ECE 5273
Test 1

Thursday, March 24, 2022
4:30 PM - 5:45 PM

Spring 2022 Name: SOLUTION

Dr. Havlicek Student Num:________

Directions: This is an open notes test. You may use the official course lecture notes and a calculator. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (20) ________
2. (20) ________
3. (20) ________
4. (20) ________
5. (20) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:________ Date:________

1. **20 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE FALSE

____ X (a) **2 pts.** Medical imaging techniques such as ultrasound sonography, MRI, and X-ray radiology are examples of absoprtion imaging. Notes pp. 1.23-25

____ X (b) **2 pts.** In the pinhole camera model, the *focal length* is the distance from the lens center to an object in the 3-D real world that is *in focus* in the image. Notes pp. 1.31, 1.35

X ____ (c) **2 pts.** For a gray-level image $\mathbf{I}$, the histogram $H_{\mathbf{I}}$ contains only first-order information about the pixel values. Notes p. 2.17

X ____ (d) **2 pts.** The binary median filter is self-dual with respect to complementation. Notes p. 2.81

X ____ (e) **2 pts.** Histogram equalization is an example of a non-linear point operation. Notes p. 3.27

____ X (f) **2 pts.** Geometric image operations modify the image gray levels, but not the spatial positions. Notes p. 3.59

X ____ (g) **2 pts.** For any practical digital image $\mathbf{I}$ that is real-valued, the 2-D DFT $\tilde{\mathbf{I}}$ is conjugate symmetric. Notes p. 4.59

X ____ (h) **2 pts.** For any practical digital image $\mathbf{I}$ that is complex-valued, the 2-D DFT $\tilde{\mathbf{I}}$ is periodic. Notes p. 3.65

X ____ (i) **2 pts.** For any finite-sized digital image $\mathbf{I}$, the 2-D DFT $\tilde{\mathbf{I}}$ is given by samples of the discrete-space Fourier transform (DSFT) $\tilde{\mathbf{I}}_D$. Notes p. 3.108

OH MY!

____ ____ (j) **2 pt.** Vladimir Putin secretly likes to wear women's underwear.

2. **20 pts.** Consider the 4×4 image I shown below, where the allowable range of gray levels is $0 \le I(i,j) \le 15$:

$$\mathbf{I} = \begin{array}{|c|c|c|c|} \hline \mathbf{4} & \mathbf{5} & \mathbf{5} & \mathbf{11} \\ \hline \mathbf{4} & \mathbf{6} & \mathbf{10} & \mathbf{8} \\ \hline \mathbf{6} & \mathbf{7} & \mathbf{9} & \mathbf{7} \\ \hline \mathbf{10} & \mathbf{9} & \mathbf{8} & \mathbf{11} \\ \hline \end{array}$$

Construct a new image **K** by applying the histogram shaping algorithm to make the histogram more "U-like." The **desired** histogram shape is given by:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\mathrm{H}_{\mathbf{K}}(k)$	3	2	2	1	0	0	0	0	0	0	0	0	1	2	2	3

Show the new image **K** and its histogram $\mathrm{H}_{\mathbf{K}}$ in the spaces provided below.

$$\mathbf{K} = \begin{array}{|c|c|c|c|} \hline 0 & 1 & 1 & 15 \\ \hline 0 & 2 & 15 & 13 \\ \hline 2 & 3 & 14 & 3 \\ \hline 15 & 14 & 13 & 15 \\ \hline \end{array}$$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\mathrm{H}_{\mathbf{K}}(k)$	2	2	2	2	0	0	0	0	0	0	0	0	0	2	2	4

Work space is provided on the next page.

Workspace for Problem 2:

For I :

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	0	0	0	2	2	2	2	2	2	2	2	0	0	0	0
p(k)	0	0	0	0	2/16	2/16	2/16	2/16	2/16	2/16	2/16	2/16	0	0	0	0
P(k)	0	0	0	0	2/16	4/16	6/16	8/16	10/16	12/16	14/16	16/16	16/16	16/16	16/16	16/16

DESIRED :

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	3	2	2	1	0	0	0	0	0	0	0	0	1	2	2	3
p(k)	3/16	2/16	2/16	1/16	0	0	0	0	0	0	0	0	1/16	2/16	2/16	3/16
P(k)	3/16	5/16	7/16	8/16	8/16	8/16	8/16	8/16	8/16	8/16	8/16	8/16	9/16	11/16	13/16	16/16

J =

($P_I(I(m,n))$)

Notes p. 3.36

2/16	4/16	4/16	16/16
2/16	6/16	14/16	10/16
6/16	8/16	12/16	8/16
14/16	12/16	10/16	16/16

K =

Notes p. 3.43

0	1	1	15
0	2	15	13
2	3	14	3
15	14	13	15

3. **20 pts**. The 512×512 gray scale image $\mathbf{I}_1$ shown below has 8-bit pixels. This image was thresholded to obtain the binary image $\mathbf{I}_2$, which is also shown below. In $\mathbf{I}_2$, the pixel value 255 (WHITE) represents LOGIC ONE and the pixel value zero (BLACK) represents LOGIC ZERO.

$\mathbf{I}_1$

$\mathbf{I}_2$

Three binary filters were applied to the image $\mathbf{I}_2$:

- a binary morphological OPEN with a 5×5 diamond-shaped structuring element,
- a binary morphological CLOSE with a 5×5 diamond-shaped structuring element, and
- a binary median filter with a 9×9 diamond-shaped structuring element.

