

# ECE 5273

## Test 1

Thursday, March 28, 2024

4:30 PM - 5:45 PM

Spring 2024

Dr. Havlicek

Name: SOLUTION

Student Num: \_\_\_\_\_

**Directions:** This is an open notes test. You may use the official course lecture notes and a calculator. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

### GOOD LUCK!

SCORE:

1. (20) \_\_\_\_\_

2. (20) \_\_\_\_\_

3. (20) \_\_\_\_\_

4. (20) \_\_\_\_\_

5. (20) \_\_\_\_\_

\_\_\_\_\_

TOTAL (100):

\_\_\_\_\_

*On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.*

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1. 20 pts. True or False. Mark *True* only if the statement is **always** true.

TRUE FALSE

X           (a) 2 pts. The YCbCr color space was originally used for color TV so that black and white TV's could continue to function by receiving and displaying only the Y component.

          X (b) 2 pts. Any digital image can be exactly reconstructed from its histogram.

X           (c) 2 pts. For a given structuring element or window, the binary Median and Majority filters are identical.

          X (d) 2 pts. Blob coloring is a simple method for generating the histogram of a color image.

X           (e) 2 pts. The binary OPEN and CLOSE filters generally do not affect the overall sizes of objects that are sufficiently large.

X           (f) 2 pts. Snapping a picture with your cell phone camera is an example of reflection imaging.

          X (g) 2 pts. MRI is a type of ~~absorption~~ Emission imaging.

          X (h) 2 pts. The 2D centered DFT is useful and intuitive because it displays the ~~highest~~ Lowest spatial frequencies in the center of the DFT magnitude image.

X           (i) 2 pts. The 2D discrete-space Fourier transform (DSFT) of any digital image is periodic.

OH MY!           (j) 2 pts. Trump will be president of the United States **again**.

2. **20 pts.** Consider the  $4 \times 4$  image **I** shown below, where the allowable range of gray levels is  $0 \leq I(i, j) \leq 15$ :

$$\mathbf{I} = \begin{array}{|c|c|c|c|} \hline \mathbf{4} & \mathbf{5} & \mathbf{9} & \mathbf{7} \\ \hline \mathbf{6} & \mathbf{10} & \mathbf{8} & \mathbf{9} \\ \hline \mathbf{7} & \mathbf{10} & \mathbf{11} & \mathbf{8} \\ \hline \mathbf{4} & \mathbf{6} & \mathbf{5} & \mathbf{11} \\ \hline \end{array}$$

Construct a new image **K** by applying the histogram shaping algorithm to make the histogram more "V-like." The **desired** histogram shape is given by:

|                     |   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |
|---------------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| $k$                 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| $H_{\mathbf{K}}(k)$ | 3 | 0 | 2 | 0 | 1 | 0 | 1 | 0 | 0 | 2 | 0  | 0  | 3  | 0  | 0  | 4  |

Show the new image **K** and its histogram  $H_{\mathbf{K}}$  in the spaces provided below.

$$\mathbf{K} = \begin{array}{|c|c|c|c|} \hline 0 & 2 & 12 & 9 \\ \hline 4 & 15 & 12 & 12 \\ \hline 9 & 15 & 15 & 12 \\ \hline 0 & 4 & 2 & 15 \\ \hline \end{array}$$

|                     |   |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |
|---------------------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| $k$                 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| $H_{\mathbf{K}}(k)$ | 2 | 0 | 2 | 0 | 2 | 0 | 0 | 0 | 0 | 2 | 0  | 0  | 4  | 0  | 0  | 4  |

Work space is provided on the next page.

Workspace for Problem 2:

$$J = \begin{matrix} 2/16 & 4/16 & 12/16 & 8/16 \\ 6/16 & 14/16 & 10/16 & 12/16 \\ 8/16 & 14/16 & 16/16 & 10/16 \\ 2/16 & 6/16 & 4/16 & 16/16 \end{matrix} = P_I [I(m,n)]$$

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|  |  |  |  |
|  |  |  |  |

For:  
I:

| k    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8     | 9     | 10    | 11    | 12    | 13    | 14    | 15    |
|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| H(k) | 0    | 0    | 0    | 0    | 2    | 2    | 2    | 2    | 2     | 2     | 2     | 2     | 0     | 0     | 0     | 0     |
| p(k) | 0/16 | 0/16 | 0/16 | 0/16 | 2/16 | 2/16 | 2/16 | 2/16 | 2/16  | 2/16  | 2/16  | 2/16  | 0/16  | 0/16  | 0/16  | 0/16  |
| P(k) | 0/16 | 0/16 | 0/16 | 0/16 | 2/16 | 4/16 | 6/16 | 8/16 | 10/16 | 12/16 | 14/16 | 16/16 | 16/16 | 16/16 | 16/16 | 16/16 |

