ECE 5273/CS 5273
Test 2

Wednesday, April 24, 2002
5:00 PM - 6:15 PM

Spring 2002 Name: SOLUTION

Dr. Havlicek Student Num:________________

Directions: This is an open notes, open book test. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________

2. (25) ________

3. (25) ________

4. (25) ________

TOTAL (100):

1. **25 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
___	X	(a) **2 pts.** Pointwise multiplying two DFT's is equivalent to linear convolution of the images.
___	X	(b) **2 pts.** To implement the linear convolution of two $N \times N$ images by pointwise multiplication of DFT's, it is generally necessary to zero pad each image to a size of $3N \times 3N$ prior to taking the DFT's.
___	X	(c) **2 pts.** As compared to nonlinear filtering, the great thing about linear image enhancement is that it can usually remove broadband noise without blurring the image edges.
X	___	(d) **2 pts.** For a linear shift invariant system, the unit pulse response $\mathbb{H}$ and frequency response $\tilde{\mathbb{H}}$ form a DFT pair.
X	___	(e) **2 pts.** High-pass filters are typically used to enhance image details and remove blur.
X	___	(f) **3 pts.** Homomorphic filtering can be used to transform a multiplicative noise problem into an additive noise problem.
X	___	(g) **3 pts.** For the linear image restoration problem, the Wiener filter is not of much interest unless there is a blur.
___	X	(h) **3 pts.** The great thing about the pseudo-Wiener filter is that it never fails to produce a visually appealing result.
___	X	(i) **3 pts.** The main tool for analyzing nonlinear filters is the DFT.
___	X	(j) **3 pts.** The Catholic church in America has had a lot of trouble with homomorphic processing over the last 50 years.

2. **25 pts**. Short Answer.

(a) **5 pts.** Briefly explain why smoothing and enhancement are usually regarded as conflicting goals.

Noise reduction usually means that high frequencies must be reduced.
Enhancement usually means that high frequencies must be preserved or increased.

(b) **5 pts.** List the main properties of the median filter.

1. Smooths additive white noise
2. Does not degrade edges
3. Particularly good at removing impulses.

(c) **5 pts.** What is the main difference between the results that are obtained from applying the gray scale morphological filters OPEN and CLOSE?

Both smooth noise and preserve edges.
Close removes negative spikes but not positive spikes
Open removes positive spikes but not negative spikes.

(d) **5 pts.** Can the gray scale morphological filters ERODE and DILATE be described as Order Statistic Filters?

Yes: $\text{Dilate}[\mathbb{I}, \mathbb{B}] = \text{MAX}[\mathbb{I}, \mathbb{B}]$

$\text{Erode}[\mathbb{I}, \mathbb{B}] = \text{MIN}[\mathbb{I}, \mathbb{B}]$

(e) **5 pts.** $\mathbb{I}$ is a constant image corrupted by additive white Laplacian noise with variance $\sigma_{\mathbb{N}}^2 = 100$. The corrupted image is filtered with a 3×3 median filter. What is the approximate variance $\sigma_{\text{filtered}}^2$ of the filtered image?

$$\sigma^2_{N, \text{filtered}} \approx \frac{100}{18} \approx 5.\overline{555}.$$

3. **25 pts.** Pixels in the 6×6 image $\mathbb{I}$ shown below take values in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image $\mathbb{J}$ is also shown below.

$\mathbb{I}$ =

11	**11**	**12**	**12**	**13**	**13**
11	**0**	**12**	**13**	**13**	**13**
12	**12**	**13**	**13**	**13**	**14**
12	**13**	**13**	**13**	**14**	**14**
13	**13**	**13**	**14**	**0**	**14**
13	**13**	**13**	**14**	**14**	**14**

$\mathbb{J}$ =

11	**11**	**12**	**12**	**13**	**13**
11	**0**	**12**	**13**	**99**	**13**
12	**12**	**13**	**13**	**13**	**14**
12	**13**	**13**	**13**	**14**	**14**
13	**13**	**13**	**14**	**0**	**14**
13	**99**	**13**	**14**	**14**	**14**

Design a nonlinear filter to restore the received image by attenuating the noise. Handle edge effects by replication. Explain your solution. Show the restored image $\mathbb{K}$ below. There is workspace on the following page.

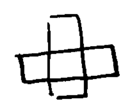
The image contains negative going spikes, so we can't remove those

The noise introduces positive going spikes that need to be removed

⇒ USE OPEN.

