ECE 5273
Test 2

Wednesday, May 6, 2009
4:30 PM - 5:45 PM

Spring 2009 Name: SOLUTION

Dr. Havlicek Student Num:________________

Directions: This is an open book, open notes test. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________

2. (25) ________

3. (25) ________

4. (25) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:________________ Date:________________

1. **25 pts**. True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
	✓	(a) **2 pts**. To implement the linear convolution of two $N \times N$ images using pointwise multiplication of FFT's, it is generally necessary to periodically extend the images to a size of $2N \times 2N$ before the FFT's are calculated.
✓		(b) **3 pts**. Any positive Boolean function can be used to define a Stack Filter.
✓		(c) **3 pts**. Homomorphic filters are most useful for transforming a multiplicative white noise problem into an additive white noise problem.
✓		(d) **3 pts**. For a white noise digital image, the autocorrelation function is given by the Kronecker delta $\delta[m, n]$.
✓		(e) **2 pts**. The main problem with watershed algorithms is that they often over segment the image.
	✓	(f) **2 pts**. The median filter is a low-pass filter that is especially effective for reducing additive Gaussian white noise.
✓		(g) **2 pts**. The gray scale morphological DILATE filter is identical to the gray scale order statistic MAX filter.
	✓	(h) **2 pts**. The main advantages of the gradient based edge detectors is low computational complexity and low sensitivity to noise.
	✓	(i) **3 pts**. Huffman coding, quantization, and run-length coding were selected for the JPEG image compression standard because the combination of these three techniques guarantees that the compression ratio reaches the theoretically optimal value for a lossless code.
	✓	(j) **3 pts**. The technical term *pixel* is a contraction of *pixie elephant*, a tiny species of winged elephant found primarily in southern India.

2. **25 pts.** A continuous optical image $I_C(x, y)$ is given by the linear convolution

$$I_C(x,y) = J_C(x,y) * K_C(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J_C(x-\alpha, y-\beta) K_C(\alpha,\beta)\, d\alpha d\beta,$$

where

$$J_C(x,y) = \exp\left[-(x^2+y^2)/36\right]$$

and

$$K_C(x,y) = \frac{\sin(5\pi x)\sin(7\pi y)}{35\pi^2 xy}.$$

The spatial coordinates x and y are expressed in units of millimeters.

Therefore $\widetilde{I}_C(\omega_x, \omega_y) = \widetilde{J}_C(\omega_x,\omega_y)\widetilde{K}_C(\omega_x,\omega_y)$, where ω_x and ω_y are in units of Hz/mm (cycles/mm).

A 1024 × 1024 digital image $I(i,j)$ is obtained by sampling $I_C(x,y)$ according to $I(i,j) = I_C(i\Delta, j\Delta)$, where the horizontal and vertical sample spacings are given by $\Delta = 0.08$mm.

Is this sampling sufficiently dense for the digital image $I(i,j)$ to have the appearance of the optical image $I_C(x,y)$ without visibly evident distortion?

$J_c(x,y)$ is a Gaussian with space constant $\sigma = 6$. From the notes p. 4.43 (with corrections), $\tilde{J}_c(\omega_x,\omega_y) = 36\pi e^{-36\pi^2(\omega_x^2+\omega_y^2)}$.

$$K_c(x,y) = \frac{\sin(5\pi x)}{5\pi x}\,\frac{\sin(7\pi y)}{7\pi y} = \mathrm{sinc}(5x)\,\mathrm{sinc}(7y).$$

Notes p. 4.43 (with corrections):

$$\tilde{K}_c(\omega_x,\omega_y) = \begin{cases} \frac{1}{35}, & |\omega_x| \le \frac{5}{2} \text{ and } |\omega_y| < 7/2, \\ 0, & \text{otherwise}. \end{cases}$$

So $\tilde{I}_c(\omega_x,\omega_y) = \tilde{J}_c(\omega_x,\omega_y)\tilde{K}_c(\omega_x,\omega_y) = \begin{cases} \frac{1}{35}\tilde{J}_c(\omega_x,\omega_y) & |\omega_x| \le \frac{5}{2} \text{ and } |\omega_y| < 7/2, \\ 0 & \text{otherwise}. \end{cases}$

$\Rightarrow \tilde{I}_c(\omega_x,\omega_y)$ is bandlimited to $\Omega_x = 5/2$ and $\Omega_y = 7/2$.

The detector spacing is $X = Y = 0.08$ mm.

So we have

$\frac{1}{2X} = \frac{1}{.16} = 6.25 > \Omega_x$ ✓ $\quad \frac{1}{2Y} = \frac{1}{.16} = 6.25 > \Omega_y$ ✓

ALIASING WILL <u>NOT</u> OCCUR. The sampling is sufficiently dense.

3. **25 pts.** Consider the *cameraman* image $\mathbb{I}$ shown below.

