ECE 5273
Test 2

Wednesday, May 2, 2007
5:15 PM - 6:15 PM

Spring 2007

Dr. Havlicek

Name: SOLUTION

Student Num:____________________

Directions: This is an open notes, open book test. You have 60 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________

2. (25) ________

3. (25) ________

4. (25) ________

TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:____________________ Date:____________________

1. **25 pts.** True or False. Mark *True* only if the statement is **always** true.

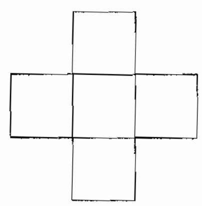
TRUE	FALSE	
	✓	(a) **3 pts.** Convolution of images is almost always implemented by pointwise multiplication of their FFT's because this reduces the required number of multiply-add operations by a factor of two.
✓		(b) **3 pts.** Applying a low-pass filter to an image tends to decrease the entropy of the image.
✓		(b) **3 pts.** Applying a high-pass filer to an image tends to decrease the entropy.
	✓	(c) **3 pts.** The Wiener filter is the best currently known method for restoring a digital image that has been corrupted by a linear blur and additive white noise.
	✓	(d) **2 pts.** Although it is computationally expensive, regularization is the preferred method of image restoration because it is theoretically guaranteed to produce a globally optimum solution.
	✓	(e) **2 pts.** Nonlinear filters are very effective for removing noise as long the filter frequency response is carefully designed to attenuate the noise spectrum at frequencies where the signal is not present.
	✓	(i) **2 pts.** The Great Pyramid of Giza appears on page 6.30 of the course notes.
✓		(j) **2 pts.** The inner average filter is robust (nearly optimal) for removing both Laplacian and Gaussian additive noise.
	✓	(k) **2 pts.** The main disadvantage of the gradient based edge detectors is their substantial computational complexity.
	✓	(l) **3 pts.** The *lena* image originally appeared in the November, 1972, issue of *Better Homes and Gardens*.

2. **25 pts.** Pixels in the 4×4 image $\mathbb{I}$ shown below take values in the range $0 \le I(i,j) \le 99$. The image is transmitted through a communication channel where it is corrupted by noise. The received image $\mathbb{J}$ is also shown below.

$$\mathbb{I} = \begin{array}{|c|c|c|c|}\hline \mathbf{72} & \mathbf{72} & \mathbf{73} & \mathbf{74} \\ \hline \mathbf{72} & \mathbf{99} & \mathbf{72} & \mathbf{74} \\ \hline \mathbf{74} & \mathbf{75} & \mathbf{71} & \mathbf{70} \\ \hline \mathbf{75} & \mathbf{71} & \mathbf{69} & \mathbf{69} \\ \hline \end{array} \qquad \mathbb{J} = \begin{array}{|c|c|c|c|}\hline \mathbf{72} & \mathbf{72} & \mathbf{73} & \mathbf{74} \\ \hline \mathbf{72} & \mathbf{99} & \mathbf{72} & \mathbf{74} \\ \hline \mathbf{74} & \mathbf{75} & \mathbf{0} & \mathbf{70} \\ \hline \mathbf{75} & \mathbf{71} & \mathbf{69} & \mathbf{69} \\ \hline \end{array}$$

Choose an appropriate morphological operation (MAJ, MED, ERODE, DILATE, OPEN, CLOSE) to restore the received image by attenuating the transmission noise. Use the structuring element $\mathbb{B}$ = CROSS(5) shown below:

$\mathbb{B}$ =



Justify your choice for the operation and show the restored image $\widehat{\mathbb{I}}$ below. Handle edge effects by replication. Work space is provided on the following page.

- The image contains a positive spike that we want to keep. But it does not contain any negative spikes.
- The noise has introduced a negative spike that we do not want to keep. The noise has not introduced any positive spikes.

Show the restored image here:

$$\widehat{\mathbb{I}} = \begin{array}{|c|c|c|c|}\hline 72 & 72 & 74 & 74 \\ \hline 72 & 99 & 74 & 74 \\ \hline 75 & 75 & 71 & 70 \\ \hline 75 & 71 & 70 & 70 \\ \hline \end{array}$$

- So an asymmetric smoother is the best choice.
- Remove negative spikes only ⇒ USE CLOSE.

$$\hat{\mathbb{I}} = \text{CLOSE}(\mathbb{J}, \mathbb{B}) = \text{ERODE}(\text{DILATE}(\mathbb{J}, \mathbb{B}), \mathbb{B})$$

Let $\mathbb{J}_1 = \text{DILATE}(\mathbb{J}, \mathbb{B}) = \text{MAX}(\mathbb{J} \circ \mathbb{B})$

Then $\hat{\mathbb{I}} = \text{ERODE}(\mathbb{J}_1, \mathbb{B}) = \text{MIN}(\mathbb{J}_1 \circ \mathbb{B})$