To minimize distortion, we want a small 2-D window: use cross(5).

$\mathbb{K} = \text{OPEN}(\mathbb{J}, \text{CROSS}(5))$



Show the restored image here:

$\mathbb{K}$ =

11	11	12	12	13	13
11	0	12	13	13	13
12	12	13	13	13	14
12	13	13	13	14	14
13	13	13	13	0	14
13	13	13	13	14	14

More Workspace for Problem 3...

ERODE JJ

11	0	11	12	12	13
0	0	0	12	13	13
11	0	12	13	13	13
12	12	13	13	0	14
12	13	13	0	0	0
13	13	13	13	0	14

IK=OPEN(JJ)

11	11	12	12	13	13
11	0	12	13	13	13
12	12	13	13	13	14
12	13	13	13	14	14
13	13	13	13	0	14
13	13	13	13	14	14

II-IK

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	-1	0
0	0	0	0	0	0
0	0	0	1	0	0
0	0	0	1	0	0

4. **25 pts.** Consider the 3×3 digital images $\mathbb{I}_1$ and $\mathbb{I}_2$ shown below. Pixels in these images take gray levels in range $\{0, 1, 2, \ldots, 255\}$. Pixel $I(i, j)$ is the pixel in ROW i and COLUMN j of image $\mathbb{I}_1$, where $0 \le i, j \le 2$.

$\mathbb{I}_1 =$

0	3	2
3	1	2
4	1	3

$I_1(-i,-j)$

3	1	4
2	1	3
2	3	0

$\mathbb{I}_2 =$

2	4	6
3	5	3
4	4	3

Let the image $\mathbb{J} = \mathbb{I}_1 * \mathbb{I}_2$ be the **linear** convolution of $\mathbb{I}_1$ and $\mathbb{I}_2$. Compute $\mathbb{J}$ directly. There is more workspace on the next page if you need it. Show your answer below.

$$J(i,j) = \sum_{m=0}^{2} \sum_{n=0}^{2} I_2(m,n)\, I_1(i-m, j-n)$$

$$J(0,0) = I_2(0,0)\, I_1(0,0) = 0 \cdot 2 = 0$$

$$J(0,1) = I_2(0,0)\, I_1(0,1) + I_2(0,1)\, I_1(0,0)$$
$$= 2 \cdot 3 + 4 \cdot 0 = 6$$

$$J(0,2) = I_2(0,0)\, I_1(0,2) + I_2(0,1)\, I_1(0,1) + I_2(0,2)\, I_1(0,0))$$
$$= 2 \cdot 2 + 4 \cdot 3 + 0 \cdot 6 = 4 + 12 = 16$$

$$J(1,0) = I_2(0,0) I_1(1,0) + I_2(1,0) I_1(0,0) = 2 \cdot 3 + 3 \cdot 0 = 6$$

$$J(1,1) = I_2(0,0) I_1(1,1) + I_2(0,1) I_1(1,0) + I_2(1,0) I_1(0,1) + I_2(1,1) I_1(0,0)$$
$$= 2 \cdot 1 + 4 \cdot 3 + 3 \cdot 3 + 5 \cdot 0 = 2 + 12 + 9 = 23$$

$$J(1,2) = 2 \cdot 2 + 4 \cdot 1 + 6 \cdot 3 + 3 \cdot 2 + 5 \cdot 3 + 3 \cdot 0$$
$$= 4 + 4 + 18 + 6 + 15 =$$

$$J(2,0) = 2 \cdot 4 + 3 \cdot 3 + 4 \cdot 0 = 8 + 9 = 17$$

$$J(2,1) = 2 \cdot 1 + 4 \cdot 4 + 3 \cdot 1 + 5 \cdot 3 + 4 \cdot 3 + 4 \cdot 0 = 2 + 16 + 3 + 15 + 12 = 48$$

$\mathbb{J} =$

0	6	16
6	23	47
17	48	74

$$J(2,2) = 2 \cdot 3 + 4 \cdot 1 + 6 \cdot 4 + 3 \cdot 2 + 5 \cdot 1 + 3 \cdot 3$$
$$+ 4 \cdot 2 + 4 \cdot 3 + 3 \cdot 0$$
$$= 6 + 4 + 24 + 6 + 5 + 9 + 8 + 12$$
$$= 74$$

More **Workspace for Problem 4...**