ECE 5273
Test 1

Monday, March 29, 2021 Thursday, April 1, 2021

Spring 2021 Name: SOLUTION

Dr. Havlicek Student Num. ____________

Directions: This is an open notes test You may use the official course lecture notes and a calculator. Other materials are not allowed. All work must be your own. You may work the test on this test paper or you may use your own blank paper. Upload a scan or photograph of your test paper to the course Canvas page no later than 11:59 PM on Thursday, April 1, 2021.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________

2 (20) ________

3. (20) ________

4. (20) ________

5. (20) ________

TOTAL (100).

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name ____________ Date: ____________

1. **20 pts**. True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE		Notes
	X	(a) **2 pts**. If two digital images have the same histogram, then they are identical up to a point operation.	p. 2.17
X		(b) **2 pts**. If two digital images have the same histogram, then they also have the same average optical density (AOD).	p. 3.4
	X	(c) **2 pts**. For histogram flattening (equalization), it is important to define the reference point to be the center of the window so that the output image is not shifted.	p. 3.38 (there is no window)
	X	(c) **2 pts**. Frame differencing is a simple but effective technique for smoothing noise in digital video frames.	pp. 3.50, 3.56
X		(e) **2 pts**. The binary CLOSE-OPEN filter removes small structures without affecting size.	p. 2.91
	X	(f) **2 pts**. The main purpose of the binary CLOSE filter (which performs dilation first) is to enlarge the objects in an image.	p. 2.88
X		(g) **2 pts**. The discrete-space Fourier transform (DSFT) of the zero padded *Peppers* image is conjugate symmetric.	pp. 4.59, 4.107 4.108
X		(h) **2 pts**. If $\mathbf{I}_C$ is a Gaussian optical image, then the digital image $\mathbf{I}$ obtained with a digital camera is aliased	p. 4.133
X		(i) **2 pt** In the "pinhole" camera model we have used, straight lines in the 3D world always project to straight lines on the 2D focal plane.	p. 1.47
OH	MY !	(j) **2 pt**. Trump got more votes than Biden.	

2. **20 pts.** A scene consisting of a black triangle against a white background is imaged with an ideal pinhole camera having a focal length of $f = 35$ mm. The world coordinates (X, Y, Z) of the three vertices of the triangle are given by

$$\begin{aligned} P_1 &= (0.0\text{ m}, 1.9\text{ m}, 1.3\text{ m}), \\ P_2 &= (-0.2\text{ m}, 0.2\text{ m}, 1.1\text{ m}), \\ P_3 &= (0.4\text{ m}, 0.1\text{ m}, 1.2\text{ m}). \end{aligned}$$

Find the *area* of the image of the triangle that is obtained on the camera focal plane.

Hint 1: note that the focal length is given in *millimeters*, while the world coordinates of the vertices are given in *meters*

Hint 2: the area of a triangle with sides of length a, b, and c is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \frac{1}{2}(a + b + c)$.

→ I am using units of meters throughout this solution.

① Find the projections of the vertices on the image plane:

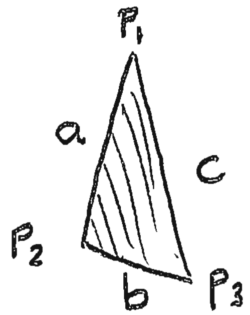
Notes p. 1.44: $(x, y) = \frac{f}{Z}(X, Y)$

P_1 : $(x_1, y_1) = \frac{0.035}{1.3}(0.0, 1.9) = (0.0, 0.0511538)$

P_2 : $(x_2, y_2) = \frac{0.035}{1.1}(-0.2, 0.2) = (-0.00636364, 0.00636364)$

P_3 : $(x_3, y_3) = \frac{0.035}{1.2}(0.4, 0.1) = (0.0116667, 0.00291667)$

② sketch the image:



③ Find lengths of sides:

$a = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(0.00636364)^2 + (0.0447902)^2}$

$= \sqrt{40.4959 \times 10^{-6} + 2.00616 \times 10^{-3}}$

$= \sqrt{2.04666 \times 10^{-3}} = 45.24 \times 10^{-3}$

More Workspace for Problem 2..

(3)... $b = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$

$= \sqrt{(-0.00636364 - 0.0116667)^2 + (0.00636364 - 0.00291667)^2}$

$= \sqrt{(-18.0303 \times 10^{-3})^2 + (3.44697 \times 10^{-3})^2}$

$= \sqrt{325.092 \times 10^{-6} + 11.8816 \times 10^{-6}} = \sqrt{336.973 \times 10^{-6}} = 18.3568 \times 10^{-3}$

$c = \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} = \sqrt{(0 - 0.011667)^2 + (0.0511538 - 0.00291667)^2}$

$= \sqrt{(0.011667)^2 + (48.2372 \times 10^{-3})^2} = \sqrt{136.111 \times 10^{-6} + 2.32683 \times 10^{-3}}$

$= \sqrt{2.46294 \times 10^{-3}} = 49.628 \times 10^{-3}$

(4) Find Area of Image:

$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(45.24 \times 10^{-3} + 18.3568 \times 10^{-3} + 49.628 \times 10^{-3})$

$= \frac{1}{2}(113.225 \times 10^{-3}) = 56.6124 \times 10^{-3}$

$A = [s(s-a)(s-b)(s-c)]^{1/2}$

$= [56.6124 \times 10^{-3}(56.6124 \times 10^{-3} - 45.24 \times 10^{-3})(56.6124 \times 10^{-3} - 18.3568 \times 10^{-3})$
$\times (56.6124 \times 10^{-3} - 49.628 \times 10^{-3})]^{1/2}$

