ECE 5273
Test 2

Wednesday, May 11, 2011
10:30 AM - 12:30 PM

Spring 2011

Dr. Havlicek

Name: SOLUTION

Student Num: ____________

Directions: This is an open notes test. You may use a clean copy of the course notes as published on the course web site. Other materials are not allowed. You have 120 minutes to complete the test. All work must be your own.

SHOW ALL OF YOUR WORK for maximum partial credit!

GOOD LUCK!

SCORE:

1. (28) ________

2. (12) ________

3. (20) ________

4. (20) ________

5. (20) ________

TOTAL (100):

____

On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this test.

Name: ____________ Date: ____________

1. **28 pts.** True or False. Mark *True* only if the statement is **always** true.

TRUE FALSE

✓ (a) **2 pts.** If two images are discrete and periodic with the same period, then their wraparound convolution equals their linear convolution.

✓ (b) **2 pts.** To implement the **linear** convolution of two 256×256 digital images by multiplying DFT's, it is generally necessary to zero pad both images to a size of 512×512.

✓ (c) **2 pts.** For convolving two $\mathrm{N} \times \mathrm{N}$ digital images, multiplying DFT's instead of directly implementing the convolution reduces the computational complexity from N^4 to $\mathrm{N}^2 \log \mathrm{N}^2$.

✓ (d) **2 pts.** A major problem with watershed algorithms is that they tend to oversegment the image.

✓ (e) **2 pts.** High-pass filters are typically used to enhance image details and remove blur.

✓ (f) **2 pts.** Homomorphic filtering can be used to transform a multiplicative noise problem into an additive noise problem.

✓ (g) **2 pts.** For the linear image restoration problem, the Wiener filter is not of much interest unless there is a blur.

✓ (h) **2 pts.** Edge thinning and edge linking are not required after application of the LoG edge detector.

✓ (i) **2 pts.** The gradient-based edge detectors are generally less sensitive to noise than the Laplacian-type edge detectors.

✓ (j) **2 pts.** One disadvantage of gradient-based edge detectors is that they usually depend on a threshold that must be selected empirically.

Problem 1, cont...

TRUE	FALSE	
✓		(k) **2 pts.** For removing additive white noise, the main advantage of anisotropic diffusion is that the diffusion coefficients tend to inhibit averaging of pixels across edge boundaries, thereby smoothing the noise while preserving edges better than a linear low-pass filter.
✓		(l) **2 pts.** The inner average filter is robust (nearly optimal) for removing both Laplacian and Gaussian additive noise.
✓		(m) **2 pts.** The basic idea behind DPCM is that pixel differencing tends to reduce the entropy of typical images like *lena* or *cameraman*.
✓		(n) **2 pts.** Any positive Boolean function can be used to define a Stack Filter.

2. **12 pts.** Short Answer.

(a) **3 pts.** Briefly explain the data constraint, the smoothness constraint, and their role in image regularization.

- The received image is $\mathbb{J} = \mathbb{G} * \mathbb{I} + \mathbb{N}$, where $\mathbb{I}$ is the original image, $\mathbb{G}$ is the linear distortion (blur), and $\mathbb{N}$ is additive noise.
- The restored image is $\hat{\mathbb{I}}$.
- The DATA CONSTRAINT says that $\hat{\mathbb{I}}$ should minimize $\|\mathbb{G} * \hat{\mathbb{I}} - \mathbb{J}\|$, so that a blurred version of the restored image agrees with the received image.
- The SMOOTHNESS CONSTRAINT says that $\hat{\mathbb{I}}$ should minimize $\|\nabla^2 \hat{\mathbb{I}}\|$, so that the restored image is smooth.
- In regularization, these two constraints are balanced by a regularization parameter λ to seek a restored image $\hat{\mathbb{I}}$ that minimizes the energy functional

$$E(\hat{\mathbb{I}}) = \|\mathbb{G} * \hat{\mathbb{I}} - \mathbb{J}\| + \lambda \|\nabla^2 \hat{\mathbb{I}}\|.$$

(b) **3 pts.** Briefly explain how the Sobel edge detector is different from the Prewitt edge detector.

