# ECE 5273
# Test 2

Wednesday, May 3, 2006
5:00 PM - 6:15 PM

Spring 2006 Name: SOLUTION

Dr. Havlicek Student Num:__________

**Directions:** This is an open notes, open book test. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

## GOOD LUCK!

SCORE:

1. (25) ______

2. (25) ______

3. (25) ______

4. (25) ______

TOTAL (100):

*On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.*

Name:__________ Date:__________

1. **25 pts**. True or False. Mark *True* only if the statement is **always** true.

| TRUE | FALSE | |
|---|---|---|
| | X | (a) **3 pts**. Convolution of images is almost always implemented by pointwise multiplication of their FFT's because this reduces the required number of multiply-add operations by a factor of two. X |
| X | | (b) **3 pts**. When implementing linear convolution of two images by pointwise multiplication of their FFT's, it is generally necessary to zero pad each image by a factor of two in each dimension. |
| | X | (c) **3 pts**. The pseudo-inverse filter is the best currently known method for restoring a digital image that has been corrupted by additive white noise. |
| | X | (d) **2 pts**. Although it is computationally expensive, regularization is the preferred method of image restoration because it is theoretically guaranteed to produce a globally optimum solution. X |
| | X | (e) **2 pts**. Nonlinear filters are very effective for removing noise as long the filter $\boxed{\text{frequency response}}$ is carefully designed to attenuate the noise spectrum at frequencies where the signal is not present. |
| | X | (h) **2 pts**. Compared to the median filter, the OPEN filter is better for attenuating noise that can contain *both* positive and negative spikes. X |
| X | | (i) **2 pts**. The Great Pyramid of Giza appears on page 6.30 of the course notes. |
| X | | (j) **2 pts**. The inner average filter is robust (nearly optimal) for removing both Laplacian and Gaussian additive noise. |
| | X | (k) **2 pts**. The main disadvantage of the gradient based edge detectors is their substantial computational complexity. |
| | X | (l) **4 pts**. The *mandril* image is a picture of the instructor's ex-girlfriend. |

2. **25 pts.** Apply local estimator-based edge detection to the $10 \times 10$ image $\mathbb{I}$ shown below to find edges within the dark boundary **only**. You do **not** have to find edges outside the dark boundary.

$\mathbb{I}$ =

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| **0** | **0** | **15** | **0** | **0** | **0** | **45** | **45** | **45** | **45** |
| **0** | **0** | **15** | **0** | **0** | **0** | **45** | **45** | **45** | **45** |
| **0** | **0** | **15** | **0** | **0** | **0** | **45** | **45** | **45** | **45** |
| **0** | **0** | **15** | **0** | **0** | **0** | **45** | **45** | **45** | **45** |
| **0** | **0** | **15** | **0** | **0** | **0** | **45** | **45** | **45** | **45** |
| **0** | **0** | **15** | **0** | **0** | **0** | **45** | **45** | **45** | **45** |
| **0** | **0** | **15** | **0** | **0** | **0** | **45** | **45** | **45** | **45** |

For your windows, use $\mathbb{B}_U = \mathbb{B}_D = \mathbb{B}_L = \mathbb{B}_R = \text{COL}(3)$. Use a median filter for the local estimator EST. Handle edge effects by replication. For the edge distance measure, use d=1. For the point operation, use $M(i,j) = \max\{|\Delta_x(i,j)|, |\Delta_y(i,j)|\}$. Use the threshold value $\tau = 10$.

You may use the blank images on the following page for work space.

The work is shown on the next page.

Show the binary edge map $\mathbb{E}$ below:

$\mathbb{E}$ =

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 |

More Workspace for Problem 2...

$\boxed{d=1}$

$\boxed{\tau=10}$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | 45 |
| 0 | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | 45 |
| 0 | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | 45 |
| 0 | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | 45 |
| 0 | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | 45 |
| 0 | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | 45 |
| 0 | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | 45 |

$\mathrm{MED}(\mathbb{I}, \mathrm{COL}(3))$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 15 | 0 | -15 | 0 | 45 | 45 | 0 | 0 | |
| | 15 | 0 | -15 | 0 | 45 | 45 | 0 | 0 | |
| | 15 | 0 | -15 | 0 | 45 | 45 | 0 | 0 | |
| | 15 | 0 | -15 | 0 | 45 | 45 | 0 | 0 | |
| | 15 | 0 | -15 | 0 | 45 | 45 | 0 | 0 | |
| | 15 | 0 | -15 | 0 | 45 | 45 | 0 | 0 | |
| | | | | | | | | | |