DESIRED:

| k    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12    | 13    | 14    | 15    |
|------|------|------|------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|
| H(k) | 3    | 0    | 2    | 0    | 1    | 0    | 1    | 0    | 0    | 2    | 0    | 0    | 3     | 0     | 0     | 4     |
| p(k) | 3/16 | 0/16 | 2/16 | 0/16 | 1/16 | 0/16 | 1/16 | 0/16 | 0/16 | 2/16 | 0/16 | 0/16 | 3/16  | 0/16  | 0/16  | 4/16  |
| P(k) | 3/16 | 3/16 | 5/16 | 5/16 | 6/16 | 6/16 | 7/16 | 7/16 | 7/16 | 9/16 | 9/16 | 9/16 | 12/16 | 12/16 | 12/16 | 16/16 |

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$$K = \arg \min_r \{ P_k(r), J(m,n) \} =$$

|   |    |    |    |
|---|----|----|----|
| 0 | 2  | 12 | 9  |
| 4 | 15 | 12 | 12 |
| 9 | 15 | 15 | 12 |
| 0 | 4  | 2  | 15 |

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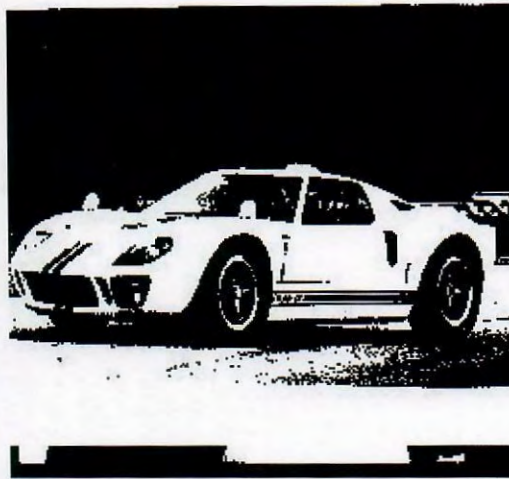
|  |  |  |  |
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3. 20 pts. The gray scale image  $I_1$  shown below has 8-bit pixels. This image was thresholded to obtain the binary image  $I_2$ , which is also shown below. In  $I_2$ , the pixel value 255 (WHITE) represents LOGIC\_ONE and the pixel value zero (BLACK) represents LOGIC\_ZERO.

$I_1$

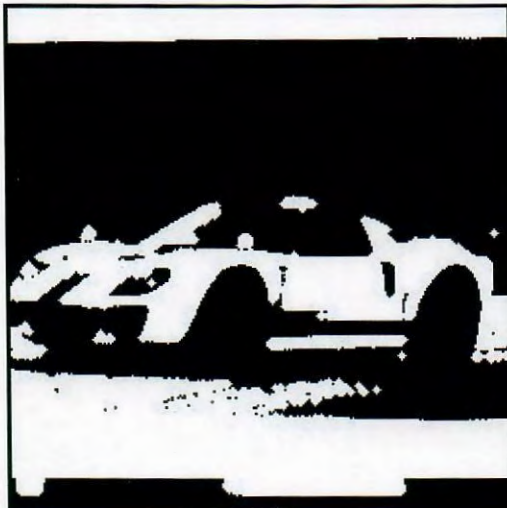


$I_2$



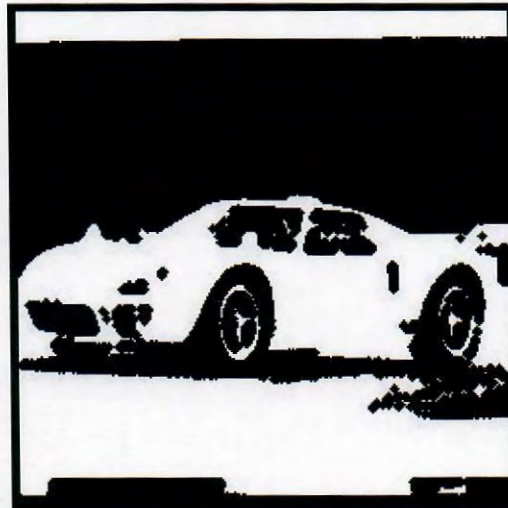
Binary morphological OPEN and CLOSE operations were performed on the image  $I_2$  using a  $5 \times 5$  diamond-shaped structuring element. The resulting images are shown as  $J_1$  and  $J_2$  below.

$J_1$



OPEN

$J_2$



CLOSE

OPEN

CLOSE

Determine which image is the result of the OPEN operation and which is the result of the CLOSE operation. Explain your answer in the space provided on the next page.

### Workspace for Problem 3...

$$J_1 = \text{OPEN}(I_2).$$

- OPEN applies the erosion first. The main effects are:

- ① Large white objects are eroded by about 2 to 3 pixels, becoming smaller.
- ② Small white objects are completely eliminated.

- OPEN then applies dilation second.

- ① restores larger white objects to approximately original size.
- ② but the smaller white objects do not come back... they are permanently removed.

