ECE 5273
Test 1

Wednesday, March 28, 2018
4:30 PM - 5:45 PM

Spring 2018

Dr. Havlicek

Name: SOLUTION

Student Num: ____________________

Directions: This is an open notes test. You may use the official course lecture notes and a calculator. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________
2. (20) ________
3. (20) ________
4. (20) ________
5. (20) ________

____

TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: ____________________ Date: ____________________

1. **20 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
X		(a) **2 pts.** X-ray computed tomography (CT scan) is an example of absorption imaging. Notes p. 1.25
	X	(b) **2 pts.** In the projection equation, the focal length f is the distance from the lens center to an object in the real world that is *in focus.* Notes p. 1.35
	X	(c) **2 pts.** Any digital image can be exactly reconstructed from its histogram. Notes p. 2.17
X		(d) **2 pts.** The binary OPEN and CLOSE filters generally do not affect the overall sizes of objects that are sufficiently large. Notes p. 2.88
X		(e) **2 pts.** If a binary morphological erosion filter is applied over and over again enough times, it will eventually reduce any binary image $\mathbf{I}$ to an image of all zeros. Notes pp. 2.73–2.75
X		(f) **2 pts.** The full-scale contrast stretch is an example of a linear point operation. Notes p. 3.14
	X	(g) **2 pts.** Frame-~~differencing~~ averaging is a simple but powerful technique for smoothing noise in digital video frames.
X		(h) **2 pts.** A geometric image operation generally requires a spatial mapping of image coordinates followed by interpolation. Notes p. 3.59
X		(i) **2 pt.** For a practical digital image $\mathbf{I}$, the 2D DFT $\tilde{\mathbf{I}}$ is given by equally spaced samples of the DSFT $\tilde{\mathbf{I}}_D$. Notes p. 4.108
OH MY!		(j) **2 pt.** Mexico will pay for Trump's border wall.

2. **20 pts.** Consider the 4×4 image **I** shown below, where the allowable range of gray levels is $0 \leq I(i,j) \leq 15$:

$$\mathbf{I} = \begin{array}{|c|c|c|c|} \hline 4 & 5 & 9 & 7 \\ \hline 6 & 10 & 8 & 9 \\ \hline 7 & 10 & 11 & 8 \\ \hline 4 & 6 & 5 & 11 \\ \hline \end{array}$$

Construct a new image **K** by applying the histogram matching algorithm to match the histogram of image **I** to the **desired** histogram given by the histogram of image **I′** shown below:

$$\mathbf{I}' = \begin{array}{|c|c|c|c|} \hline 6 & 15 & 12 & 9 \\ \hline 9 & 15 & 15 & 12 \\ \hline 0 & 12 & 15 & 4 \\ \hline 0 & 0 & 2 & 2 \\ \hline \end{array}$$

Show the new image **K** and its histogram H_K in the spaces provided below.

$$\mathbf{K} = \begin{array}{|c|c|c|c|} \hline 0 & 2 & 12 & 9 \\ \hline 4 & 15 & 12 & 12 \\ \hline 9 & 15 & 15 & 12 \\ \hline 0 & 4 & 2 & 15 \\ \hline \end{array}$$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H_K(k)$	2	0	2	0	2	0	0	0	0	2	0	0	4	0	0	4

Work space is provided on the next page.

Workspace for Problem 2:

$I =$

4	5	9	7
6	10	8	9
7	10	11	8
4	6	5	11

$I' =$

6	15	12	9
9	15	15	12
0	12	15	4
0	0	2	2

For I:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	0	0	0	2	2	2	2	2	2	2	2	0	0	0	0
p(k)	0/16	0/16	0/16	0/16	2/16	2/16	2/16	2/16	2/16	2/16	2/16	2/16	0/16	0/16	0/16	0/16
P(k)	0/16	0/16	0/16	0/16	2/16	4/16	6/16	8/16	10/16	12/16	14/16	16/16	16/16	16/16	16/16	16/16

DESIRED (I'): $\{P_k\}$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	3	0	2	0	1	0	1	0	0	2	0	0	3	0	0	4
p(k)	3/16	0/16	2/16	0/16	1/16	0/16	1/16	0/16	0/16	2/16	0/16	0/16	3/16	0/16	0/16	4/16
P(k)	3/16	3/16	5/16	5/16	6/16	6/16	7/16	7/16	7/16	9/16	9/16	9/16	12/16	12/16	12/16	16/16

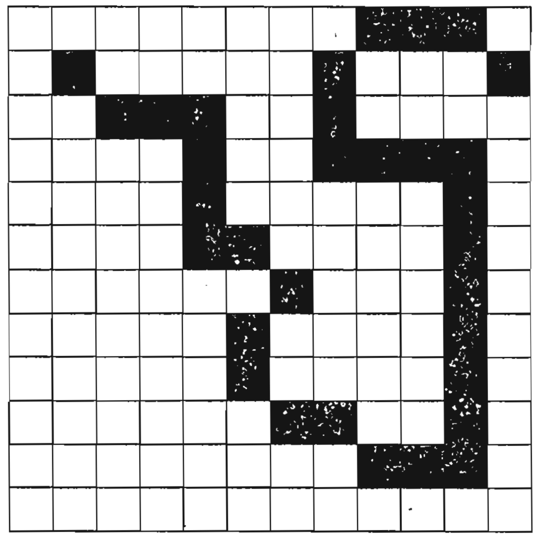
$J = P_I\left(I(m,n)\right) =$

2/16	4/16	12/16	8/16
6/16	14/16	10/16	12/16
8/16	14/16	16/16	10/16
2/16	6/16	4/16	16/16

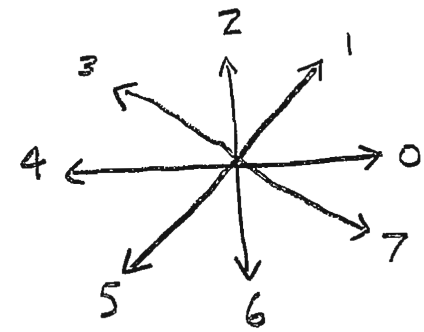
$K = \arg\min_r \{P_K(r) \geq J(m,n)\} =$

0	2	12	9
4	15	12	12
9	15	15	12
0	4	2	15

3. **20 pts.** Consider the binary contour image shown below, where white represents LOGIC_ZERO and black represents LOGIC_ONE.



