ECE 5273
Test 1

Wednesday, March 4, 1998, 4:30 PM - 5:45 PM

Spring 1998 — Name: SOLUTION

Dr. Havlicek — Student Num: ____________

Directions: There are **seven** problems on this test. You have 75 minutes to complete the test. All work must be your own. You may use one 8.5 × 11 inch two-sided note sheet.

All Students: Work problems 1 and 2.

Students enrolled for undergraduate credit: Work **three** problems out of problems 3 through 7. Each of these problems counts **20 pts**. Below, circle the numbers of the **three** problems you wish to have graded.

Students enrolled for graduate credit: Work **four** problems out of problems 3 through 7. Each of these problems counts **15 pts**. Below, circle the numbers of the **four** problems you wish to have graded.

SHOW ALL OF YOUR WORK for maximum partial credit! Good Luck!

Circle the numbers of the problems you wish to have graded:

1. 2. 3. 4. 5. 6. 7.

SCORE:

1. (25) ________	4. (20/15) ________	7. (20/15) ________
2. (15) ________	5. (20/15) ________	
3. (20/15) ________	6. (20/15) ________	

TOTAL (100):

1. **25 pts**. True or False. Mark the correct answer. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
___	X	(a) **3 pts**. The number of bits in a 256×256 image with 8-bits per pixel is 524,286.
___	X	(b) **2 pts**. Because of the symmetries in the DFT and IDFT equations, the original image is always conjugate symmetric.
___	X	(c) **2 pts**. For viewing the DFT of a real-valued image, it is usually most useful to display the logarithm of the DFT phase.
X	___	(d) **2 pts**. An image can be exactly recovered from its DFT.
___	X	(e) **2 pts**. An image can be exactly recovered from its histogram.
X	___	(f) **2 pts**. If a continuous image $\mathbb{I}_C$ is sampled using sampling frequencies greater than the Nyquist rates, then the DFT of the sampled image can be derived by sampling the continuous Fourier transform of $\mathbb{I}_C$.
X	___	(g) **2 pts**. The OPEN-CLOSE operation can be implemented as a sequence of ERODE and DILATE operations.
___	X	(h) **2 pts**. When histogram equalization (histogram flattening) is applied to a digital image, the resulting histogram is always perfectly flat.
___	X	(h) **2 pts**. Run-length coding always reduces the amount of memory needed to store a digital image.
X	___	(h) **2 pts**. Any 8×8 digital image can be written uniquely as a sum of 64 8×8 complex exponential images.
___	X	(h) **2 pts**. For computing the DFT of an image, the FFT algorithm provides a noticeable speedup in the computation only if the image is 512×512 or larger.
X	___	(j) **3 pts**. The stories that most young American children are told about Santa Claus are myths.

2. **15 pts**. Short answer.

(a) **3 pts**. Synthetic aperture radar and electron microscopy are two types of reflection imaging. Name one type of emission imaging and one type of absorption imaging.

Emission: thermal, infrared, astronomy, nuclear

Absorption: X-ray, optical microscopy, tomography (CAT, PET, NMR), Vibra-size.

(b) **3 pts**. Name a 2D function that has the same functional form as its DFT.

The 2D Gaussian.

(c) **3 pts**. Which binary morphological operation tends to link neighboring objects together: CLOSE-OPEN or OPEN-CLOSE?

CLOSE-OPEN.

(d) **3 pts**. Name a binary image processing operator that is idempotent.

OPEN and CLOSE are both idempotent.

(e) **3 pts**. Explain why a binary image of all logical zeros is a *root signal* of the ERODE operation.

The output of a binary erode is the "And" of all pixels covered by the structuring element.

If the image is all zeros, the result of the "AND" operation will be ZERO at every pixel.

Therefore, the output image is all ZERO's.

So any number of repeated applications of ERODE does not change the image.

3. **UG 20 pts. / G 15 pts.** Find the DFT of the 256×256 image $\mathbb{I}$ defined by

$$I(i,j) = 40\cos\left[\frac{\pi}{64}(10i + 20j)\right] + 105 + 58\delta(i,j),$$

where $\delta(i,j)$ is the unit pulse.

$N = 256$

$$I(i,j) = 40\cos\left[\frac{2\pi}{256}(20i + 40j)\right] + 105 + 58\delta(i,j)$$

$$\mathrm{DFT}\left\{40\cos\left[\frac{2\pi}{256}(20i+40j)\right]\right\} = \frac{40(256)}{2}\left[\delta(u-20, v-40) + \delta(u+20, v+40)\right]$$

$$= 5120\left[\delta(u-20, v-40) + \delta(u+20, v+40)\right]$$

$$\mathrm{DFT}\{105\} = (105)(256)\delta(u,v) = 26880\,\delta(u,v)$$

$$\mathrm{DFT}\{58\,\delta(i,j)\} = \frac{58}{256} = \frac{29}{128}$$

$$\boxed{\tilde{I}(u,v) = 5120\left[\delta(u-20, v-40) + \delta(u+20, v+40)\right] + 26880\,\delta(u,v) + \frac{29}{128}}$$

4. **UG 20 pts. / G 15 pts.** Pixels in the 4×4 image $\mathbb{I}$ shown below take gray levels in the range $\{0, 1, 2, \ldots, 15\}$. Perform histogram equalization (histogram flattening) on the image so that the new image $\mathbb{J}$ has a flatter histogram.

$$\mathbb{I} = \begin{array}{|c|c|c|c|} \hline 2 & 2 & 3 & 4 \\ \hline 2 & 3 & 2 & 4 \\ \hline 5 & 6 & 8 & 11 \\ \hline 5 & 5 & 14 & 14 \\ \hline \end{array}$$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H_{\mathbb{I}}(k)$	0	0	4	2	2	3	1	0	1	0	0	1	0	0	2	0
p(k)	0	0	$\frac{4}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	0	$\frac{1}{16}$	0	0	$\frac{1}{16}$	0	0	$\frac{2}{16}$	0

