ECE 5273
Test 1

Wednesday, March 25, 2015
4:30 PM - 5:45 PM

Spring 2015 Name: SOLUTION

Dr. Havlicek Student Num:__________

Directions: This is an open notes test. You may use the official course lecture notes and a calculator. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________
2. (20) ________
3. (20) ________
4. (20) ________
5. (20) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:__________ Date:__________

1. **20 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
	F	(a) **2 pts.** Positron emission tomogrraphy (PET) is an example of emission imaging. Notes p. 1.24 Absorption
	F	(b) **2 pts.** In the projection equation, the focal length f is the distance from the lens center to an object in the real world that is *in focus.* Distance from image plane to lens Notes p. 1.34
T		(c) **2 pts.** For a binary morphological filter, the main reason that we usually choose the reference point to be the center of the structuring element is to ensure that the output image is not shifted relative to the input image. Notes p. 2.61
T		(d) **2 pts.** The binary median filter is self-dual with respect to complementation. Notes p. 2.80
	F	(e) **2 pts.** The binary OPEN filter removes small holes and gaps better than the binary MEDIAN filter, but not objects or peninsulas. Notes p. 2.87
	F	(f) **2 pts.** Run-length coding is a powerful compression technique that always reduces the size of a binary image. Notes p. 2.102
	F	(g) **2 pts.** If two images have the same histogram, then they are identical up to a linear point operation. Notes p. 2.17
	F	(h) **2 pts.** The DFT of any real-valued digital image $\mathbf{I}$ is even symmetric. Conjugate Symmetric: Notes p. 4.59
T		(i) **2 pts.** For a practical digital image $\mathbf{I}$, the 2D DFT $\widetilde{\mathbf{I}}$ is given by equally spaced samples of the DSFT $\widetilde{\mathbf{I}}_D$. Notes p. 4.108
	F	(j) **2 pts.** The famous *Lena* image is actually a photo of OU president David Boren from a Halloween party in 1961.

2. **20 pts.** Consider the 4×4 images $\mathbf{I}$ and $\mathbf{I}'$ shown below, where the allowable range of gray levels is $0 \leq I(m,n), I'(m,n) \leq 15$:

$$\mathbf{I} = \begin{array}{|c|c|c|c|} \hline 15 & 13 & 11 & 9 \\ \hline 11 & 9 & 7 & 5 \\ \hline 9 & 7 & 2 & 3 \\ \hline 9 & 5 & 0 & 0 \\ \hline \end{array} \qquad \mathbf{I}' = \begin{array}{|c|c|c|c|} \hline 2 & 1 & 2 & 15 \\ \hline 1 & 0 & 2 & 14 \\ \hline 15 & 13 & 13 & 14 \\ \hline 15 & 15 & 14 & 15 \\ \hline \end{array}$$

Construct a new image $\mathbf{K}$ by applying the histogram matching algorithm to shape the histogram of image $\mathbf{I}$, where the desired shape is given by the histogram of the image $\mathbf{I}'$.

Show the new image $\mathbf{K}$ and its histogram $\mathrm{H}_{\mathbf{K}}$ in the spaces provided below.

For this problem, you are **not** required to perform a full-scale contrast stretch at the end.

$$\mathbf{K} = \begin{array}{|c|c|c|c|} \hline 15 & 15 & 15 & 15 \\ \hline 15 & 15 & 13 & 2 \\ \hline 15 & 13 & 1 & 2 \\ \hline 15 & 2 & 1 & 1 \\ \hline \end{array}$$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\mathrm{H}_{\mathbf{K}}(k)$	0	3	3	0	0	0	0	0	0	0	0	0	0	2	0	8

Work space is provided on the next page.

Workspace for Problem 2:

$I =$

15	13	11	9
11	9	7	5
9	7	2	3
9	5	0	0

$I' =$

2	1	2	15
1	0	2	14
15	13	13	14
15	15	14	15

For I:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	2	0	1	1	0	2	0	2	0	4	0	2	0	1	0	1
p(k)	2/16	0/16	1/16	1/16	0/16	2/16	0/16	2/16	0/16	4/16	0/16	2/16	0/16	1/16	0/16	1/16
P(k)	2/16	2/16	3/16	4/16	4/16	6/16	6/16	8/16	8/16	12/16	12/16	14/16	14/16	15/16	15/16	16/16

For I'

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	1	2	3	0	0	0	0	0	0	0	0	0	0	2	3	5
p(k)	1/16	2/16	3/16	0/16	0/16	0/16	0/16	0/16	0/16	0/16	0/16	0/16	0/16	2/16	3/16	5/16
P(k)	1/16	3/16	6/16	6/16	6/16	6/16	6/16	6/16	6/16	6/16	6/16	6/16	6/16	8/16	11/16	16/16

$$J(m,n) = \sum_{k=0}^{I(m,n)} p_I(k) = P_I(k)$$

16/16	15/16	14/16	12/16
14/16	12/16	8/16	6/16
12/16	8/16	3/16	4/16
12/16	6/16	2/16	2/16

$$K(m,n) = \min_r \left\{ P_{I'}(r) \geq J(m,n) \right\}$$

15	15	15	15
15	15	13	2
15	13	1	2
15	2	1	1

3. **20 pts**. The 512×512 gray scale image $\mathbf{I}_1$ shown below has 8-bit pixels. This image was thresholded to obtain the binary image $\mathbf{I}_2$, which is also shown below. In $\mathbf{I}_2$, the pixel value 255 (WHITE) represents LOGIC ONE and the pixel value zero (BLACK) represents LOGIC ZERO.