- The Prewitt edge detector averages the horizontal gradient estimates equally across three rows and the vertical gradient estimates equally across three columns to reduce noise.
- The Sobel edge detector also averages the horizontal gradient estimates across three rows and the vertical gradient estimates across three columns to reduce noise, but the averaging is <u>unequal</u>. With the Sobel detector, the gradient estimates from the center row and column are weighted more heavily to improve spatial localization relative to the Prewitt detector.

Problem 2, cont...

(c) **4 pts.** For the window SQUARE(9), give the filter weights (coefficients) $\mathbf{A}^T$ for the following OS filters:

Median Filter: $\mathbf{A}^T = [0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]^T$

EROSION: $\mathbf{A}^T = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$

DILATION: $\mathbf{A}^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]^T$

INNER_AVE$_3$: $\mathbf{A}^T = [0\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ \frac{1}{7}\ 0]^T$

$P = 3$

$2P + 1 = 7$

(d) **2 pts.** Briefly explain why the gray scale morphological ERODE and DILATE operations are only approximate inverses of one another.

- Erode shrinks large objects and eliminates objects that are smaller than the structuring element.
- Dilate enlarges all objects and eliminates small gaps and bays.
- Both Erode and Dilate smooth the gray levels.
- A CLOSE operation applies Dilation followed by Erosion. Large objects are enlarged and then shrunk so that their overall size doesn't change much. But the gray levels are changed (smoothed). Moreover, small gaps and bays are are permanently filled in by the Dilation, while small objects including peninsulas are permanently removed by the Erosion. Thus, the Dilation and Erosion are only approximate inverses.
- An analogous dual argument applies for OPEN, which is Erosion followed by Dilation.

3. **20 pts**. Consider the 1D "image" (signal) $\mathbb{I} = [3\ 2\ 4\ 1\ 3]$. Thus, $I(0) = 3$, $I(1) = 2$, $\ldots$ $I(4) = 3$. This image has **three**-bit pixels.

(a) **6 pts.** Apply a three-point gray scale morphological EROSION to $\mathbb{I}$ to obtain the result $\mathbb{J} = \text{ERODE}[\mathbb{I}, \text{ROW}(3)]$. Handle edge effects by replication. Show the result "image" $\mathbb{J}$.

$\mathbb{I} = 3 \mid 3\ 2\ 4\ 1\ 3 \mid 3$

$\mathbb{J} = [2\ 2\ 1\ 1\ 1]$

(b) **14 pts.** Now obtain the same result by implementing the EROSION filter as a stack filter. **Hint:** compute the binary signals that make up the threshold decomposition of $\mathbb{I}$, apply a binary EROSION filter to each threshold signal, and then combine the results using the stacking property to get the final gray scale answer.

	3	3 2 4 1 3	3		
≥ 4:	0	0 0 1 0 0	0	$\longrightarrow$	0 0 0 0 0
≥ 3:	1	1 0 1 0 1	1	$\longrightarrow$	0 0 0 0 0
≥ 2:	1	1 1 1 0 1	1	$\longrightarrow$	1 1 0 0 0
≥ 1:	1	1 1 1 1 1	1	$\longrightarrow$	1 1 1 1 1
≥ 0:	1	1 1 1 1 1	1	$\longrightarrow$	1 1 1 1 1

$\mathbb{J} = [2\ 2\ 1\ 1\ 1]$

4. **20 pts.** Pixels in the 6×6 image $\mathbb{I}$ shown below take values in the range $\{0, 1, 2, \ldots, 99\}$. The image is sent through a communication channel where it is corrupted by additive noise. The received image $\mathbb{J}$ is also shown below.

$\mathbb{I} =$

11	**11**	**12**	**12**	**13**	**13**
11	**0**	**12**	**13**	**13**	**13**
12	**12**	**13**	**13**	**13**	**14**
12	**13**	**13**	**13**	**14**	**14**
13	**13**	**13**	**14**	**0**	**14**
13	**13**	**13**	**14**	**14**	**14**

$\mathbb{J} =$

11	11	11	12	12	13	13	13
11	**11**	**11**	**12**	**12**	**13**	**13**	13
11	**11**	**0**	**12**	**13**	**99**	**13**	13
12	**12**	**12**	**13**	**13**	**13**	**14**	14
12	**12**	**13**	**13**	**13**	**14**	**14**	14
13	**13**	**13**	**13**	**14**	**0**	**14**	14
13	**13**	**99**	**13**	**14**	**14**	**14**	14
13	13	99	13	14	14	14	14

Design a nonlinear filter to restore the received image by attenuating the noise. Handle edge effects by replication. Explain your solution. Show the restored image $\mathbb{K}$ below and compute the ISNR. There is workspace on the following page.