$= [56.6124 \times 10^{-3}(11.3724 \times 10^{-3})(38.2556 \times 10^{-3})(6.98443 \times 10^{-3})]^{1/2}$

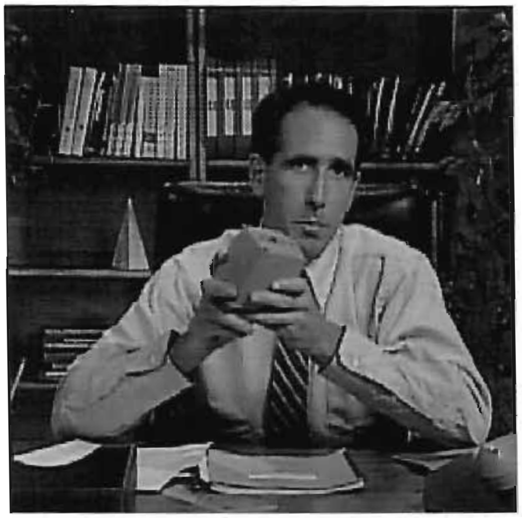
$= [172.024 \times 10^{-9}]^{1/2}$

$= 414.758 \times 10^{-6}\ m^2$

$\boxed{\text{AREA of Image} = 414.758 \times 10^{-6}\ m^2}$

3. **20 pts**. The gray scale image $\mathbf{I}_1$ shown below has 8-bit pixels. This image was thresholded to obtain the binary image $\mathbf{I}_2$, which is also shown below. In $\mathbf{I}_2$, the pixel value 255 (WHITE) represents LOGIC_ONE and the pixel value zero (BLACK) represents LOGIC_ZERO.

$\mathbf{I}_1$



$\mathbf{I}_2$

Binary morphological OPEN and CLOSE operations were performed on the image $\mathbf{I}_2$ using a 5×5 diamond-shaped structuring element. The resulting images are shown as $\mathbf{J}_1$ and $\mathbf{J}_2$ below.

$\mathbf{J}_1$ = OPEN

$\mathbf{J}_2$ = CLOSE

Determine which image is the result of the OPEN operation and which is the result of the CLOSE operation. Explain your answer in the space provided on the next page.

Workspace for Problem 3. .

J_1 is the <u>OPEN</u>.

- The stripes on his tie are obliterated by the erosion. The dilation cannot bring them back, so they are absent in the filtered image J_1.
- Several of the gaps between his fingers are enlarged by the erosion but then returned to their original size by the dilation.

J_2 is the <u>CLOSE</u>.

- The stripes on his tie are preserved in the filtered image J_2. They are enlarged by the dilation, but then the erosion returns them to their original sizes.
- The gaps between his fingers are obliterated by the dilation. Therefore they cannot be restored by the erosion and they are absent from the filtered image J_2.

4. **20 pts.** Consider a 6×6 digital image $\mathbf{I}$ given by

$$I(m,n) = 4 - 2\delta(m,n) + \cos\left[\frac{2\pi}{6}(m+n)\right] + \sin\left[\frac{2\pi}{6}(-2m+n)\right],$$

where m = column and n = row.

(a) **10 pts.** Find a closed form expression for the DFT $\tilde{\mathbf{I}}$.

Notes p. 4.126: $4 \xleftrightarrow{DFT} 4 \cdot 6 \cdot 6\, \delta(u,v) = 144\delta(u,v)$

Notes p. 4.127: $-2\delta(m,n) \xleftrightarrow{DFT} -2$

Notes p. 4.128: $\cos\left[\frac{2\pi}{6}(m+n)\right] \xleftrightarrow{DFT} (\tfrac{1}{2})(6 \cdot 6)\left[\delta(u-1,v-1) + \delta(u+1,v+1)\right]$

$$= 18\left[\delta(u-1,v-1) + \delta(u+1,v+1)\right]$$

Notes p. 4.129: $\sin\left[\frac{2\pi}{6}(-2m+n)\right] \xleftrightarrow{DFT} j(\tfrac{1}{2})(6 \cdot 6)\left[\delta(u-2,v+1) - \delta(u+2,v-1)\right]$

$$= j18\left[\delta(u-2,v+1) - \delta(u+2,v-1)\right]$$

$$\boxed{\tilde{I}(u,v) = 144\delta(u,v) - 2 + 18\left[\delta(u-1,v-1) + \delta(u+1,v+1)\right] + j18\left[\delta(u-2,v+1) - \delta(u+2,v-1)\right]}$$

(b) **10 pts.** Show the real and imaginary parts of the **centered** DFT array in the space provided below:

$$\tilde{\mathbf{I}} = \begin{array}{c} u \backslash v \\ \end{array}
\begin{array}{cccccc|c}
-3 & -2 & -1 & 0 & 1 & 2 & \\
\hline
-2 & -2 & -2 & -2 & -2 & -2 & -3 \\
-2 & -2 & -2 & -2 & -2 & -2 & -2 \\
-2 & -2 & 16 & -2 & -2 & -2 & -1 \\
-2 & -2 & -2 & 142 & -2 & -2 & 0 \\
-2 & -2 & -2 & -2 & 16 & -2 & 1 \\
-2 & -2 & -2 & -2 & -2 & -2 & 2
\end{array}
\quad + \quad j \times \quad
\begin{array}{cccccc|c}
-3 & -2 & -1 & 0 & 1 & 2 & \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & -3 \\
0 & 0 & 0 & 0 & 0 & 0 & -2 \\
0 & 0 & 0 & 0 & 0 & 18 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -18 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 2
\end{array}$$

5. **20 pts.** Draw lines to match the images with their log-magnitude DFT spectra.