ECE 5273
Test 1

Wednesday, March 30, 2011
4:30 PM - 5:45 PM

Spring 2011

Dr. Havlicek

Name: SOLUTION

Student Num:________________

Directions: This is an open notes test. You may use the official course lecture notes and a calculator. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________
2. (20) ________
3. (20) ________
4. (20) ________
5. (20) ________

TOTAL (100):

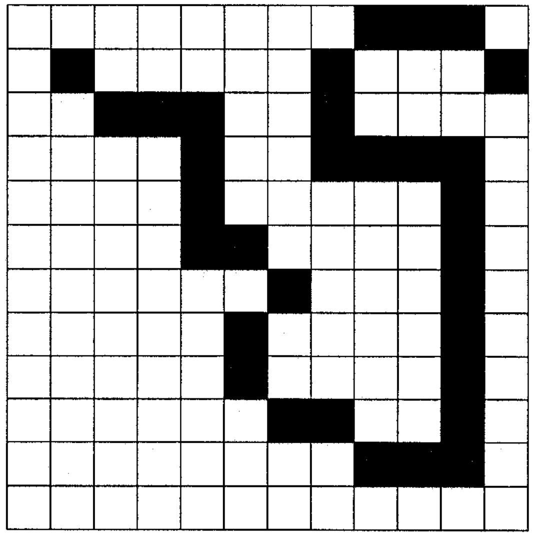
On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:________________ Date:________________

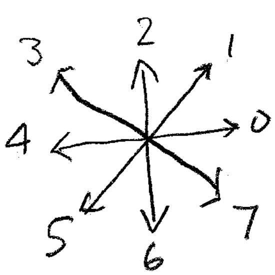
1. **20 pts**. True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
	X	(a) **3 pts**. Medical X-ray images are an example of reflection absorption imaging.
	X	(b) **3 pts**. Medical CAT scans are an example of emission absorption imaging.
	X	(c) **3 pts**. Any image $\mathbb{I}$ can be exactly reconstructed from its histogram $H_{\mathbb{I}}(k)$.
	X	(d) **3 pts**. The full-scale contrast stretch is an example of a ~~geometric image~~ operation. point
	X	(e) **3 pts**. Binary erosion removes holes of insufficient size. objects
X		(f) **3 pts**. Because of the inherent periodicity of the 2D DFT and IDFT equations, the original image is always implied to be periodic.
	X	(g) **2 pts**. The famous *Lena* image originally appeared in the November, 1972 issue of *Better Homes and Gardens*.

2. **20 pts**. Consider the binary contour image shown below, where white represents LOGIC_ZERO and black represents LOGIC_ONE.



(a) **10 pts.** Let the upper left pixel have coordinates (row,col) = (0,0) and consider that the LOGIC_ONE pixel located at (1,1) is the initial pixel of the contour. Give a chain code for the contour.

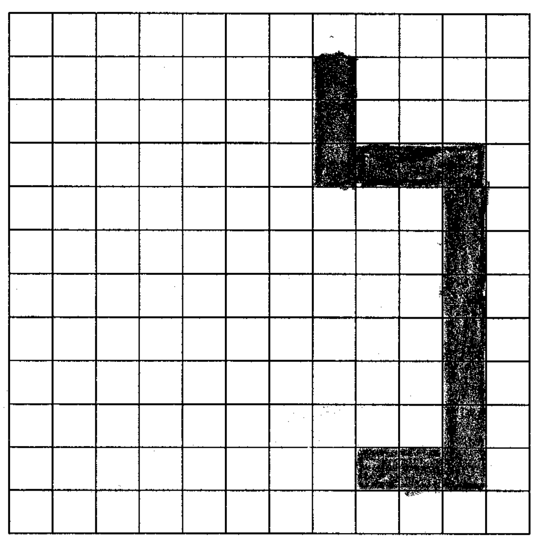


$\boxed{1,1}$ 7 0 0 6 6 6 0 7 5 6 7 0 7 0 0
2 2 2 2 2 2 2 4 4 4 2 2 1 0 0 7 $\boxed{3}$

Workspace for Problem 2...

(b) **10 pts.** Four-conntected Blob Coloring (connected components analysis) is applied to the image with minor region removal. Show the result below.

Since this is a 4-connected algorithm, the diagonal connections in the contour are breaks between blobs. After coloring and minor region removal, only the largest blob will remain.