The noise introduces positive spikes that need to be removed. But the image contains negative spikes that need to be retained. So we need to use an asymmetric smoother that attenuates positive spikes only. To minimize distortion, we need to use a small 2D window. USE OPEN with $\mathbb{B}$= CROSS(5).

Show the restored image here:

$\mathbb{K} =$

11	11	12	12	13	13
11	0	12	13	13	13
12	12	13	13	13	14
12	13	13	13	14	14
13	13	13	13	0	14
13	13	13	13	14	14

ISNR= 38.69 dB

More Workspace for Problem 4...

$J_1 = \text{ERODE}(J)$

	11	0	11	12	12	13	
11	11	0	11	12	12	13	13
0	0	0	0	12	13	13	13
11	11	0	12	13	13	13	13
12	12	12	13	13	0	14	14
12	12	13	13	0	0	0	0
13	13	13	13	13	0	14	14
	13	13	13	13	0	14	

$K = \text{DILATE}(J_1)$

11	11	12	12	13	13
11	0	12	13	13	13
12	12	13	13	13	14
12	13	13	13	14	14
13	13	13	13	0	14
13	13	13	13	14	14

$|K - I|$

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	1	0	0
0	0	0	1	0	0

$|J - I|$

0	0	0	0	0	0
0	0	0	0	86	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	86	0	0	0	0

$$MSE(K) = \frac{\sum_{i=0}^{5}\sum_{j=0}^{5}[K(i,j) - I(i,j)]^2}{36} = \frac{1+1}{36} = \frac{2}{36}$$

$$MSE(J) = \frac{\sum_{i=0}^{5}\sum_{j=0}^{5}[J(i,j) - I(i,j)]^2}{36} = \frac{86^2 + 86^2}{36} = \frac{14{,}792}{36}$$

