ECE 5273
Test 2

Wednesday, April 28, 2010
4:30 PM - 5:45 PM

Spring 2010 Name: SOLUTION

Dr. Havlicek Student Num:__________

Directions: This is an open notes test. You may use the official course notes pack, including the Notes Supplements from the handouts section of the course web site. Other materials are not allowed. You have 75 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (25) ________

2. (25) ________

3. (25) ________

4. (25) ________

TOTAL (100):

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name:__________ Date:__________

1. **25 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE	FALSE	
	✓	(a) **3 pts.** The most efficient way to implement an inner average OS filter is by pointwise multiplying the DFT's of the zero padded image and filter impulse response.
✓		(b) **3 pts.** For removing additive white noise, the main advantage of anisotropic diffusion is that the diffusion coefficients tend to inhibit averaging of pixels across edge boundaries, thereby smoothing the noise while preserving edges better than a linear low-pass filter.
	✓	(c) **3 pts.** To implement linear convolution of two $N \times N$ images by multiplying DFT's, it is generally necessary to zero pad the images to a size of $2N \times 2N$. Since this makes the images four times larger, it is almost always faster to implement linear convolution directly in the space domain without using DFT's.
✓		(d) **2 pts.** For a white noise digital image, the autocorrelation function is given by the Kronecker delta $\delta[m, n]$.
✓		(e) **2 pts.** The grayscale OPEN-CLOSE morphological filter eradicates both positive-going and negative-going impulses.
	✓	(f) **2 pts.** The inverse filter is a good choice for performing deconvolution when the image has been blurred and also corrupted by primarily high frequency additive noise.
✓		(g) **2 pts.** The median filter is optimal for removing salt-and-pepper noise from an image.
	✓	(h) **2 pts.** Baseline JPEG achieves lossless compression by quantizing the 8×8 block DCT coefficients.
	✓	(i) **3 pts.** Baseline JPEG achieves lossy compression by applying run-length coding and Huffman coding.
		(j) **3 pts.** Al Bovik, the second Editor-in-Chief of IEEE TRANS. IMAGE PROCESS., received his nickname "Big Al" because of his many similarities to the *South Park* character "Big Gay Al."

2. **25 pts.** Short Answer.

(a) **5 pts.** For the window SQUARE(9), give the filter weights (coefficients) $\mathbf{A}^T$ of: a median filter, an OS filter to perform grayscale morphological EROSION, and the inner average OS filter INNER_AVE$_2$:

Median Filter: $\mathbf{A}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

EROSION: $\mathbf{A}^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

INNER_AVE$_2$: $\mathbf{A}^T = \begin{bmatrix} 0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 \end{bmatrix}$

$\rightarrow p = 2, \quad 2P+1 = 5$

(b) **5 pts.** Briefly explain why smoothing and enhancement are usually regarded as conflicting goals.

Smoothing seeks to average out noise... needs a low pass filter

Enhancement seeks to undo a "blur"... needs a high pass filter.

(c) **5 pts.** For a digital image $\mathbb{I}$, briefly explain why we usually look at the DFT log magnitude spectrum $\log\left[1 + |\widetilde{\mathbb{I}}(u,v)|\right]$ instead of at the actual DFT magnitude $|\widetilde{\mathbb{I}}(u,v)|$ itself.

The DC coefficient of $\widetilde{\mathbb{I}}$ is usually many orders of magnitude larger than the rest. If you display $|\widetilde{\mathbb{I}}|$ on an 8-bit display, you just see a bright dot in the center. With the log, dynamic range is compressed and you can see the rest.

(d) **5 pts.** A digital image $\mathbb{I}$ has been corrupted with additive white noise $\mathbb{N}$ having a very large noise power η. A linear low-pass filter is applied to smooth the noise. Should this filtering increase or decrease the entropy of the image? Briefly explain your answer.

The noise power η is the variance... when large, causes the histogram to be widely spread.
Low-Pass filter = averaging → reduces variance.
→ makes histogram more narrow or concentrated.
⇒ LOWERS ENTROPY

(e) **5 pts.** A digital image $\mathbb{I}$ with 8-bit pixels is transmitted through two communications channels where it is corrupted by additive random noise.

For the first channel, the received image is $\mathbb{J}_1 = \mathbb{I} + \mathbb{N}_1$, where the noise $\mathbb{N}_1$ is IID and zero mean. To smooth the noise in $\mathbb{J}_1$, would you use a median filter, a morphological OPEN filter, or a morphological CLOSE filter? Briefly state the reason for your choice.

