ECE 5273
Test 2

Wednesday, April 30, 2003
5:00 PM - 6:15 PM

Spring 2003

Dr. Havlicek

Name: SOLUTION

Student Num: _____

Directions: This is an open notes, open book test. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) _____
2. (25) _____
3. (25) _____
4. (25) _____

TOTAL (100):

---

1. **25 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
____	X	(a) **2 pts.** Contour coding is an integral part of the JPEG image compression standard.
X	____	(b) **2 pts.** Run-length coding is an integral part of the JPEG image compression standard.
____	X	(c) **2 pts.** For edge detection, the Laplacian operator is less sensitive to noise than the gradient operator.
X	____	(d) **2 pts.** The Laplacian-of-Gaussian (LoG) operator is isotropic.
____	X	(e) **2 pts.** Salt-and-pepper noise is best modeled as an additive noise.
____	X	(f) **2 pts.** A "good" mother wavelet $\psi(t)$ is a low-pass function, whereas the scaling function $\phi(t)$ is high-pass.
____	X	(g) **2 pts.** The translates of the scaling function $\phi(t)$ at the scale 2^j form an orthonormal basis for the detail space O_{2^j}.
____	X	(h) **2 pts.** Laplacian noise is the most common type of noise that occurs in nature.
X	____	(i) **2 pts.** The WMMR filters are a subset of the OS filters.
____	X	(j) **2 pts.** Homomorphic filtering is most useful for processing images that have been corrupted by additive noise.
____	X	(k) **2 pts.** Edge thinning and edge linking are always required after application of the LoG edge detector.
____	X	(l) **3 pts.** The camera man used to be an ECE professor at OU, but he flunked tenure.

2. **25 pts.** Short Answer.

(a) **3 pts.** A very noisy image is smoothed with a low-pass filter. Should this operation increase or decrease the entropy of the image?

Decrease. Low-pass filtering introduces increased spatial correlations between pixels. In other words, it tends to make the values of different pixels more similar. This narrows the histogram and lowers the entropy.

(b) **3 pts.** Name an effective technique for lossless entropy reduction.

- Huffman coding
- DPCM.

(c) **3 pts.** Explain what people mean when they describe the median filter as a "low-pass" filter. Strictly speaking, is this correct? What is the frequency response of the median filter?

They mean that the median filter tends to remove small objects which are rich in high frequencies. However, this is not strictly correct because the MF is nonlinear... and therefore has no frequency response.

(d) **3 pts.** Can the gray scale morphological opening be implemented by multiplication of DFT's? Briefly explain.

NO. OPEN is a nonlinear filter. It does not have a DFT frequency domain interpretation.

Problem 2, cont...

(e) **3 pts.** Which DFT coefficient in $\tilde{\mathbb{I}}$ is numerically equal to N times the average optical density AOD($\mathbb{I}$)?

$\tilde{\mathbb{I}}(0,0)$: the "DC" coefficient

(f) **3 pts.** $\mathbb{I}$ is a constant image corrupted by additive white Gaussian noise with variance $\sigma_N^2 = 77$. The corrupted image is filtered with a 7×7 median filter. What is the approximate variance $\sigma^2_{\text{filtered}}$ of the filtered image?

$2M+1 = 49$

$$\sigma^2_{\text{filtered}} \approx \frac{\sigma^2_{IN}}{(.75)(2M+1)} = \frac{77}{(.75)(49)} = 2.095$$

(g) **7 pts.** The meaning of the figure on page 6.30 of the notes is that, for certain filtering operations, there is a relationship between the binary version of the filter and the gray scale version. Specifically, the class of filters known as *Stack Filters* includes all of the ones for which the gray scale filter can be implemented by: 1) using thresholds to decompose the input image into a series of *binary* threshold signals (images), 2) applying the binary version of the filter at each threshold level, and 3) combining the resulting binary output images to obtain the gray scale output image.

What requirement does the binary version of a filter have to satisfy in order for the corresponding gray scale filter to be a stack filter? **Hint:** the median filter is a stack filter. The minimum sum of products expression for a binary median filter with binary inputs a, b, and c is $y = ab + ac + bc$.

The equation for the binary filter must be a positive boolean function.

In other words, there can be no complemented variables in its minimum sum of products formulation.

3. **25 pts.** For a vector `X` (horizontal or vertical), the matlab command `median(X)` returns the median of the elements in `X`.

However, for a $N \times N$ matrix `A`, the matlab command `median(A)` returns an N-element row vector M. Each element of M is the median of the corresponding column of `A`. For example, $M(2)$ is the median of the second column of `A`.

In homework 7, you were asked to implement a 2D 3×3 median filter. Many of you did this as follows. First, you collected up the nine pixels in the 3×3 window set into a 3×3 matrix `W`. Then you used a call such as:

```
J(row,col) = median(median(W));
```

Is this a correct implementation of the 2D median filter? Why or why not? In answering, you may wish to consider the following window set:

$$\mathtt{W} = \begin{matrix} 1 & 4 & 5 \\ 2 & 50 & 70 \\ 3 & 60 & 80 \end{matrix}$$

Sorting the set W given above, we obtain

1 2 3 4 5 50 60 70 80

↑

So the median is 5.

Applying median(median(W)) = median(2 50 70) we obtain 70, which is not the median.

So this implementation is incorrect.