You may use this image as a work space:

$$\mathbb{J}_1 = \begin{array}{|c|c|c|c|} \hline \frac{4}{16} & \frac{4}{16} & \frac{6}{16} & \frac{8}{16} \\ \hline \frac{4}{16} & \frac{6}{16} & \frac{4}{16} & \frac{8}{16} \\ \hline \frac{11}{16} & \frac{12}{16} & \frac{13}{16} & \frac{14}{16} \\ \hline \frac{11}{16} & \frac{11}{16} & \frac{16}{16} & \frac{16}{16} \\ \hline \end{array}$$

Show the new image here:

$$\mathbb{J} = \begin{array}{|c|c|c|c|} \hline 4 & 4 & 6 & 8 \\ \hline 4 & 6 & 4 & 8 \\ \hline 10 & 11 & 12 & 13 \\ \hline 10 & 10 & 15 & 15 \\ \hline \end{array}$$

5. **UG 20 pts. / G 15 pts.** Consider sampling the continuous image $\mathbb{I}_C$ defined by

$$I_C(x,y) = 100\,\mathrm{sinc}(20x)\,\mathrm{sinc}(30y),$$

where x and y have units of miles. According to the sampling theorem, what are the maximum sample spacings X and Y that can be used without the occurrence of aliasing?

$$\tilde{I}_c(\omega_x, \omega_y) = \begin{cases} 100, & |\omega_x| \le 20 \text{ and } |\omega_y| \le 30 \\ 0, & \text{otherwise} \end{cases}$$

So $\Omega_x = 20$ and $\Omega_y = 30$.

To prevent aliasing, we need

$$\frac{1}{X} \geq 2\Omega_x \qquad \frac{1}{Y} \geq 2\Omega_y$$

$$\frac{1}{X} \geq 40 \qquad \frac{1}{Y} \geq 60$$

$$1 \geq 40X \qquad 1 \geq 60Y$$

$$X \le \tfrac{1}{40} \text{ mile} \qquad Y \le \tfrac{1}{60} \text{ mile}$$

The maximum sample spacings for no aliasing are

$$\boxed{X = \tfrac{1}{40} \text{ mile} \quad \text{and} \quad Y = \tfrac{1}{60} \text{ mile}}$$

6. **UG 20 pts. / G 15 pts.** A 3D scene consisting of a black line is imaged with a camera having a focal length of 35mm. In 3D space, the endpoints of the line are P_1 and P_2.

The 3D space coordinates of P_1 are (X,Y,Z) = (1.0m,2.0m,2.5m).

The 3D space coordinates of P_2 are (X,Y,Z) = (1.5m,1.0m,2.0m).

In the image plane, find the 2D image coordinates of the projections of P_1 and P_2.

In the image plane, what is the length of the image of the line?

$f = 35mm$

project P_1: $(x_1, y_1) = \frac{f}{Z}(X_1, Y_1) = \frac{35mm}{2.5m}(1m, 2m) = (14mm, 28mm)$

project P_2: $(x_2, y_2) = \frac{f}{Z}(X_2, Y_2) = \frac{35mm}{2m}(1.5m, 1m) = (26.25mm, 17.5mm)$

Find length in image plane:

$x_2 - x_1 = 26.25mm - 14mm = 12.25mm$

$y_2 - y_1 = 17.5mm - 28mm = -10.5mm$

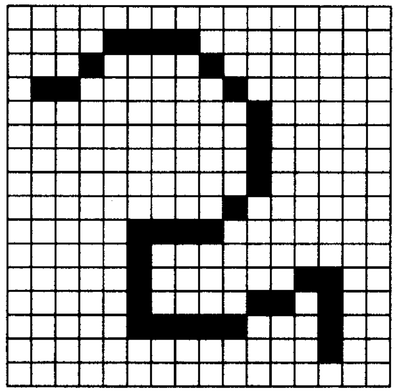
$length = [12.25^2 + (-10.5)^2]^{1/2} = 16.134203mm$

P_1: $(x_1, y_1) = (14mm, 28mm)$

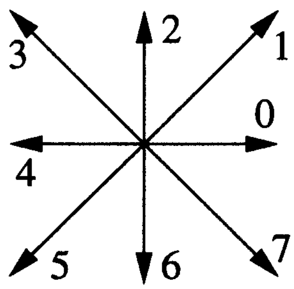
P_2: $(x_2, y_2) = (26.25mm, 17.5mm)$

Length = 16.134203 mm

7. **UG 20 pts. / G 15 pts.** Consider the 16×16 binary contour image $\mathbb{I}$ shown below. In the image, white pixels have the value logical ZERO and black pixels have the value logical ONE.

$\mathbb{I} =$ 

The upper left corner of the image is $I(0,0)$, the upper right corner is $I(15,0)$, and the lower right corner is $I(15,15)$. The image contains a single contour with initial point $I(1,3)$ (col=1, row=3) and final point $I(13,14)$. Find a chain code for the contour using the 8-neighbor direction codes shown below:



As a flag to indicate the end of the contour, give the code for the direction that "goes back on itself." For example, if the final direction code is 0, then the flag is 4.

$i_0 = 1$, $j_0 = 3$.

CODE: 1, 3, 0, 1, 1, 0, 0, 0, 7, 7, 7, 6, 6, 6, 5, 5, 4, 4, 4,
6, 6, 6, 6, 0, 0, 0, 0, 1, 0, 1, 0, 6, 6, 6, 2