MEDIAN. Removes both positive & negative impulses (noise is centered at zero).
→ OPEN & CLOSE each remove only one kind of impulse.

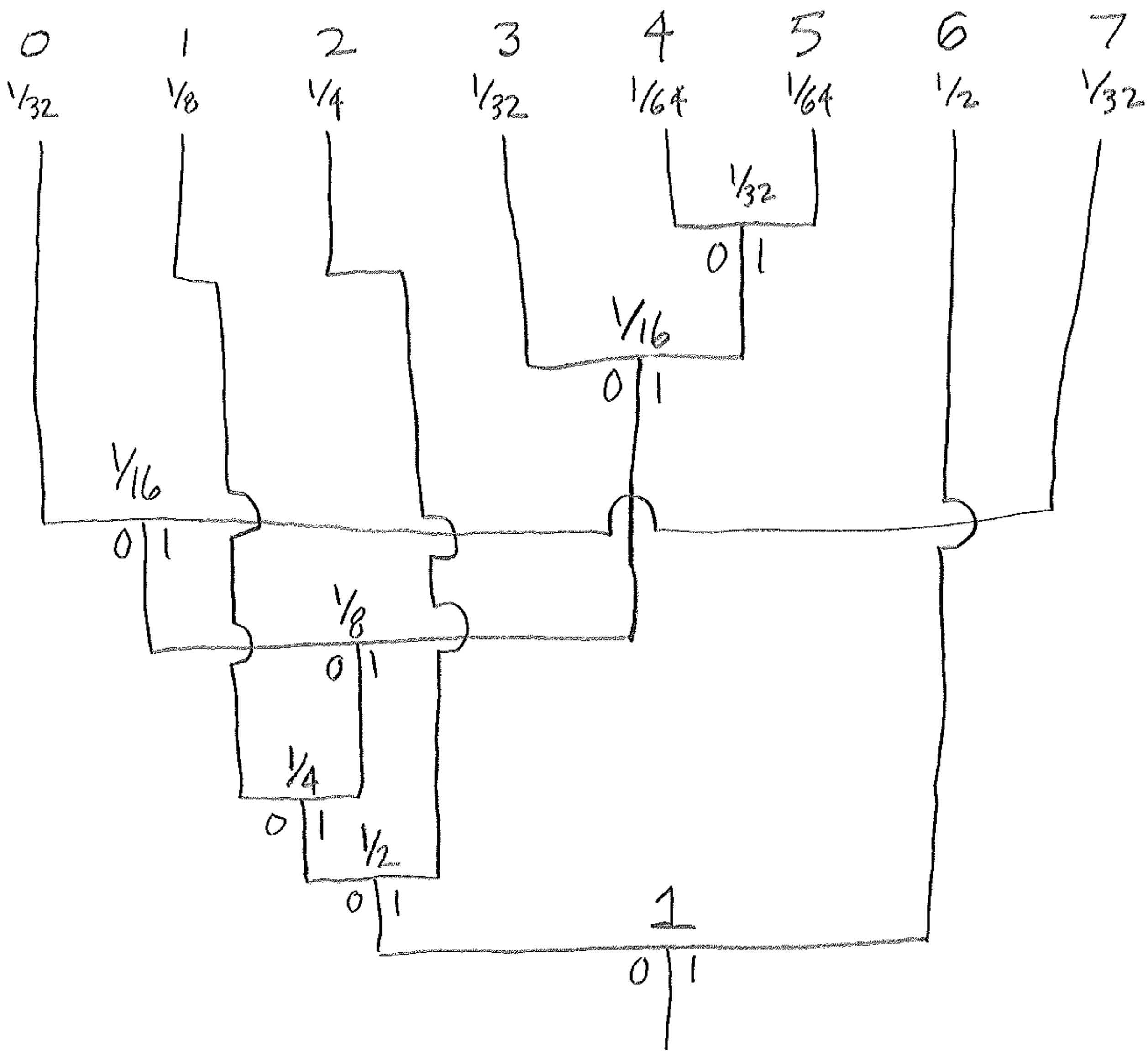
For the second channel, the received image $\mathbb{J}_2$ is given by $J_2(i,j) = I(i,j)$ with probability 0.99 and $J_2(i,j) = I(i,j) + N_2(i,j)$ with probability 0.01, where $N_2(i,j)$ is a random variable with mean 196. Would you use a median filter, a morphological OPEN filter, or a morphological CLOSE filter to smooth the noise in $\mathbb{J}_2$? Briefly state the reason for your choice.