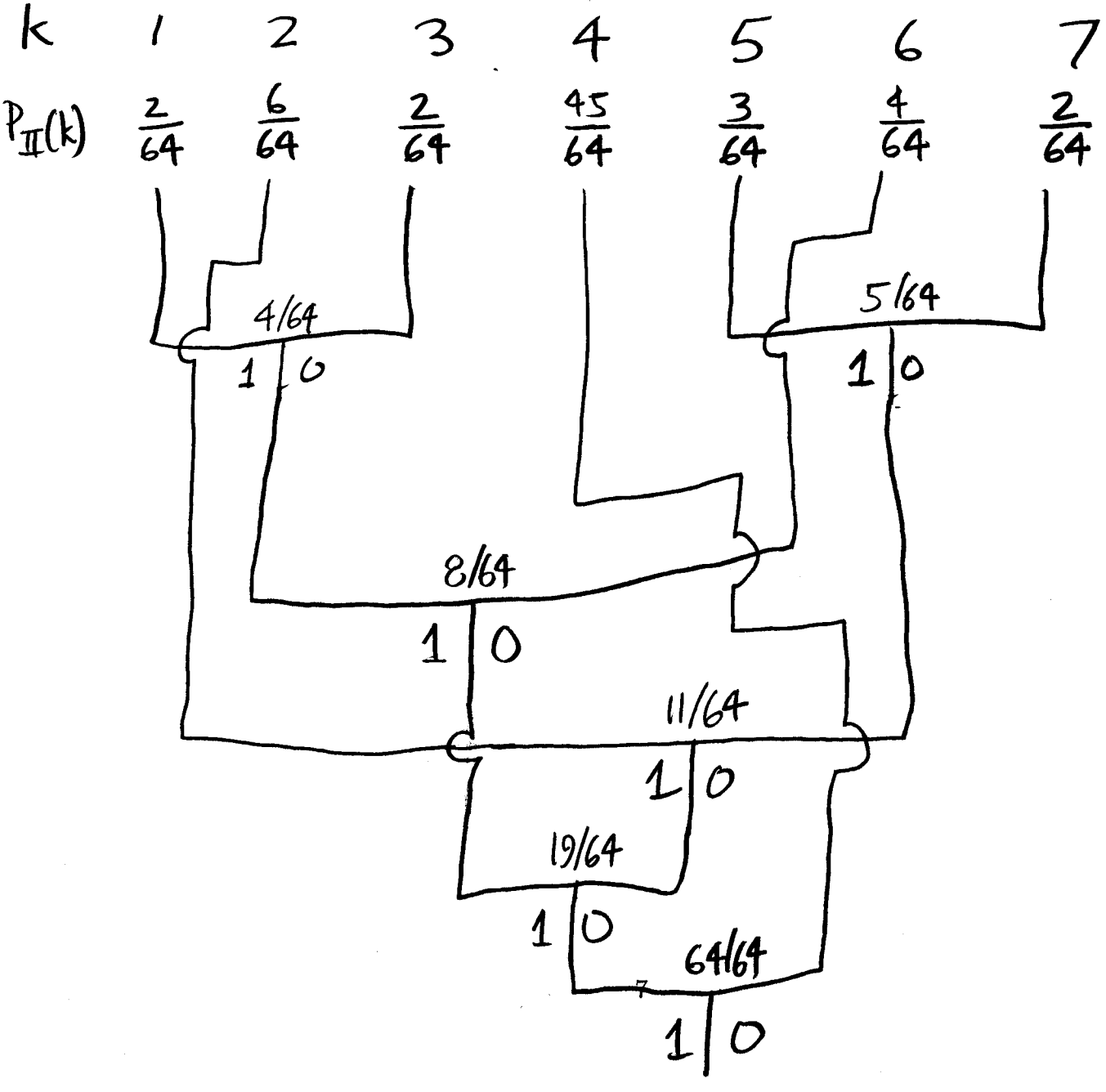
In fact, this is the separable median filter, which outputs the median of the column medians. For large window sizes, it is much faster than the median filter. However, unless the true median is also the median of its column, this filter does not give the same result as the MF.

4. **25 pts.** Gray scale digital images $\mathbb{I}$ with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

k	0	1	2	3	4	5	6	7
$p_{\mathbb{I}}(k)$	0	$\frac{2}{64}$	$\frac{6}{64}$	$\frac{2}{64}$	$\frac{45}{64}$	$\frac{3}{64}$	$\frac{4}{64}$	$\frac{2}{64}$

(a) **15 pts.** Design a Huffman code to encode these images.

Since $P_{\mathbb{I}}(0)=0$, the symbol $k=0$ does not need a Huffman code.



Problem 4 cont...

k	1	2	3	4	5	6	7
$\hat{k}_k$	1111	101	1110	0	1001	110	1000
$L(\hat{k}_k)$	4	3	4	1	4	3	4

Problem 4 cont...

(b) **5 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbb{I})$.

Original BPP = 3

$$\text{Coded BPP} = \left(\tfrac{2}{64}\right)(4) + \left(\tfrac{6}{64}\right)(3) + \left(\tfrac{2}{64}\right)(4) + \left(\tfrac{45}{64}\right)(1) + \left(\tfrac{3}{64}\right)(4) + \left(\tfrac{4}{64}\right)(3) + \left(\tfrac{2}{64}\right)(4)$$

$$= \frac{111}{64} = \underline{1.73438}$$

$$CR = \frac{3}{1.73438} = \underline{1.72973}$$

(c) **5 pts.** Does your code reach the theoretical bound on maximum entropy reduction? Explain why or why not.

$$\text{Source entropy} = -\sum_{k=0}^{7} P_{\mathbb{I}}(k) \log_2 P_{\mathbb{I}}(k) = 1.60316$$

The coded BPP is > 1.60316; the code does not reach the theoretical bound. The theoretical bound can only be reached if all of the source symbol probabilities are powers of 2_y which is not the case here.