.

$\mathbb{J} =$

14	11	8	5
14	11	8	14
11	14	14	14
8	5	14	14

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H_{\mathbb{J}}(k)$	0	0	0	0	0	2	0	0	3	0	0	3	0	0	8	0

Work is shown on the next page.

Workspace for Problem 3... J

Work Space:

$J_1 =$

16/16	8/16	5/16	2/16
11/16	8/16	5/16	16/16
8/16	11/16	14/16	14/16
5/16	2/16	11/16	14/16

14	11	8	5
14	11	8	14
11	14	14	14
8	5	14	14

Histogram of I

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H(k)$	0	2	3	3	3	0	0	0	0	3	2	0	0	0	0	0
$p(k)$	0/16	2/16	3/16	3/16	3/16	0/16	0/16	0/16	0/16	3/16	2/16	0/16	0/16	0/16	0/16	0/16
$P(k)$	0/16	2/16	5/16	8/16	11/16	11/16	11/16	11/16	11/16	14/16	16/16	16/16	16/16	16/16	16/16	16/16

Histogram of I'

k'	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H(k)$	0	0	1	0	0	2	0	0	3	0	0	4	0	0	6	0
$p'(k)$	0/16	0/16	1/16	0/16	0/16	2/16	0/16	0/16	3/16	0/16	0/16	4/16	0/16	0/16	6/16	0/16

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$P'(n)$	0	0	1/16	1/16	1/16	3/16	3/16	3/16	6/16	6/16	6/16	10/16	10/16	10/16	16/16	16/16

$$J_1(i,j) = \sum_{k=0}^{I(i,j)} p(k)$$ as shown above.

$$J(i,j) = \arg\min_n \left\{ P'(n) \geq J_1(i,j) \right\}$$ as shown above

$I(i,j)$	$J_1(i,j)$	$\min P'(n)$	$n = J(i,j)$
1	2/16	3/16	5
2	5/16	6/16	8
3	8/16	10/16	11
4	11/16	16/16	14
9, 10	14/16, 16/16	16/16	14

~~16~~ 25

4. **16 pts**. A continuous optical image $I_C(x,y)$ is given by the linear convolution

$$I_C(x,y) = J_C(x,y) * K_C(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J_C(x-\alpha, y-\beta)K_C(\alpha,\beta)\, d\alpha d\beta,$$

where

$$J_C(x,y) = \exp\left[-(x^2+y^2)/36\right]$$

and

$$K_C(x,y) = \frac{\sin(5\pi x)\sin(7\pi y)}{35\pi^2 xy}.$$

Should say cycles/mm or Hz/mm

The spatial coordinates x and y are expressed in units of millimeters.

Therefore $\widetilde{I}_C(\omega_x,\omega_y) = \widetilde{J}_C(\omega_x,\omega_y)\widetilde{K}_C(\omega_x,\omega_y)$, where ω_x and ω_y are in units of rad/mm.

A 1024×1024 digital image $I(i,j)$ is obtained by sampling $I_C(x,y)$ according to $I(i,j) = I_C(i\Delta, j\Delta)$, where the horizontal and vertical sample spacings are given by $\Delta = 0.08$mm.

Is this sampling sufficiently dense for the digital image $I(i,j)$ to have the appearance of the optical image $I_C(x,y)$ without visibly evident distortion?

$J_c(x,y)$ is a Gaussian with space constant $\sigma = \sqrt{36} = 6$.

Notes (w/ corrections): $\widetilde{J}_c(\omega_x,\omega_y) = 36\pi \exp\left[-36\pi^2(\omega_x^2+\omega_y^2)\right]$

$$K_c(x,y) = \frac{\sin(5\pi x)}{5\pi x} \cdot \frac{\sin(7\pi y)}{7\pi y} = \text{sinc}(5x)\,\text{sinc}(7y)$$

Notes (w/ corrections): $\widetilde{K}_c = \begin{cases} \frac{1}{35}, & |\omega_x| \le 5/2 \text{ and } |\omega_y| \le \frac{7}{2} \\ 0, & \text{otherwise} \end{cases}$

$$\text{So } \widetilde{I}_c(\omega_x,\omega_y) = \begin{cases} \frac{1}{35}\widetilde{J}_c(\omega_x,\omega_y), & |\omega_x| \le \frac{5}{2} \text{ and } |\omega_y| \le \frac{7}{2} \\ 0, & \text{otherwise.} \end{cases}$$

$\Rightarrow I_c(x,y)$ is a bandlimited image w/ $\Omega_x = \frac{5}{2}$ and $\Omega_y = \frac{7}{2}$.

We have $X = Y = \Delta = 0.08$ mm for the detector pitch.

$$\frac{1}{2X} = \frac{1}{.16} = 6.25 > \Omega_x \checkmark$$

$$\frac{1}{2Y} = \frac{1}{.16} = 6.25 > \Omega_y \checkmark$$

Aliasing will not occur.
→ Distortion will not occur.
The sampling is sufficiently dense.