$\mathbb{J}$

72	72	72	73	74	74
72	72	72	73	74	74
72	72	99	72	74	74
74	74	75	0	70	70
75	75	71	69	69	69
75	75	71	69	69	69

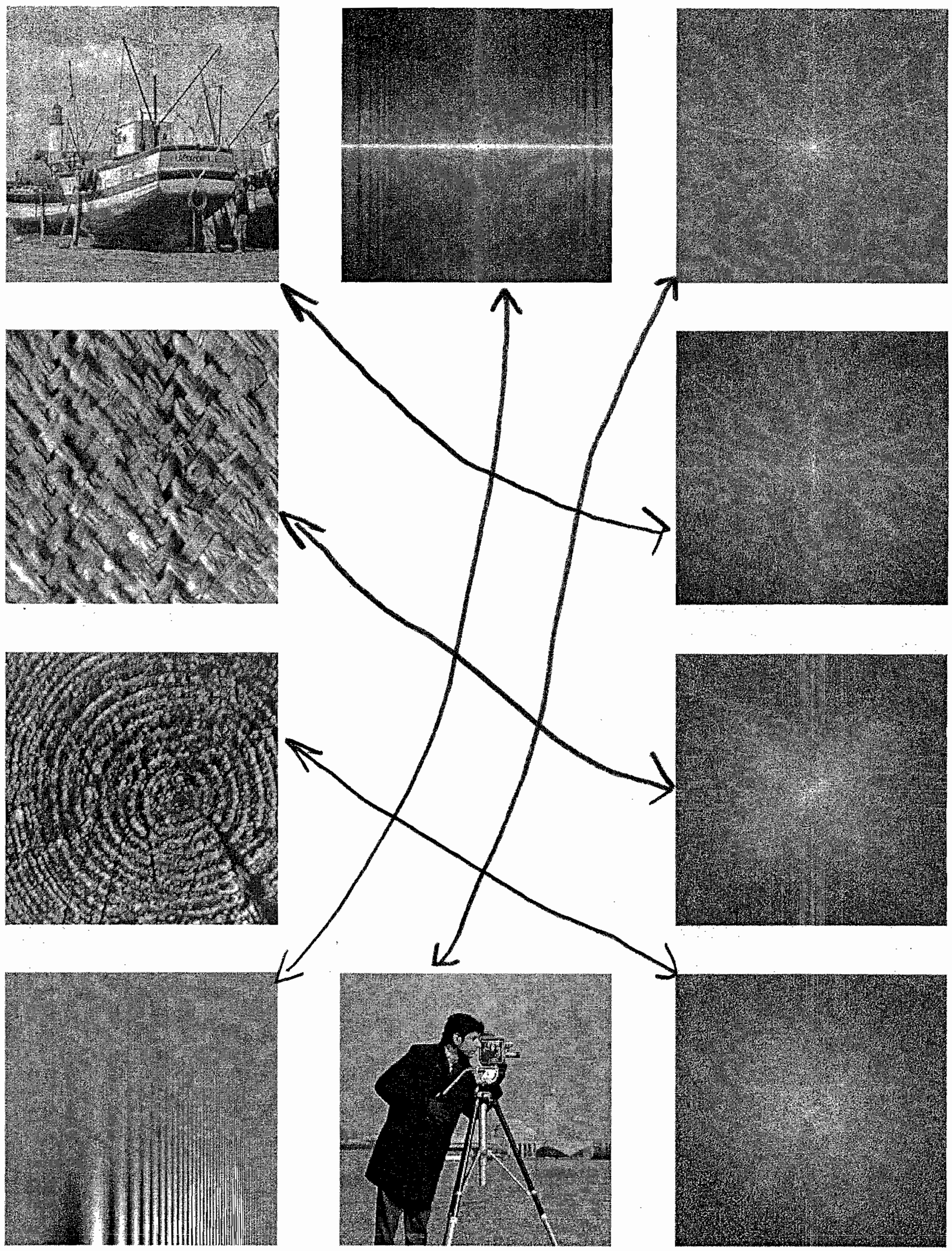
$\mathbb{J}_1$

72	72	99	74	74	74
72	72	99	74	74	74
99	99	99	99	74	74
75	75	99	75	74	74
75	75	75	71	70	70
75	75	75	71	70	70