⇒ This can be seen, e.g., in the wheels and windows of the car in  $J_1$ .

$$J_2 = \text{CLOSE}(I_2).$$

- CLOSE applies the dilation first. This expands all white objects by about 2 to 3 pixels. For the black objects;

- ① Larger black objects become smaller by about 2 to 3 pixels.
- ② smaller black objects are completely eliminated.

- CLOSE then applies the erosion second.

- ① restores white objects to approximately original size.
- ② restores larger black objects to approximately original size.
- ③ but the smaller black objects do not come back... they are permanently removed.

⇒ This can be seen, e.g., in the racing stripes and headlamps of the car in  $J_2$ .

4. 20 pts. Consider a  $6 \times 6$  digital image  $I$  given by

$$I(m, n) = 4 - 144\delta(m, n) + 2 \cos \left[ \frac{2\pi}{6}(m + 2n) \right] + \sin \left[ \frac{2\pi}{6}(-2m + n) \right],$$

where  $m = \text{column}$  and  $n = \text{row}$ .

(a) 10 pts. Find a closed form expression for the DFT  $\tilde{I}$ .

Notes p. 4.126:  $4 \xrightarrow{\text{DFT}} 4(6)(6)\delta(u, v) = 144\delta(u, v)$

Notes p. 4.127:  $-144\delta(m, n) \xleftrightarrow{\text{DFT}} -144$

Notes p. 4.128:  $2 \cos \left[ 2\pi \left( \frac{1}{6}m + \frac{2}{6}n \right) \right] \xleftrightarrow{\text{DFT}} \left( \frac{2}{2} \right) (6)(6) [\delta(u-1, v-2) + \delta(u+1, v+2)]$   
 $= 36 [\delta(u-1, v-2) + \delta(u+1, v+2)]$

Notes p. 4.129:  $\sin \left[ 2\pi \left( -\frac{2}{6}m + \frac{1}{6}n \right) \right] \xleftrightarrow{\text{DFT}} j \left( \frac{1}{2} \right) (6)(6) [\delta(u-2, v+1) - \delta(u+2, v-1)]$   
 $= j18 [\delta(u-2, v+1) - \delta(u+2, v-1)]$

$$\tilde{I}(u, v) = 144\delta(u, v) - 144 + 36 [\delta(u-1, v-2) + \delta(u+1, v+2)] + j18 [\delta(u-2, v+1) - \delta(u+2, v-1)]$$

(b) 10 pts. Show the real and imaginary parts of the centered DFT array in the space provided below:

|     |     |    |      |    |      |   |   |
|-----|-----|----|------|----|------|---|---|
|     | $u$ |    |      |    |      |   |   |
| $v$ |     | -3 | -2   | -1 | 0    | 1 | 2 |
| -3  | A   | A  | A    | A  | A    | A |   |
| -2  | A   | A  | -108 | A  | A    | A |   |
| -1  | A   | A  | A    | A  | A    | A |   |
| 0   | A   | A  | A    | 0  | A    | A |   |
| 1   | A   | A  | A    | A  | A    | A |   |
| 2   | A   | A  | A    | A  | -108 | A |   |

|     |     |     |    |    |   |    |   |
|-----|-----|-----|----|----|---|----|---|
|     | $u$ |     |    |    |   |    |   |
| $v$ |     | -3  | -2 | -1 | 0 | 1  | 2 |
| -3  | 0   | 0   | 0  | 0  | 0 | 0  |   |
| -2  | 0   | 0   | 0  | 0  | 0 | 0  |   |
| -1  | 0   | 0   | 0  | 0  | 0 | 18 |   |
| 0   | 0   | 0   | 0  | 0  | 0 | 0  |   |
| 1   | 0   | -18 | 0  | 0  | 0 | 0  |   |
| 2   | 0   | 0   | 0  | 0  | 0 | 0  |   |

$A = -144$

5. 20 pts. Draw lines to match the images with their log-magnitude DFT spectra.

The image displays a matching exercise between four original images and their corresponding log-magnitude DFT spectra. The original images are arranged vertically on the left, and their spectra are arranged vertically on the right. Colored arrows indicate the correct matches:

- Top image (Ship):** A black and white photograph of a fishing boat named "LACONIQUE" docked at a pier. Its DFT spectrum (top-right) shows a central peak with a complex, multi-lobed radial pattern.
- Second image (Putin):** A black and white portrait of Vladimir Putin against a map of Russia. Its DFT spectrum (middle-right) shows a central peak with a radial pattern that is more uniform than the ship's.
- Third image (Trump):** A black and white portrait of Donald Trump with an American flag in the background. Its DFT spectrum (bottom-right) shows a central peak with a very simple, sparse radial pattern.
- Bottom image (Clock):** A black and white photograph of a round analog clock. Its DFT spectrum (top-left) shows a central peak with a radial pattern that is very similar to the ship's spectrum.

Connections shown by arrows:

- Red arrow from Ship to its spectrum.
- Red arrow from Putin to its spectrum.
- Blue arrow from Trump to its spectrum.
- Green arrow from Clock to its spectrum.