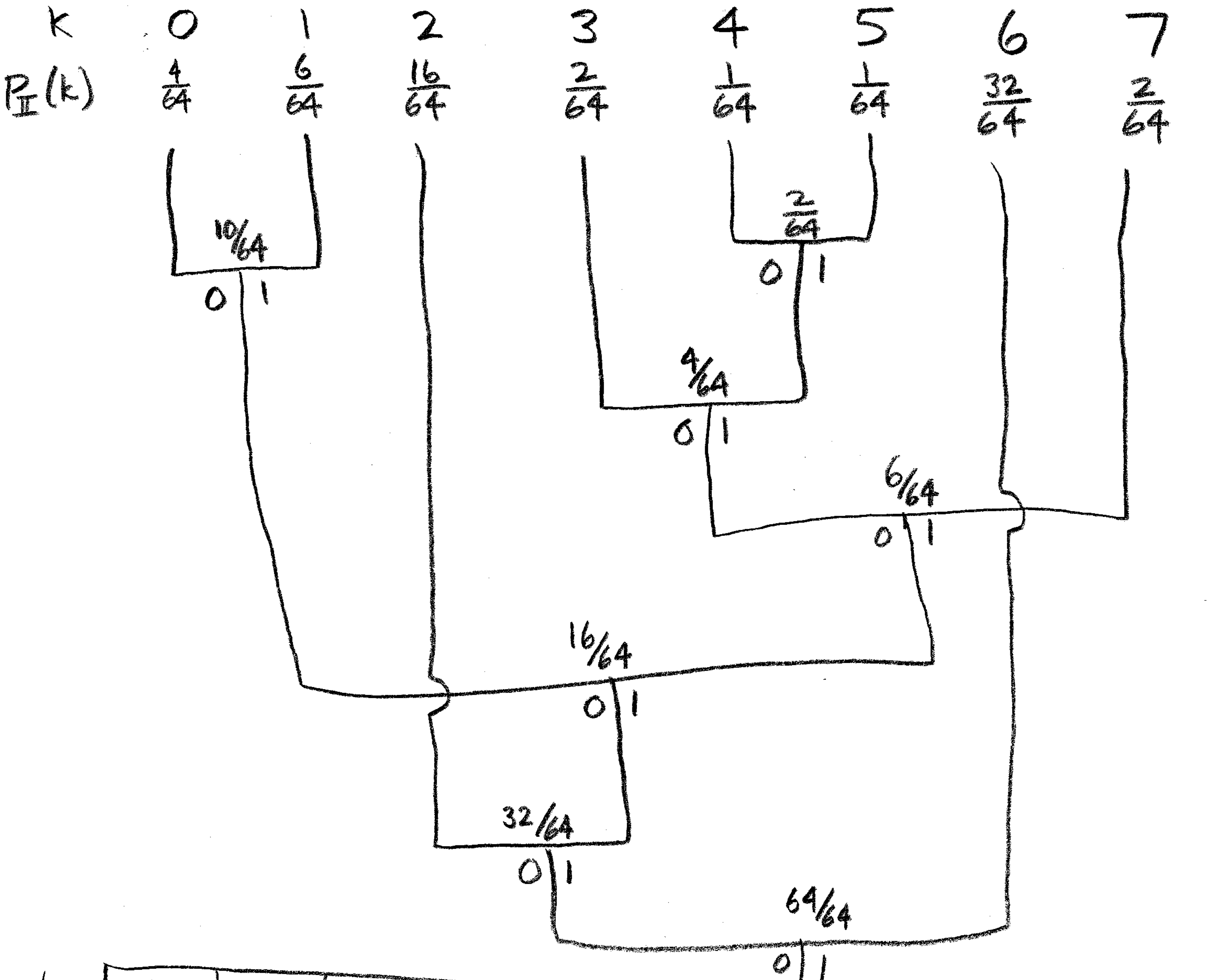
$$ISNR = 10\log_{10}\frac{MSE(J)}{MSE(K)}$$

$$= 10\log_{10}\frac{14{,}792}{2} = 38.69\,\text{dB}$$

5. **20 pts.** Gray scale digital images $\mathbb{I}$ with 3 bits per pixel and gray levels in the range $\{0, 1, \ldots, 7\}$ are modeled as coming from an information source with the following source symbol probabilities (normalized histogram):

k	0	1	2	3	4	5	6	7
$p_{\mathbb{I}}(k)$	$\frac{2}{32}$	$\frac{3}{32}$	$\frac{1}{4}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{1}{2}$	$\frac{1}{32}$

(a) **14 pts.** Design a Huffman code to encode these images.



k	0	1	2	3	4	5	6	7
$\hat{k}_k$	0100	0101	00	01100	011010	011011	1	0111
$L(\hat{k}_k)$	4	4	2	5	6	6	1	4

Problem 5 cont...

(b) **3 pts.** Find the expected BPP (bits per pixel) and CR (compression ratio) for the coded images $C(\mathbb{I})$.

Original BPP = 3

$$\text{Coded BPP} = 4\left(\tfrac{4}{64}\right)+4\left(\tfrac{6}{64}\right)+2\left(\tfrac{16}{64}\right)+5\left(\tfrac{2}{64}\right)+6\left(\tfrac{1}{64}\right)+6\left(\tfrac{1}{64}\right)+1\left(\tfrac{32}{64}\right)+4\left(\tfrac{2}{64}\right)$$

$$= \frac{4\cdot 4 + 4\cdot 6 + 2\cdot 16 + 5\cdot 2 + 6 + 6 + 32 + 8}{64}$$

$$= \frac{16+24+32+10+6+6+32+8}{64}$$

$$= \frac{134}{64} = \underline{\underline{2.09375}}$$

$$CR = \frac{3}{2.09375} = \underline{\underline{1.43284}}$$

(c) **3 pts.** Does your code achieve the theoretical lower bound on BPP for the coded images? Explain why or why not.

No. The source symbol "1" has probability $\frac{3}{32}$ which is <u>not</u> an integer power of 2.