OPEN. Noise has huge expected value; will create positive-going spikes.
OPEN removes these, CLOSE does not.
Median removes positive-going, but also negative-going spikes

3. **25 pts.** Gray scale digital images $\mathbb{I}$ with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

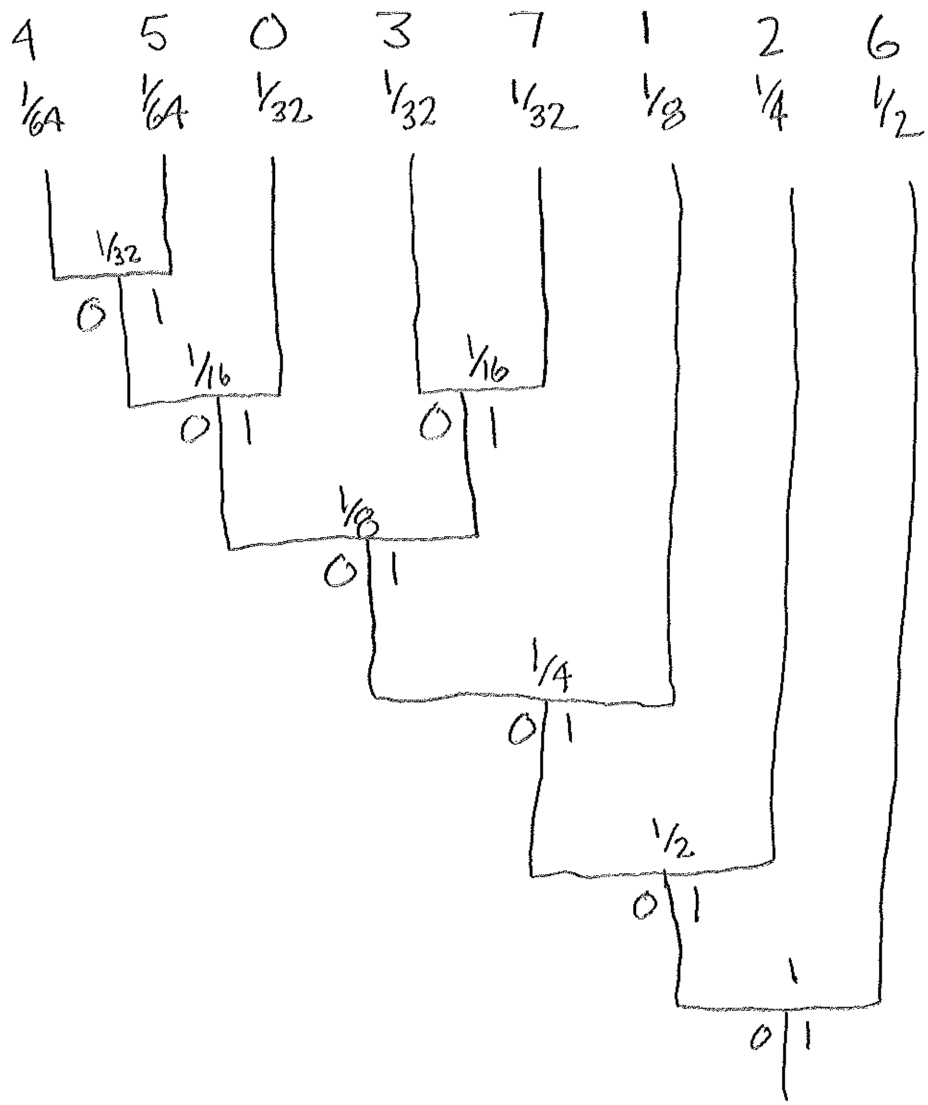
k	0	1	2	3	4	5	6	7
$p_{\mathbb{I}}(k)$	$\frac{1}{32}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{2}$	$\frac{1}{32}$

(a) **15 pts.** Design a Huffman code to encode these images.



	0	1	2	3	4	5	6	7
$\hat{k}_K$	00100	000	01	00110	001110	001111	1	00101
$L(\hat{k}_K)$	5	3	2	5	6	6	1	5

Equivalent solution (same code rate) with the symbols sorted to avoid "crossings!"



K	0	1	2	3	4	5	6	7
$\hat{K}_k$	00001	001	01	00010	000000	000001	1	00011
$L(\hat{k}_k)$	5	3	2	5	6	6	1	5

Problem 3 cont...

(b) **5 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbb{I})$.