$\mathbf{I}_1$

$\mathbf{I}_2$

Three binary filters were applied to the image $\mathbf{I}_2$:

- a binary morphological OPEN with a 5×5 diamond-shaped structuring element,
- a binary morphological CLOSE with a 5×5 diamond-shaped structuring element, and
- a binary median filter with a 9×9 diamond-shaped structuring element.

The three resulting output images $\mathbf{J}_1$ - $\mathbf{J}_3$ are shown on the next page.

(a) **15 pts**. Determine which output image resulted from each filtering operation. Explain your answer.

(b) **5 pts**. Explain why a structuring element size of 5×5 for the binary morphological OPEN and CLOSE filters was compared to a structuring element size of 9×9 for the binary median filter.

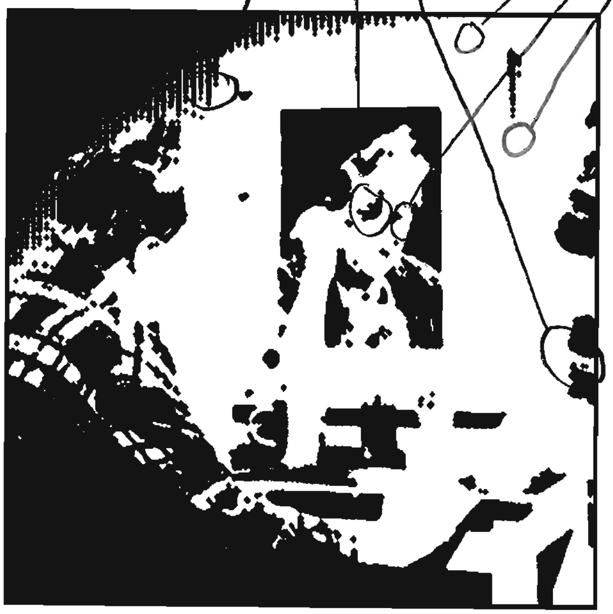
Problem 3 cont...

Small white structures remain

small black structures are killed

$\mathbf{J}_1$

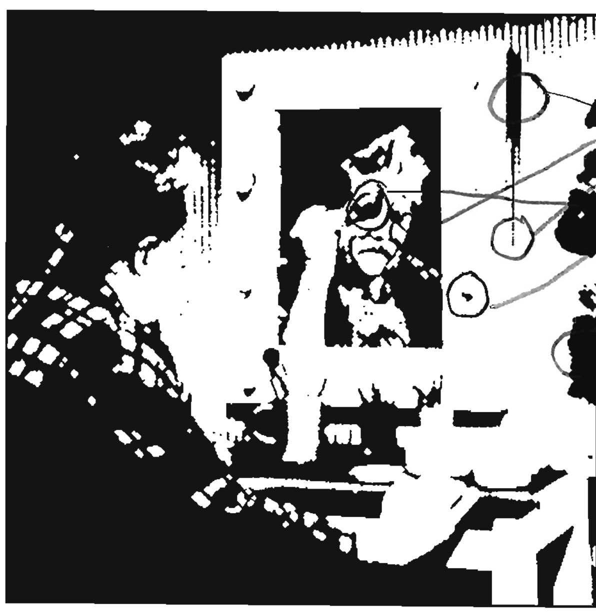
CLOSE



$\mathbf{J}_2$

small white <u>and</u> black structures are killed; MEDIAN

$\mathbf{J}_3$



small black structures remain

Small white structures are killed

↓

OPEN

(space for your answers is provided on the next page)

More space for Problem 3:

a) In J_1, small white structures like lines on the wall, a spot above the eye, and horizontal stripes at the right edge of the image are all preserved. But small black structures like lines on the wall and thin contours around the eyes are killed. This is because DILATE was done first, followed by ERODE. The dilation kills small black structures and they are not restored by the erosion. Small white structures are enlarged by the dilation and then returned to their approximate original sizes by the erosion.

$\longrightarrow J_1$ is <u>CLOSE</u>.

In J_2, any small structure is killed. It doesn't matter if it is black or white. This is the action of the binary MEDIAN filter.

$\longrightarrow J_2$ is <u>MEDIAN</u>.

In J_3, small black structures like lines on the wall and thin contours around the eyes are preserved. Small white structures like lines on the wall and a spot above the eye are killed. This is because ERODE was done first. The erosion kills small white structures and they are not restored by dilation. Small black structures are enlarged by the erosion and then returned to their approximate original sizes by the dilation.