The size of the image is 256×256 pixels and each pixel has eight bits. Five grayscale morphological filters are applied, all with respect to the structuring element $\mathbb{B} = \text{SQUARE}(9)$, to define five new filtered images according to

$$\begin{aligned}
\mathbb{J}_M &= \text{MED}(\mathbb{I}, \mathbb{B}), \\
\mathbb{J}_E &= \text{ERODE}(\mathbb{I}, \mathbb{B}), \\
\mathbb{J}_D &= \text{DILATE}(\mathbb{I}, \mathbb{B}), \\
\mathbb{J}_O &= \text{OPEN}(\mathbb{I}, \mathbb{B}), \\
\mathbb{J}_C &= \text{CLOSE}(\mathbb{I}, \mathbb{B}).
\end{aligned}$$

Label the five output images shown on the next page.

Problem 3 cont...

CLOSE →

overall size of most objects not affected, but small dark objects are removed.

← DILATE

The bright objects grow.

ERODE →

The dark objects grow.

← MEDIAN

Small objects both light and dark are removed.

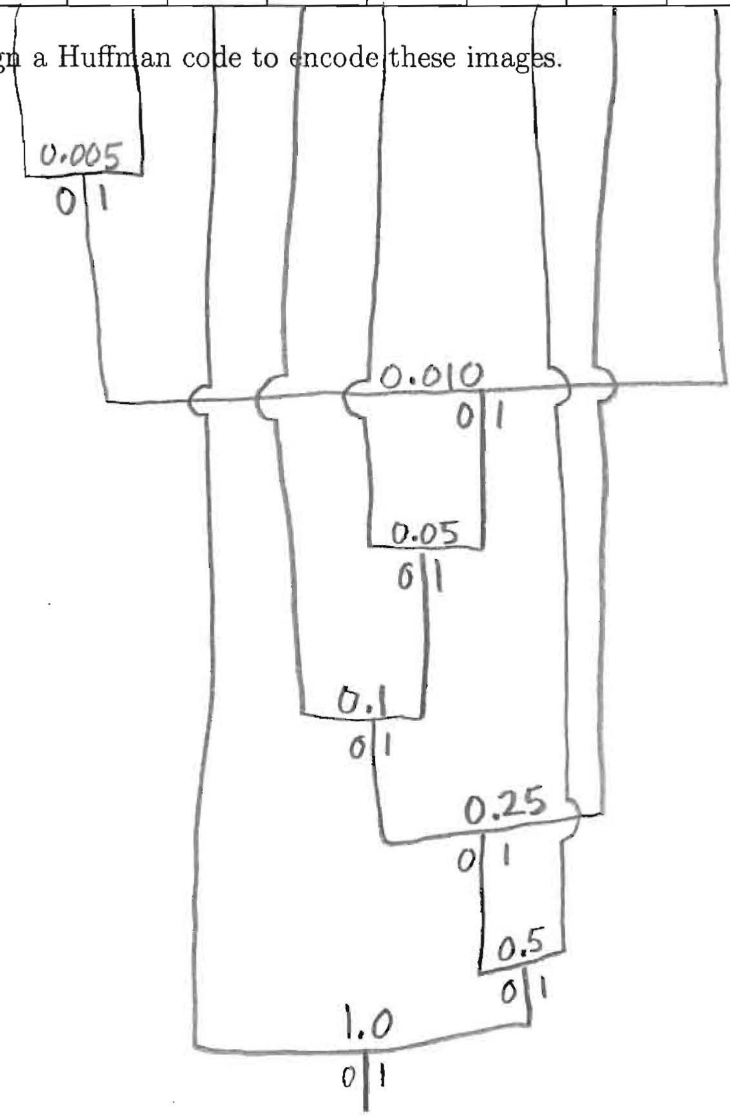
← OPEN

overall size of most objects is not affected much, but small bright objects are removed.

4. **25 pts.** Gray scale digital images $\mathbb{I}$ with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

k	0	1	2	3	4	5	6	7
$p_{\mathbb{I}}(k)$	0.001	0.004	0.500	0.050	0.040	0.250	0.150	0.005

(a) **15 pts**. Design a Huffman code to encode these images.



K	0	1	2	3	4	5	6	7
$\hat{K}_K$	1001100	1001101	0	1000	10010	11	101	100111
$L(\hat{K}_K)$	7	7	1	4	5	2	3	6

Problem 4 cont...

(b) **5 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbb{I})$.

Original BPP = 3.

New BPP = 7(.001) + 7(.004) + 1(.5) + 4(.05) + 5(.04) + 2(.25) + 3(.15) + 6(.005) = 1.915

$$CR = \frac{3}{1.915} = 1.56658 : 1$$

(c) **5 pts.** Does your code reach the theoretical bound on maximum entropy reduction? Explain why or why not.

No. To reach the theoretical bound, all the symbol probabilities must be integer powers of two. In this case, we have, e.g., p(6) = 0.15 which is not a power of two. Therefore this code does not reach the theoretical bound.