Original BPP = 3

$$\text{Coded BPP} = \sum_{k=0}^{7} p_{\mathbb{I}}(k) L(\hat{r}_k)$$

$$= \frac{1}{32}5 + \frac{1}{8}3 + \frac{1}{4}2 + \frac{1}{32}5 + \frac{1}{64}6 + \frac{1}{64}6 + \frac{1}{2}1 + \frac{1}{32}5$$

$$= \frac{10}{64} + \frac{24}{64} + \frac{32}{64} + \frac{10}{64} + \frac{6}{64} + \frac{6}{64} + \frac{32}{64} + \frac{10}{64}$$

$$= \frac{130}{64} = 2.03125$$

$$CR = \frac{3}{2.03125} = 1.4769$$

(c) **5 pts.** Does your code achieve the theoretical lower bound on BPP for the coded images? Explain why or why not.

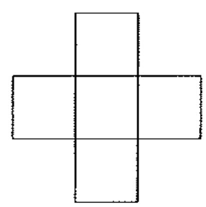
YES. All the symbol probabilities are integer powers of 2 $\Rightarrow$ code achieves lower bound.

4. **25 pts.** The 4×4 image $\mathbb{I}$ shown below has 8-bit pixels that take values in the range $0 \le I(i,j) \le 255$. The image is transmitted through a communication channel where it is corrupted by IID additive white Laplacian noise with parameter $\sigma = 4$. The received image $\mathbb{J}$ is also shown below. Work space is provided on the following page.

$$\mathbb{I} = \begin{array}{|c|c|c|c|} \hline 32 & 32 & 32 & 32 \\ \hline 120 & 120 & 32 & 32 \\ \hline 120 & 120 & 120 & 32 \\ \hline 120 & 120 & 120 & 120 \\ \hline \end{array} \qquad \mathbb{J} = \begin{array}{c|c|c|c|c|c} & 30 & 32 & 28 & 35 & \\ \hline 30 & 30 & 32 & 28 & 35 & 35 \\ \hline 125 & 125 & 134 & 39 & 36 & 36 \\ \hline 117 & 117 & 115 & 107 & 27 & 27 \\ \hline 123 & 123 & 120 & 122 & 119 & 119 \\ \hline & 123 & 120 & 122 & 119 & \end{array}$$

(a) **20 pts.** Choose the best OS or morphological filter (MED, AVE, INNER_AVE, ERODE, DILATE, OPEN, CLOSE) to restore the received image by attenuating the transmission noise. Use the structuring element $\mathbb{B}$ = CROSS(5) shown below. **Show** the restored image $\widehat{\mathbb{I}}$ in the space provided at the bottom of this page. Handle edge effects by replication.

Laplacian noise
→ USE MEDIAN.

$\mathbb{B}$ = 

(b) **5 pts.** **Compute the ISNR** for your restored image.

$$\text{ISNR} = 10\log_{10} \frac{MSE(\mathbb{J})}{MSE(\hat{\mathbb{I}})} = 10\log_{10} \frac{34.8125}{21.3125}$$

$$= 10\log_{10} 1.63343 = 2.13101 \text{ dB}$$

Show the restored image here:

$$\widehat{\mathbb{I}} = \begin{array}{|c|c|c|c|} \hline 30 & 32 & 32 & 35 \\ \hline 125 & 115 & 39 & 36 \\ \hline 117 & 117 & 107 & 36 \\ \hline 123 & 120 & 120 & 119 \\ \hline \end{array}$$

ISNR = 2.13101 dB

Work Space for Problem 4...

$$MSE(\mathbb{J}) = \frac{1}{16}\sum(\mathbb{J}-\mathbb{I})^2 = \frac{557}{16} = 34.8125$$

$$MSE(\hat{\mathbb{I}}) = \frac{1}{16}\sum(\hat{\mathbb{I}}-\mathbb{I}) = \frac{341}{16} = 21.3125$$

$(\hat{\mathbb{I}}-\mathbb{I})^2 =$

4	0	0	9
25	25	49	16
9	9	169	16
9	0	0	1

$(\mathbb{J}-\mathbb{I})^2$

4	0	16	9
25	16	49	16
9	25	169	25
9	0	4	1

$\hat{\mathbb{I}}-\mathbb{I}$

-2	0	0	3
5	-5	7	4
-3	-3	-13	4
3	0	0	-1

$\mathbb{J}-\mathbb{I}$

-2	0	-4	3
5	4	7	4
-3	-5	-13	-5
3	0	2	-1