Direction codes from Notes p. 2-109:



(a) **10 pts.** Let the upper left pixel have coordinates (row,col) = (0,0) and consider that the LOGIC_ONE pixel located at (1,1) is the initial pixel of the contour. Give a chain code for the contour. Don't forget to include the coordinates of the initial pixel and the end-of-code "flag."

(1,1) 7 0 0 6 6 6 0 7 5 6 7 0 7 0 0

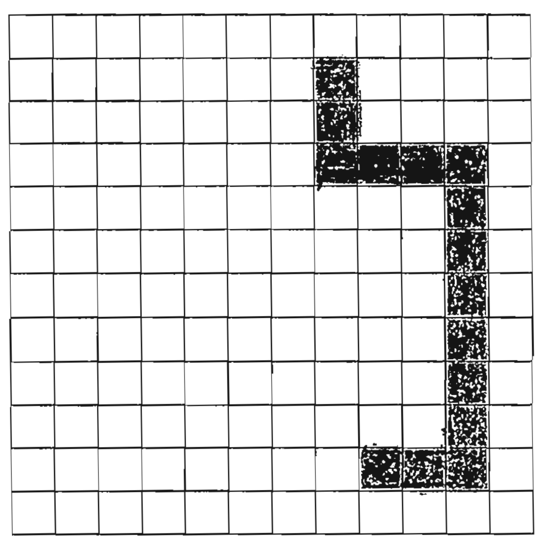
2 2 2 2 2 2 2 4 4 4 2 2 1 0 0 7 $\boxed{3}$

Workspace for Problem 3...

(b) **10 pts.** Four-conntected Blob Coloring (connected components analysis) is applied to the image with minor region removal. Show the result below.

For 4-connected topology, the diagonal connections are breaks between separate blobs.

After blob coloring, blob counting, and minor region removal, only the largest blob will remain.



4. **20 pts.** Consider a 6×6 digital image I given by

$$I(m,n) = 5 + 10\delta(m,n) + 2\cos\left[\frac{2\pi}{6}(2m-3n)\right] + \sin\left[\frac{2\pi}{6}(3m+2n)\right],$$

where m = column and n = row.

$M = N = 6$

(a) **10 pts.** Find a closed form expression for the DFT $\tilde{\text{I}}$.

Notes p. 4.126: $5 \overset{DFT}{\longleftrightarrow} 5(6)(6)\delta(u,v) = 180\delta(u,v)$

Notes p. 4.127: $10\delta(m,n) \overset{DFT}{\longleftrightarrow} 10$

Notes p. 4.128: $2\cos\left[2\pi\left(\frac{2}{6}m - \frac{3}{6}n\right)\right] \overset{DFT}{\longleftrightarrow} \left(\frac{2}{2}\right)(6)(6)\left[\delta(u-2,v+3)+\delta(u+2,v-3)\right]$

$= 36\left[\delta(u-2,v+3)+\delta(u+2,v-3)\right]$

Notes p. 4.129: $\sin\left[2\pi\left(\frac{3}{6}m+\frac{2}{6}n\right)\right] \overset{DFT}{\longleftrightarrow} \left(\frac{j}{2}\right)(6)(6)\left[\delta(u+3,v+2)-\delta(u-3,v-2)\right]$

$= j18\left[\delta(u+3,v+2)-\delta(u-3,v-2)\right]$

So $\tilde{I}(u,v) = 180\delta(u,v) + 10 + 36\left[\delta(u-2,v+3)+\delta(u+2,v-3)\right] + j18\left[\delta(u+3,v+2)-\delta(u-3,v-2)\right]$

→ But, see notes p. 4.64: this 2D DFT must be horizontally periodic with period $M=6$ and vertically periodic with period $N=6$.

→ So, accounting for the periodicity, we have:

$$\boxed{\begin{aligned}\tilde{I}(u,v) = 180\delta(u,v) + 10 + 36\left[\delta(u-2,v\pm3)+\delta(u+2,v\pm3)\right] \\ + j18\left[\delta(u\pm3,v+2)-\delta(u\pm3,v-2)\right]\end{aligned}}$$

(b) **10 pts.** Show the real and imaginary parts of the **centered** DFT array in the space provided below:

$\tilde{\text{I}}$ =

v \ u	−3	−2	−1	0	1	2
−3	10	46	10	10	10	46
−2	10	10	10	10	10	10
−1	10	10	10	10	10	10
0	10	10	10	190	10	10
1	10	10	10	10	10	10
2	10	10	10	10	10	10

+ j ×

v \ u	−3	−2	−1	0	1	2
−3	0	0	0	0	0	0
−2	18	0	0	0	0	0
−1	0	0	0	0	0	0
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	−18	0	0	0	0	0

NOTE: this answer can be directly verified using Matlab.

5. **20 pts.** Draw lines to match the images with their log-magnitude DFT spectra.