$\Delta_x(i,j) = J(i,j+1) - J(i,j-1)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | |
| | 0 | 15 | 0 | 0 | 0 | 45 | 45 | 45 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

$\Delta_y(i,j) = J(i+1,j) - J(i-1,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | | |
| | | 15 | 0 | 0 | 0 | 45 | 45 | | |
| | | 15 | 15 | 0 | 45 | 45 | 45 | | |
| | | 0 | 15 | 0 | 45 | 45 | 0 | | |
| | | 0 | 15 | 0 | 45 | 45 | 0 | | |
| | | 0 | 15 | 0 | 45 | 45 | 0 | | |
| | | 0 | 15 | 0 | 45 | 45 | 0 | | |
| | | | | | | | | | |
| | | | | | | | | | |

$M(i,j) = \max(|\Delta_x(i,j)|, |\Delta_y(i,j)|)$

3. **25 pts.** Consider the $2 \times 2$ digital images $\mathbb{I}_1$ and $\mathbb{I}_2$ shown below.

$$\mathbb{I}_1 = \begin{array}{|c|c|} \hline \mathbf{0} & \mathbf{1} \\ \hline \mathbf{4} & \mathbf{2} \\ \hline \end{array} \qquad \mathbb{I}_2 = \begin{array}{|c|c|} \hline \mathbf{2} & \mathbf{3} \\ \hline \mathbf{1} & \mathbf{0} \\ \hline \end{array}$$

Compute the circular (wraparound) convolution $\mathbb{J} = \mathbb{I}_1 \circledast \mathbb{I}_2$. Additional workspace is provided on the next page. Give your answer $\mathbb{J}$ in the space provided below:

$$\mathbb{J} = \begin{array}{|c|c|} \hline 7 & 4 \\ \hline 14 & 17 \\ \hline \end{array}$$

Work is given on the next page.

More Workspace for Problem 3. . .

periodic extension of $I_2$: $\longrightarrow$

Fund. Period shown w/ Bold outline

| | (·,−1) | (·,0) | (·,1) | (·,2) |
|---|---|---|---|---|
| (−1,·) | −1,−1: 0 | −1,0: 1 | −1,1: 0 | −1,2: 1 |
| (0,·) | 0,−1: 3 | 0,0: 2 | 0,1: 3 | 0,2: 2 |
| (1,·) | 1,−1: 0 | 1,0: 1 | 1,1: 0 | 1,2: 1 |
| (2,·) | 2,−1: 3 | 2,0: 2 | 2,1: 3 | 2,2: 2 |

$I_1$:

| | (·,0) | (·,1) |
|---|---|---|
| (0,·) | 0,0: 0 | 0,1: 1 |
| (1,·) | 1,0: 4 | 1,1: 2 |

| | |
|---|---|
| 7 | 4 |
| 14 | 17 |

$J$

Top of p. 5.6: $J(i,j) = \sum_{m=0}^{1} \sum_{n=0}^{1} I_1(m,n)\, I_2(i-m, j-n)$

$$= \underbrace{I_1(0,0)}_{0} I_2(i,j) + \underbrace{I_1(0,1)}_{1} I_2(i,j-1) + \overbrace{I_1(1,0)}^{4} I_2(i-1,j) + \underbrace{I_1(1,1)}_{2} I_2(i-1,j-1)$$