3. **20 pts.** Consider the 4×4 image $\mathbb{I}$ shown below, where the allowable gray levels are $0 \leq I(i,j) \leq 15$:

$$\mathbb{I} = \begin{array}{|c|c|c|c|} \hline \mathbf{15} & \mathbf{14} & \mathbf{13} & \mathbf{14} \\ \hline \mathbf{10} & \mathbf{12} & \mathbf{9} & \mathbf{0} \\ \hline \mathbf{10} & \mathbf{9} & \mathbf{12} & \mathbf{10} \\ \hline \mathbf{9} & \mathbf{11} & \mathbf{15} & \mathbf{15} \\ \hline \end{array}$$

Construct a new image $\mathbb{J}$ by applying the histogram flattening (equalization) algorithm to $\mathbb{I}$. Show the new image $\mathbb{J}$ and its histogram $H_{\mathbb{J}}$ in the spaces provided below:

$$\mathbb{J} = \begin{array}{|c|c|c|c|} \hline 15 & 12 & 10 & 12 \\ \hline 7 & 9 & 4 & 1 \\ \hline 7 & 4 & 9 & 7 \\ \hline 4 & 8 & 15 & 15 \\ \hline \end{array}$$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H_{\mathbb{J}}(k)$	0	1	0	0	3	0	0	3	1	2	1	0	2	0	0	3

$K = 16$

$\mathbb{I}$	$\mathbb{J}_1$	$(K-1)\mathbb{J}_1 + 0.5$	$\mathbb{J}$
0	1/16	1.4375	1
9	4/16	4.2500	4
10	7/16	7.0625	7
11	8/16	8.0000	8
12	10/16	9.8750	9
13	11/16	10.8125	10
14	13/16	12.6875	12
15	16/16	15.5000	15

Work Space:

$$\mathbb{J}_1 = \begin{array}{|c|c|c|c|} \hline 16/16 & 13/16 & 11/16 & 13/16 \\ \hline 7/16 & 10/16 & 4/16 & 1/16 \\ \hline 7/16 & 4/16 & 10/16 & 7/16 \\ \hline 4/16 & 8/16 & 16/16 & 16/16 \\ \hline \end{array}$$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H(k)$	1	0	0	0	0	0	0	0	0	3	3	1	2	1	2	3
$p(k)$	1/16	0	0	0	0	0	0	0	0	3/16	3/16	1/16	2/16	1/16	2/16	3/16
$\sum_k p(k)$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	4/16	$\frac{7}{16}$	$\frac{8}{16}$	$\frac{10}{16}$	$\frac{11}{16}$	$\frac{13}{16}$	$\frac{16}{16}$

4. **20 pts**. Consider the continuous (optical) image

$$I_C(x,y) = e^{-(2x+3y)}u(x)u(y),$$

where $u(x)$ is the unit step function given by

$$u(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Find the continuous Fourier transform $\widetilde{I}(\omega_x, \omega_y)$.

Hint: $\int e^{ax}\,dx = \frac{1}{a}e^{ax}$

$$\widetilde{I}_c(\omega_x,\omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_c(x,y)\, e^{-j2\pi(x\omega_x + y\omega_y)}\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(2x+3y)}\,u(x)u(y)\,e^{-j2\pi(x\omega_x+y\omega_y)}\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-2x}e^{-j2\pi x\omega_x}u(x)\,e^{-3y}e^{-j2\pi y\omega_y}u(y)\,dx\,dy$$

$$= \int_{-\infty}^{\infty} e^{-(2+j2\pi\omega_x)x}u(x)\,dx \int_{-\infty}^{\infty} e^{-(3+j2\pi\omega_y)y}u(y)\,dy$$

$$= \int_{0}^{\infty} e^{-(2+j2\pi\omega_x)x}\,dx \int_{0}^{\infty} e^{-(3+j2\pi\omega_y)y}\,dy$$

$$= \left[\frac{-1}{2+j2\pi\omega_x}\,e^{-(2+j2\pi\omega_x)x}\Big|_{x=0}^{\infty}\right]\left[\frac{-1}{3+j2\pi\omega_y}\,e^{-(3+j2\pi\omega_y)y}\Big|_{y=0}^{\infty}\right]$$

$$= \left[\frac{-e^{-j2\pi x\omega_x}}{2+j2\pi\omega_x}\,e^{-2x}\Big|_{x=0}^{\infty}\right]\left[\frac{-e^{-j2\pi y\omega_y}}{3+j2\pi\omega_y}\,e^{-3y}\Big|_{y=0}^{\infty}\right]$$

$$= \lim_{A\to\infty}\frac{-e^{-j2\pi x\omega_x}}{2+j2\pi\omega_x}\,e^{-2x}\Big|_{x=0}^{A}\ \lim_{B\to\infty}\frac{-e^{-j2\pi y\omega_y}}{3+j2\pi\omega_y}\,e^{-3y}\Big|_{y=0}^{B}$$

$$= \left\{0 + \frac{1}{2+j2\pi\omega_x}\right\}\left\{0 + \frac{1}{3+j2\pi\omega_y}\right\}$$

$$= \frac{1}{2+j2\pi\omega_x}\ \frac{1}{3+j2\pi\omega_y} \quad \boxed{= \frac{1}{(2+j2\pi\omega_x)(3+j2\pi\omega_y)}}$$

More Workspace for Problem 4...

5. **20 pts.** Draw lines to match the images with their log-magnitude DFT spectra.