$\hat{\mathbb{I}} =$

72	72	74	74
72	99	74	74
75	75	71	70
75	71	70	70

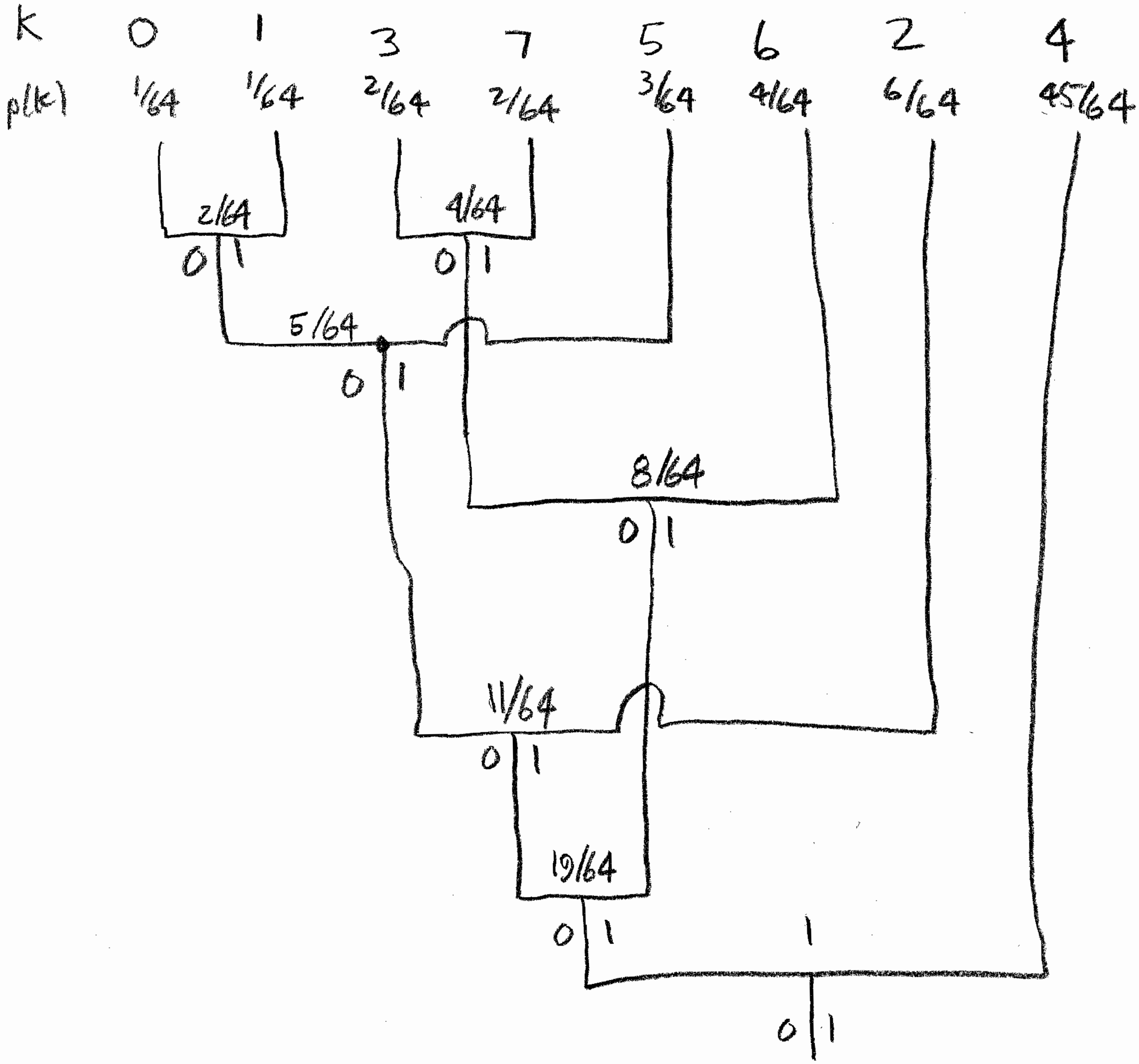
3. **25 pts.** Draw lines to match the images with their log magnitude DFT spectra.



4. **25 pts.** Gray scale digital images $\mathbb{I}$ with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

k	0	1	2	3	4	5	6	7
$p_{\mathbb{I}}(k)$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{6}{64}$	$\frac{2}{64}$	$\frac{45}{64}$	$\frac{3}{64}$	$\frac{4}{64}$	$\frac{2}{64}$

(a) **15 pts**. Design a Huffman code to encode these images.



Problem 4 cont…

K	0	1	2	3	4	5	6	7
$\hat{K}_K$	00000	00001	001	0100	1	0001	011	0101
$L(\hat{K}_K)$	5	5	3	4	1	4	3	4

Problem 4 cont...

(b) **5 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbb{I})$.

$$BPP = \sum_{k=0}^{7} p_{\mathbb{I}}(k) L(\hat{k}_k)$$

$$= \frac{1}{64}(1\cdot5 + 1\cdot5 + 6\cdot3 + 2\cdot4 + 1\cdot45 + 3\cdot4 + 4\cdot3 + 2\cdot4)$$

$$= \frac{1}{64}(5+5+18+8+45+12+12+8)$$

$$= \frac{113}{64} \approx 1.7656$$

$$CR = \frac{B}{BPP} = \frac{3}{\frac{113}{64}} = \frac{3\cdot64}{113} \approx 1.6991$$

(c) **5 pts.** Does your code reach the theoretical bound on maximum entropy reduction? Explain why or why not.

No: Some of the symbol probabilities are not integer powers of 2, e.g. 45/64.

Thus, the Huffman code cannot achieve the entropic lower bound on BPP.