$\longrightarrow J_3$ is <u>OPEN</u>.

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b) The Structuring Element:

$(-2,0)$

$(-1,-1)(-1,0)\ (-1,1)$

$(0,-2)(0,-1)(0,0)\ (0,1)\ (0,2)$

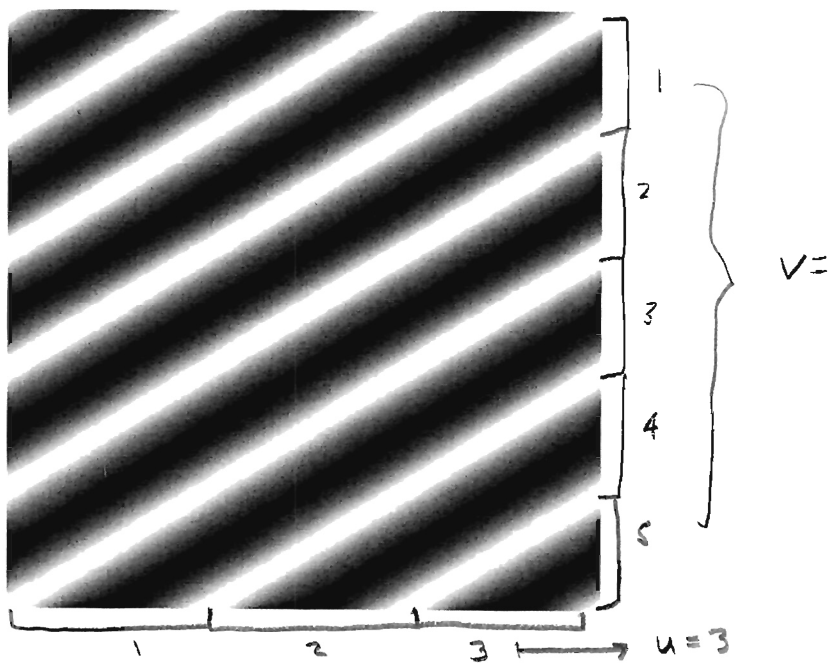
$(1,-1)(1,0)\ (1,1)$

$(2,0)$

- Let $J = OPEN(I_2, B) = DILATE[ERODE(I_2,B), B]$.
- Let $K = ERODE(I_2, B)$.
- Then $J = OPEN(I_2, B) = DILATE(K, B)$.
- Each pixel $K(m,n)$ in the ERODED image K contains contributions from pixels $I_2(m-2,n)$, $I_2(m+2,n)$, $I_2(m,n-2)$ and $I_2(m,n+2)$ in the original I_2 image.
- Similarly, each pixel $J(m,n)$ in the OPENED image contains contributions from pixels $K(m-2,n)$, $K(m+2,n)$, $K(m,n-2)$, and $K(m,n+2)$ in the ERODED image K.

- But $K(m-2,n)$ contains a contribution from $I_2(m-4,n)$. And $K(m+2,n)$ contains a contribution from $I_2(m+4,n)$. Likewise, $K(m,n-2)$ contains a contribution from $I_2(m,n-4)$; and $K(m,n+2)$ contains a contribution from $I_2(m,n+4)$.
- So, overall, pixel (m,n) in the OPENED image J has contributions from $I_2(m-4,n)$, $I_2(m+4,n)$, $I_2(m,n-4)$, and $I_2(m,n+4)$ in the original I_2 image.
- Thus, the EFFECTIVE spatial extent of the overall OPEN filter is like a 9×9 diamond shaped structuring element.
- Since the binary MEDIAN filter involves only a <u>single</u> pass of the structuring element, it is necessary to use a 9×9 structuring element with MEDIAN to obtain a filter that is directly comparable to the OPEN and CLOSE, so that pixel (m,n) in the MEDIAN filtered image also depends on $I_2(m+4,n)$, $I_2(m-4,n)$, $I_2(m,n+4)$, and $I_2(m,n-4)$.

4. **20 pts.** Consider the 256×256 sinusoidal grating image **I** shown below:



The value of the pixel in the upper left corner of the image is $I(0,0) = 1.0$.

Write closed form analytical expressions for the image pixels $I(m,n)$ and for the DFT $\widetilde{I}(u,v)$.

Since $I(0,0) = 1$, it is a <u>cosine</u> image.

$$I(m,n) = \cos\left[\frac{2\pi}{256}(3m + 5n)\right] \quad //$$

Notes, page 4.128:

$$\widetilde{I}(u,v) = \frac{256^2}{2}\left[\delta(u-3, v-5) + \delta(u+3, v+5)\right]$$

$$= 32{,}768\left[\delta(u-3, v-5) + \delta(u+3, v+5)\right] \quad //$$

5. **20 pts.** Draw lines to match the images with their log-magnitude DFT spectra.