The three resulting output images $\mathbf{J}_1$ - $\mathbf{J}_3$ are shown on the next page.

(a) **15 pts**. Determine which output image resulted from each filtering operation. Explain your answer.

(b) **5 pts**. Explain why a structuring element size of 5×5 for the binary morphological OPEN and CLOSE filters was compared to a structuring element size of 9×9 for the binary median filter.

Problem 3 cont...

Small white objects remain

Small black objects are killed

Small white and black objects are killed

$\mathbf{J}_1$

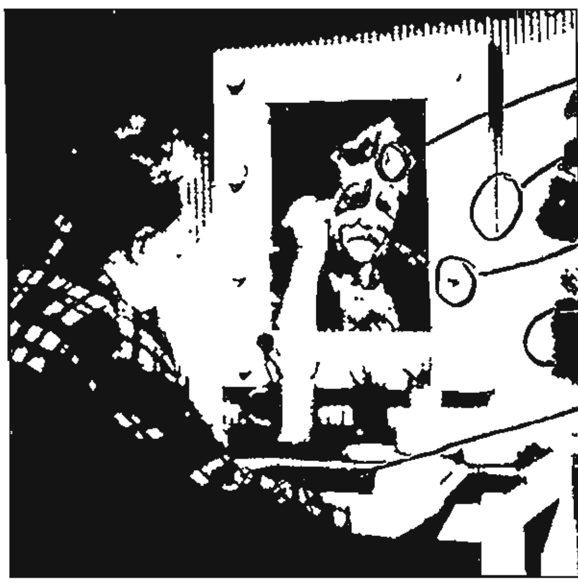
CLOSE

$\mathbf{J}_2$

MEDIAN

$\mathbf{J}_3$

OPEN



small black objects remain

small white objects are killed

(space for your answers is provided on the next page)

More space for Problem 3:

a) In J_1, small white objects (LOGIC ONE) like thin white lines on the wall, a small spot above the left eye in the mirror, and small white horizontal stripes along the right edge of the image are preserved. But small black objects (LOGIC ZERO) like thin black lines on the wall and contours around the eyes are killed. ⟶ **J_1 is CLOSE**

In J_2, any small object is killed... it doesn't matter if it's white or black. ⟶ **J_2 is MEDIAN**

In J_3, small white objects like thin white lines on the wall and sleeve are killed. But small black objects like thin black lines on the wall and mirror edge and a small black spot in the hair persist. ⟶ **J_3 is OPEN**

---

b) The reason is that OPEN and CLOSE each involve two passes of the structuring element, whereas MEDIAN involves only one pass. For OPEN and CLOSE with a 5x5 structuring element, this increases the effective spatial extent of the operation from 5x5 to 9x9.

For example, OPEN is ERODE followed by DILATE. In the EROSION result, pixel (m,n) depends on $I(m-2,n)$, $I(m-1,n)$, $I(m,n)$, $I(m+1,n)$ and $I(m+2,n)$... and others. The DILATION result at (m,n) then depends on the ERODED pixels, at $(m-2,n)$ through $(m+2,n)$. But these depend on the original image pixels $I(m-4,n)$ through $I(m+4,n)$. Thus, the effective spatial extent of the OPEN is 9x9.

4. **20 pts.** Consider a 6×6 digital image I given by

$$I(m,n) = 3 + \delta(m,n) + \cos\left[\frac{2\pi}{6}(m+2n)\right] + \sin\left[\frac{2\pi}{6}(-2m-2n)\right],$$

where m = column and n = row.

(a) **10 pts.** Find a closed form expression for the DFT $\tilde{\mathrm{I}}$.

Notes p. 4.126: $2 \longleftrightarrow 2 \cdot 6 \cdot 6\,\delta(u,v) = 72\delta(u,v)$

Notes p. 4.127: $\delta(m,n) \longleftrightarrow 1$

Notes p. 4.128: $\cos\left[\frac{2\pi}{6}(m+2n)\right] \longleftrightarrow \frac{1}{2}\cdot 6 \cdot 6\left[\delta(u-1,v-2)+\delta(u+1,v+2)\right]$

$= 18\left[\delta(u-1,v-2)+\delta(u+1,v+2)\right]$

Notes p. 4.129: $\sin\left[\frac{2\pi}{6}(-2m-2n)\right] \longleftrightarrow j\frac{1}{2}\cdot 6 \cdot 6\left[\delta(u-2,v-2)-\delta(u+2,v+2)\right]$

$= j18\left[\delta(u-2,v-2)-\delta(u+2,v+2)\right]$

$$\boxed{\tilde{I}(u,v) = 72\delta(u,v) + 1 + 18\left[\delta(u-1,v-2)+\delta(u+1,v+2)\right] + j18\left[\delta(u-2,v-2)-\delta(u+2,v+2)\right]}$$

(b) **10 pts.** Show the real and imaginary parts of the centered DFT array in the space provided below:

$\tilde{\mathrm{I}}$ =

$v \backslash u$	-3	-2	-1	0	1	2
-3	1	1	1	1	1	1
-2	1	1	19	1	1	1
-1	1	1	1	1	1	1
0	1	1	1	73	1	1
1	1	1	1	1	1	1
2	1	1	1	1	19	1

$+ \quad j \times$

$v \backslash u$	-3	-2	-1	0	1	2
-3	0	0	0	0	0	0
-2	0	-18	0	0	0	0
-1	0	0	0	0	0	0
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	18

5. **20 pts.** Draw lines to match the images with their log-magnitude DFT spectra.