$$= I_2(i,j-1) + 4I_2(i-1,j) + 2I_2(i-1,j-1)$$

$i=0, j=0$: $J(0,0) = I_2(0,-1) + 4I_2(-1,0) + 2I_2(-1,-1)$

$$= 3 + 4\cdot 1 + 2\cdot 0 = 3 + 4 = \underline{\underline{7}}$$

$i=0, j=1$: $J(0,1) = I_2(0,0) + 4I_2(-1,1) + 2I_2(-1,0)$

$$= 2 + 4\cdot 0 + 2\cdot 1 = 2 + 2 = \underline{\underline{4}}$$

$i=1, j=0$: $J(1,0) = I_2(1,-1) + 4I_2(0,0) + 2I_2(0,-1)$

$$= 0 + 4\cdot 2 + 2\cdot 3 = 8 + 6 = \underline{\underline{14}}$$

$i=1, j=1$: $J(1,1) = I_2(1,0) + 4I_2(0,1) + 2I_2(0,0)$

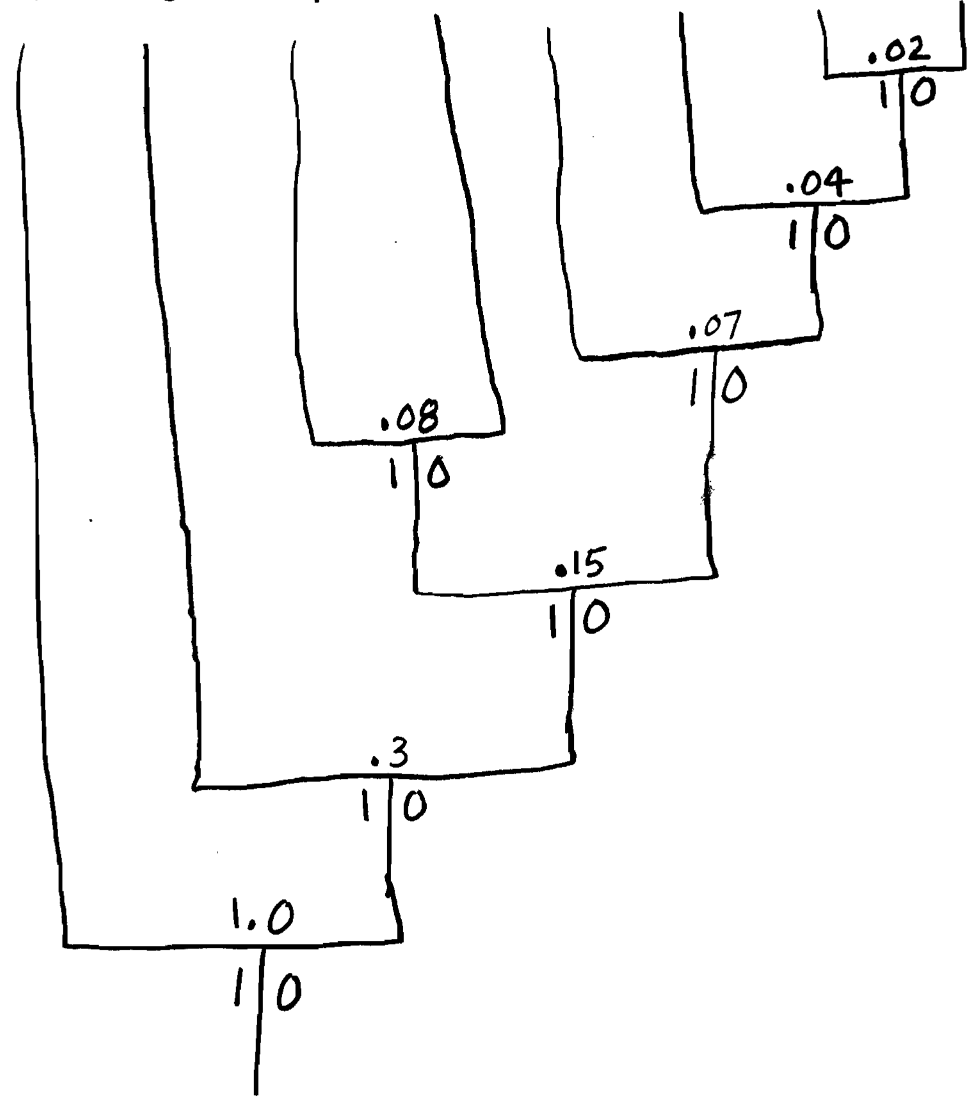
$$= 1 + 4\cdot 3 + 2\cdot 2 = 1 + 12 + 4 = 17$$

4. **25 pts.** Gray scale digital images $\mathbb{I}$ with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p_{\mathbb{I}}(k)$ | 0.70 | 0.15 | 0.04 | 0.04 | 0.03 | 0.02 | 0.01 | 0.01 |

Design a Huffman code to encode these images.

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p_{\mathbb{I}}(k)$ | .7 | .15 | .04 | .04 | .03 | .02 | .01 | .01 |




| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $\hat{K}_k$ | 1 | 01 | 0011 | 0010 | 0001 | 00001 | 000001 | 000000 |
| $L(\hat{K}_k)$ | 1 | 2 | 4 | 4 | 4 | 5 | 6 | 6 |